一次元光学系での「自然吸収」: 単一光子による量子状態操作

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OUTLINE

(1) Charm of 1D optical systems

(2) Single-photon response of impedance-matched Λ system

(3) Implementation by circuit QED: theory & experiment

(4) Summary

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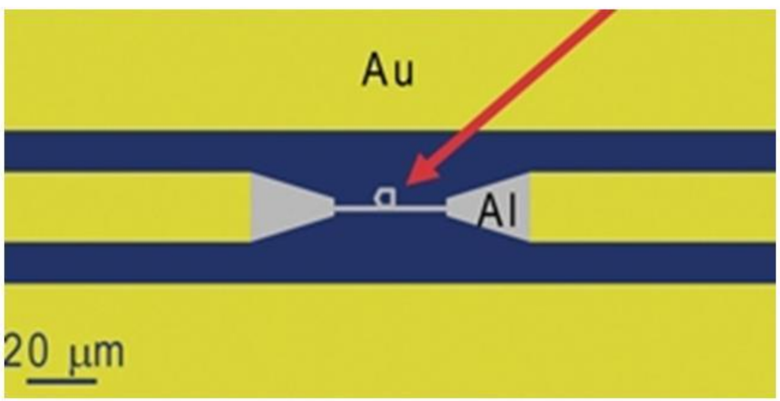
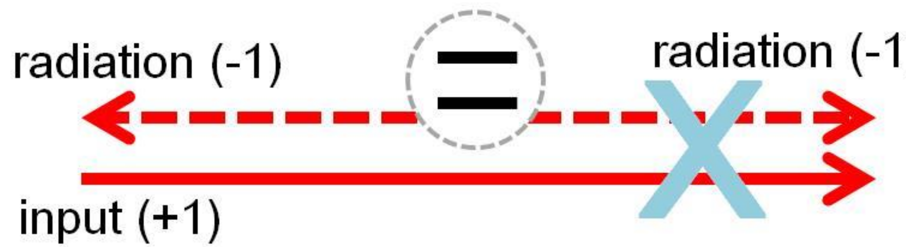
In 1D optical systems (waveguide QED), interaction between the photon and qubit is drastically enhanced due to destructive interference between input field and radiation.

ex) Two-level atom in 1D line→ perfect reflection





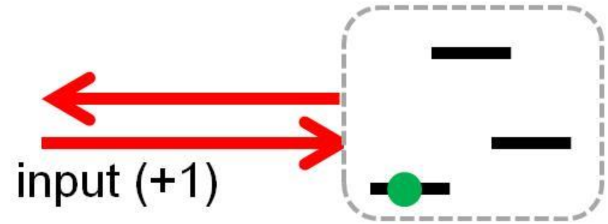
Astafiev et al, Science **327**, 840 (2010)

In 1D optical systems (waveguide QED), interaction between the photon and qubit is drastically enhanced due to destructive interference between input field and radiation.

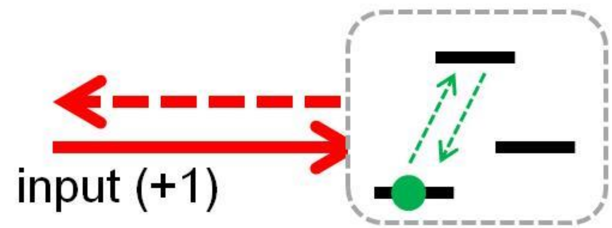
ex) Two-sided cavity→ perfect transmission

When 1D photon is reflected by a Λ system, there are 3 possibilities.

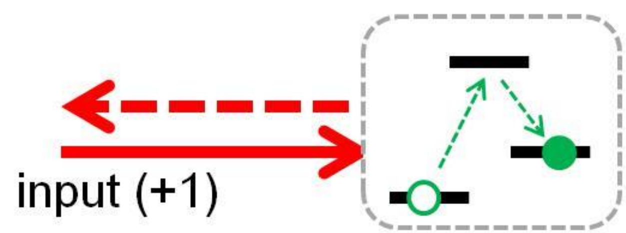
(a) simple reflection



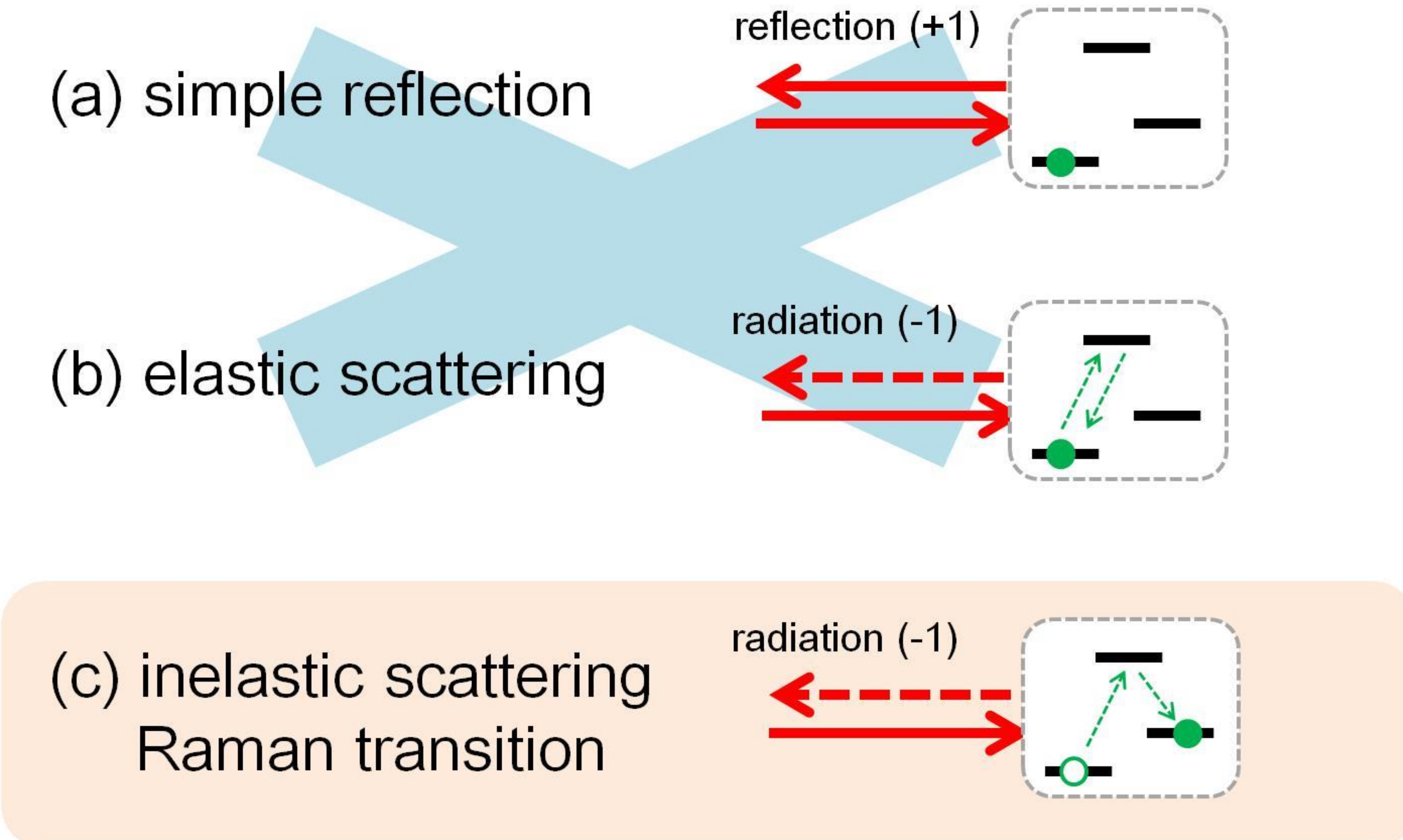
(b) elastic scattering



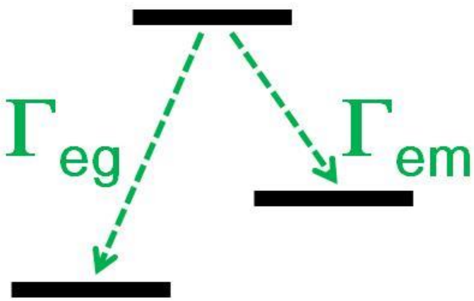
(c) inelastic scattering
Raman transition



If Λ system has identical decay rates, (a) and (b) cancel out each other.



We call Λ system having identical decay rates as impedance-matched Λ system

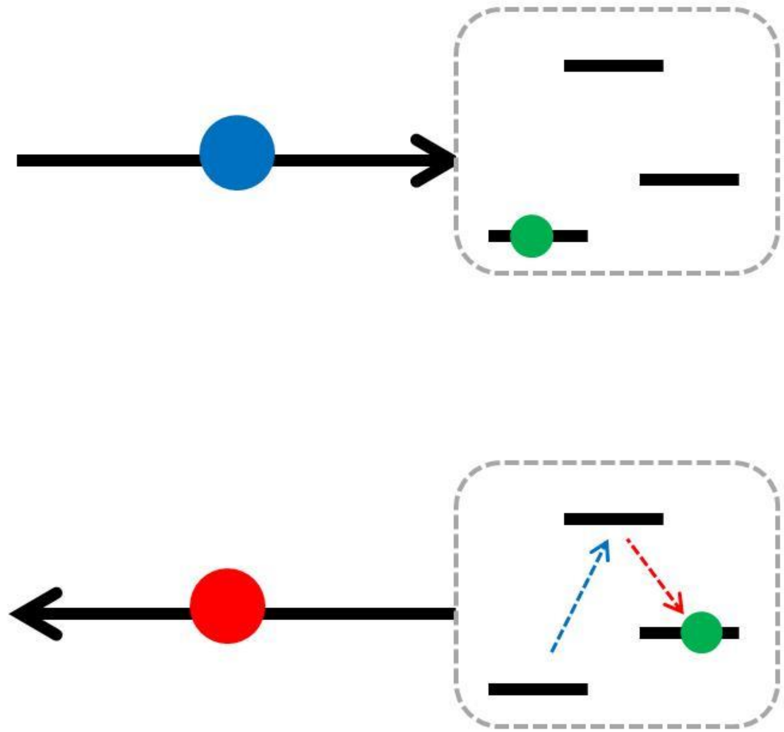


$$\Gamma_{eg}=\Gamma_{em}$$

When a photon is reflected by imp-matched Λ system,

- Λ system is switched (Raman transition)
- photon frequency is converted

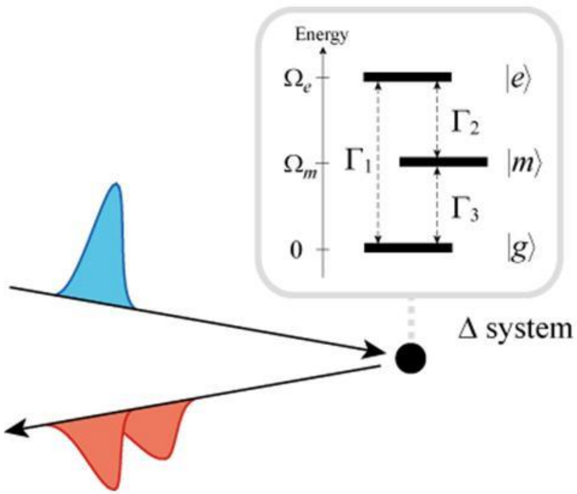
deterministic



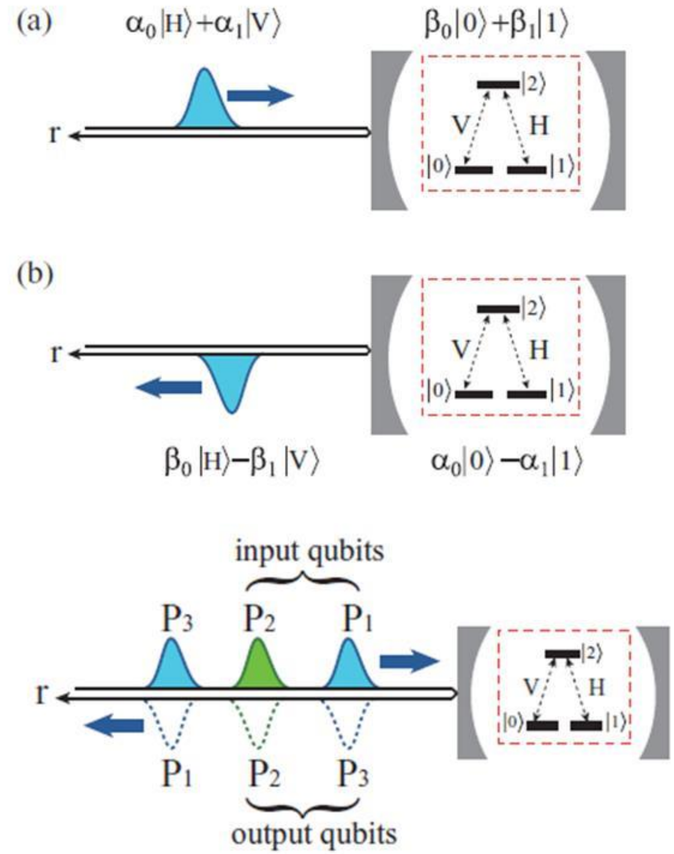
Applications

(1) Down-converter of single photons

(2) Quantum memory
(3) Photon-photon gate
(4) Microwave photon detection



Koshino, PRA **79**, 013804



Koshino et al, PRA **82**, 010301(R)

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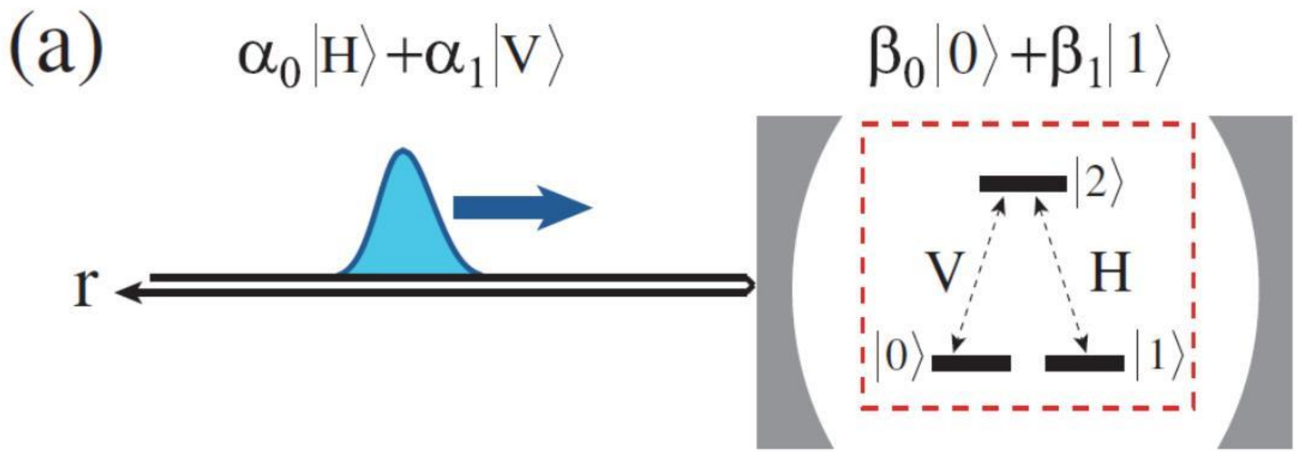
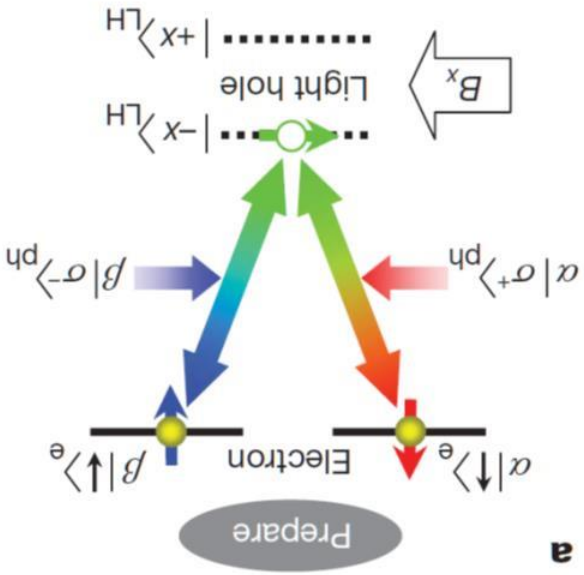
System(1)

- “3 level Λ system + 1D photon” in reflection geometry
  - Two degenerate ground states |0>,|1> excited state |2>
  - Two polarization states |H>,|V>
  - |0>—|2> transition ⇔ |V> photon
  - |1>—|2> transition ⇔ |H> photon

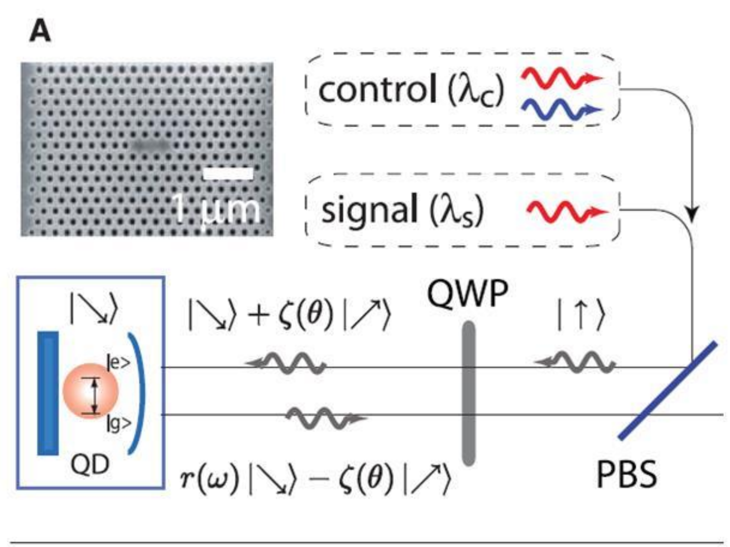


System(2)

- Candidates
  - Charged QD + photonic crystal nanocavity
  - Diamond NV center + spherical/toroidal cavity + fiber
  - superconducting qubit + microwave (circuit QED)



Kosaka et al (Tohoku), Nature **457** 702 (2009)

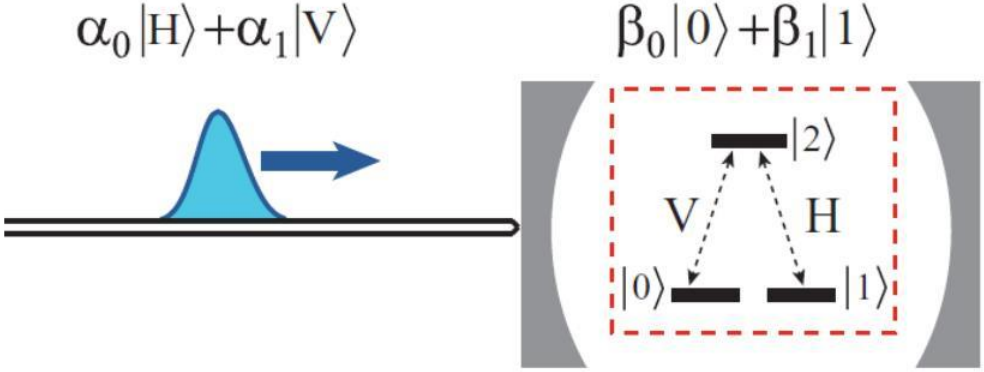


Fushman et al (Stanford), Science **320** 769 (2008)

System(3)

- Hamiltonian
  - Radiative decay rates: $\Gamma_{\mathrm{H}}, \Gamma_{\mathrm{V}}$
  - Transition frequency: Ω

$$\mathcal{H} = \Omega\sigma_{22} + \int dk \left[k h_k^\dagger h_k + i\sqrt{\frac{\Gamma_H}{2\pi}}(\sigma_{21}h_k - h_k^\dagger \sigma_{12}) \right] + \int dk \left[k v_k^\dagger v_k + i\sqrt{\frac{\Gamma_V}{2\pi}}(\sigma_{20}v_k - v_k^\dagger \sigma_{02}) \right],$$



Input state

- Initial state vector
  - atom…ground states (superposition of 0 &1)
  - single photon… (superposition of H & V)
- 4 basis states

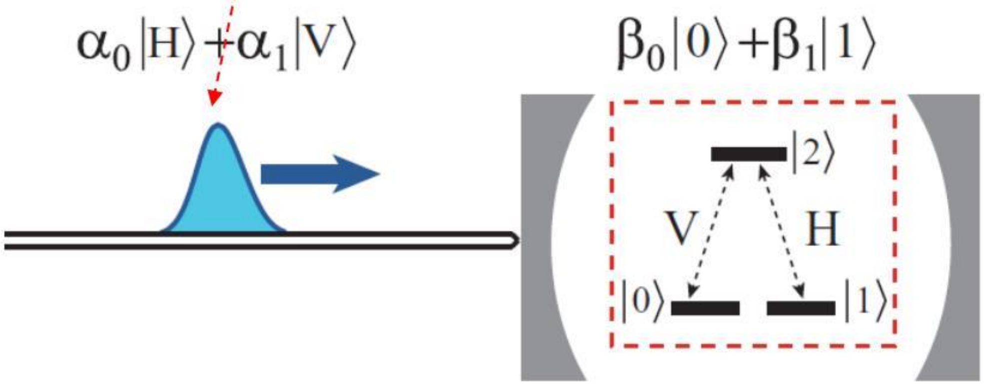
Wavepacket of input photon

$$|H,0\rangle = \int dr\, f(r)\tilde{h}_r^\dagger|0\rangle$$

$$|H,1\rangle = \int dr\, f(r)\tilde{h}_r^\dagger|1\rangle$$

$$|V,0\rangle = \int dr\, f(r)\tilde{v}_r^\dagger|0\rangle$$

$$|V,1\rangle = \int dr\, f(r)\tilde{v}_r^\dagger|1\rangle$$



Heisenberg equations

- Real-space representation of 1D field

$$\widetilde{h}_r = (2\pi)^{-1/2} \int dk e^{ikr} h_k$$

r<0 incoming field
r>0 outgoing field

- Input-output relation

$$\widetilde{h}_r(t) = \widetilde{h}_{r-t}(0) - \sqrt{\Gamma_H}\theta(r)\theta(t-r)\sigma_{12}(t-r)$$

$$\widetilde{v}_r(t) = \widetilde{v}_{r-t}(0) - \sqrt{\Gamma_V}\theta(r)\theta(t-r)\sigma_{02}(t-r)$$

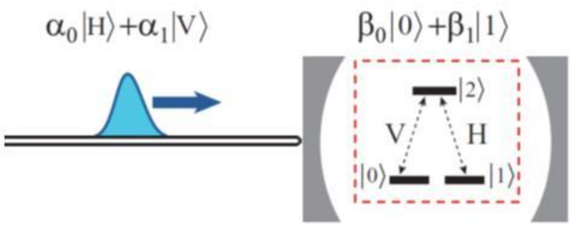
- Heisenberg equation for Λ system

$$\frac{d}{d\tau}\sigma_{12} = \left(-i\Omega - \frac{\Gamma_H + \Gamma_V}{2}\right)\sigma_{12} + \sqrt{\Gamma_H}(\sigma_{11} - \sigma_{22})\widetilde{h}_{-\tau}(0) + \sqrt{\Gamma_V}\sigma_{10}\widetilde{v}_{-\tau}(0)$$

$$\frac{d}{d\tau}\sigma_{02} = \left(-i\Omega - \frac{\Gamma_H + \Gamma_V}{2}\right)\sigma_{02} + \sqrt{\Gamma_V}(\sigma_{00} - \sigma_{22})\widetilde{v}_{-\tau}(0) + \sqrt{\Gamma_H}\sigma_{01}\widetilde{h}_{-\tau}(0)$$

Output states (1)

- |H0>, |V1>… no interaction (simple reflection)
- |H1>, |V0>…Raman transition may take place



$$
\begin{aligned}
|H,0\rangle &\rightarrow \int dr g_1(r,t)\widetilde{h}_r^\dagger|0\rangle, \\
|H,1\rangle &\rightarrow \int dr g_3(r,t)\widetilde{h}_r^\dagger|1\rangle - \int dr g_2(r,t)\widetilde{v}_r^\dagger|0\rangle \\
|V,0\rangle &\rightarrow \int dr g_4(r,t)\widetilde{v}_r^\dagger|0\rangle - \int dr g_2(r,t)\widetilde{h}_r^\dagger|1\rangle \\
|V,1\rangle &\rightarrow \int dr g_1(r,t)\widetilde{v}_r^\dagger|1\rangle,
\end{aligned}
$$

- Output wavepackets

$$
\begin{aligned}
g_1(r,t) &= f(r-t), \\
g_2(r,t) &= \sqrt{\Gamma_H\Gamma_V}s(t-r), \\
g_3(r,t) &= f(r-t) - \Gamma_H s(t-r) \\
g_4(r,t) &= f(r-t) - \Gamma_V s(t-r)
\end{aligned}
$$

reflection

radiation

reflection + radiation

Output states (2)

- Equation of motion for s (polarization)

$$\frac{d}{dt}s(t) = \left(-i\Omega - \frac{\Gamma_H + \Gamma_V}{2}\right) s(t) + f(-t)$$

damped oscillator (linear)

- adiabatic solution in the long pulse limit

$$s(t) = \frac{2}{\Gamma_H + \Gamma_V - 2i\omega} f(-t)$$

ω: detuning
(photon energy~$\Omega+\omega$)

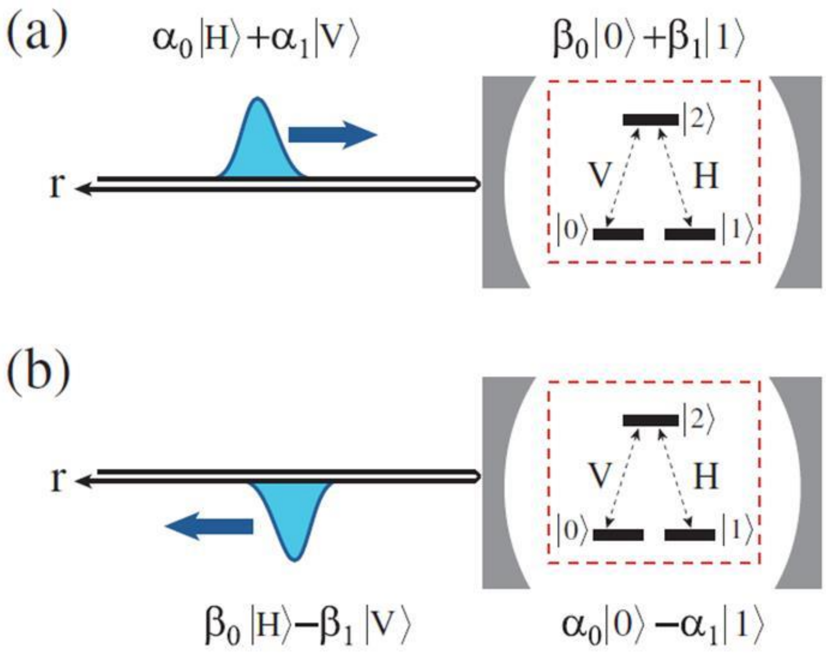
- Input—output relation

$$\begin{pmatrix} |H,0\rangle \\ |H,1\rangle \\ |V,0\rangle \\ |V,1\rangle \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\Gamma_V - \Gamma_H - 2i\omega}{\Gamma_V + \Gamma_H - 2i\omega} & -\frac{2\sqrt{\Gamma_V \Gamma_H}}{\Gamma_V + \Gamma_H - 2i\omega} & 0 \\ 0 & -\frac{2\sqrt{\Gamma_V \Gamma_H}}{\Gamma_V + \Gamma_H - 2i\omega} & \frac{\Gamma_H - \Gamma_V - 2i\omega}{\Gamma_H + \Gamma_V - 2i\omega} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |H,0\rangle \\ |H,1\rangle \\ |V,0\rangle \\ |V,1\rangle \end{pmatrix}$$

Gate matrix = unitary 4x4 matrix

Atom-photon SWAP gate (1)

- Useful as quantum logic gates, when symmetric ($\Gamma_H=\Gamma_V$)
- Resonant case (no detuning, $\omega=0$)
  - deterministic Raman transition $|H1\rangle \rightarrow -|V0\rangle$, $|V0\rangle \rightarrow -|H1\rangle$

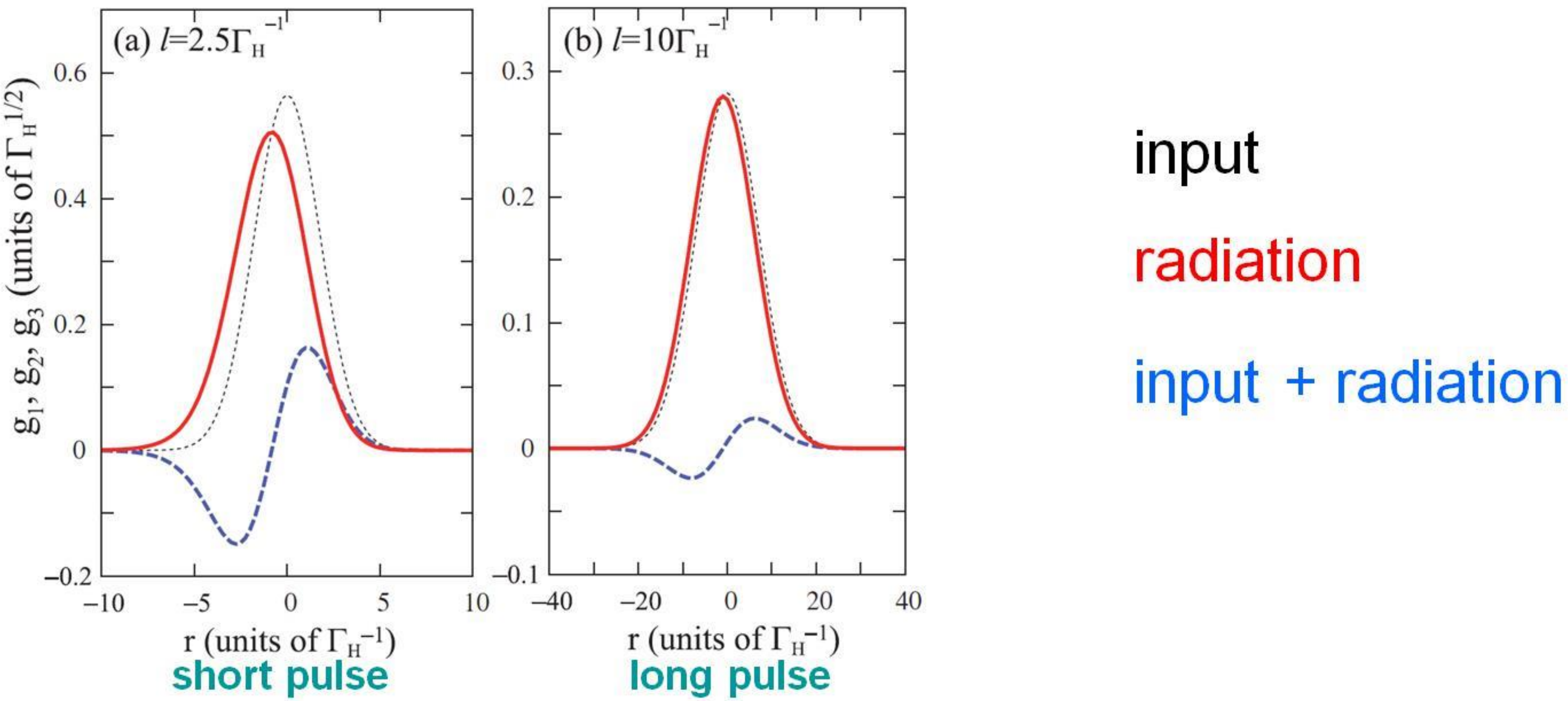


SWAP gate

photon and atom qubits are swapped by reflection

Atom-photon SWAP gate (3)

- Shapes of output pulses

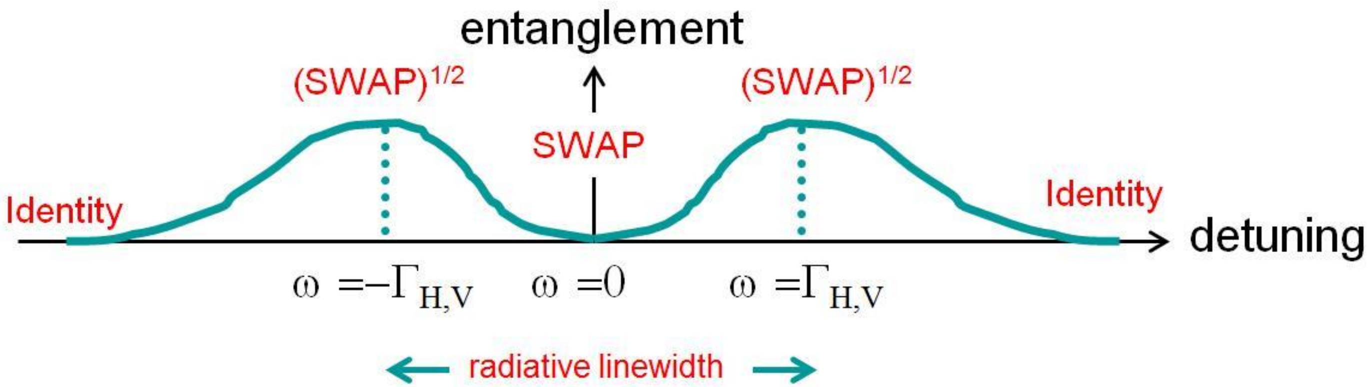


Delay of "radiation" from "input" due to absorption & re-emission (~ Γ^{-1})

In a long pulse case, this delay becomes relatively small →complete cancelation

Atom-photon $(SWAP)^{1/2}$ gate

- Effects of detuning



- $(SWAP)^{1/2}$ gate when $\omega = -\Gamma_{H,V}, +\Gamma_{H,V}$

$$\begin{pmatrix} |H,0\rangle \\ |H,1\rangle \\ |V,0\rangle \\ |V,1\rangle \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |H,0\rangle \\ |H,1\rangle \\ |V,0\rangle \\ |V,1\rangle \end{pmatrix}$$

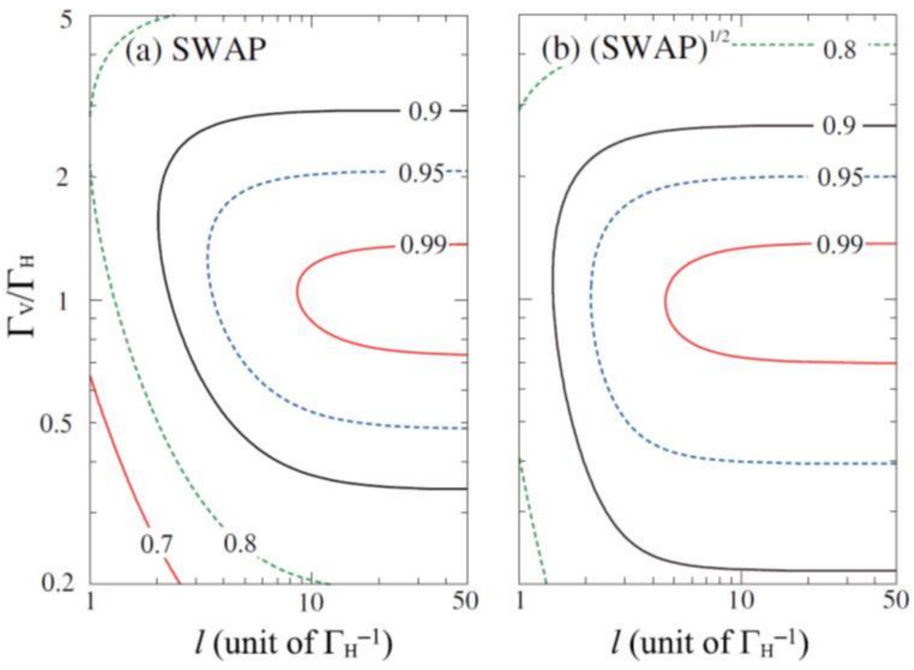
  - maximum atom--photon entanglement (Bell state)
  - universal gate set (~CNOT)

Fidelity of Atom-photon gates

- Averaged gate fidelity

$$\bar{F}_{\mathrm{SWAP}} = \frac{1 + \left|1 + \int dr g_2^* g_1\right|^2}{5}$$

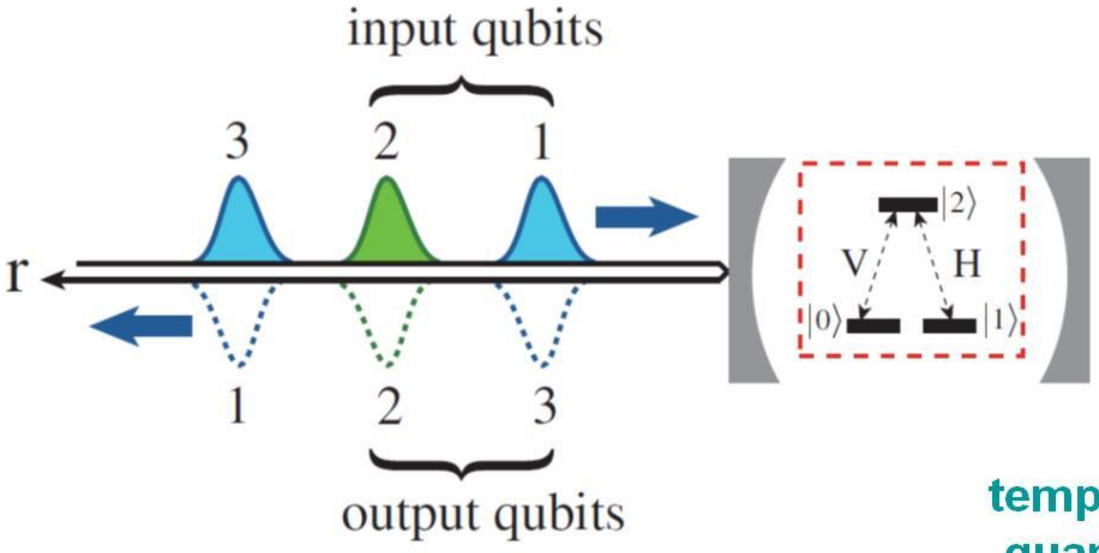
$$\bar{F}_{\sqrt{\mathrm{SWAP}}} = \frac{1 + \left|1 + \frac{1+i}{4} \int dr (g_3^* + g_4^* - 2i g_2^*) g_1\right|^2}{5}$$



Conditions for high fidelity
- symmetric ($\Gamma_H = \Gamma_V$)
- long pulse

Photon-photon $(SWAP)^{1/2}$ gate (1)

- Successive input of 3 photons
  - Photon 1 (resonant), photon 2 (off-resonant), photon 3 (resonant)
  - Input qubits: photon 1 & 2
  - Output qubits: photon 3 & 2



temporary quantum memory

Photon-photon $(\mathrm{SWAP})^{1/2}$ gate (2)

- Input-output relation

$$\begin{pmatrix} |H\rangle_1|H\rangle_2 \\ |H\rangle_1|V\rangle_2 \\ |V\rangle_1|H\rangle_2 \\ |V\rangle_1|V\rangle_2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |H\rangle_3|H\rangle_2 \\ |H\rangle_3|V\rangle_2 \\ |V\rangle_3|H\rangle_2 \\ |V\rangle_3|V\rangle_2 \end{pmatrix}$$

- Atom → photon 1, photon 3 → atom
  - These qubits are unentangled with relevant qubits, so we can discard them.

$$\begin{aligned} \alpha_0|0\rangle_a + \alpha_1|1\rangle_a &\rightarrow \alpha_0|H\rangle_1 - \alpha_1|V\rangle_1, \\ \beta_0|H\rangle_3 + \beta_1|V\rangle_3 &\rightarrow \beta_0|0\rangle_a - \beta_1|1\rangle_a, \end{aligned}$$

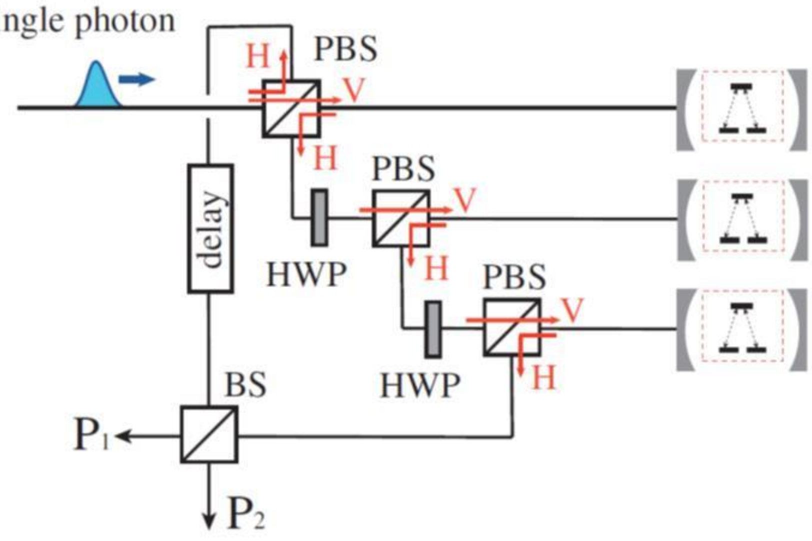
Photon-photon $(SWAP)^{1/2}$ gate (3)

- Merits of this scheme
  - Free from optical nonlinearity
    - No need for pulse shape control.
  - Free from path interference
    - No need for high stability of optical paths.
  - Atom is used completely passively *as catalyst*
    - No need for active control such as π pulses.
    - Initial states may be arbitrary, including mixed states.

- Suitable for Scalable quantum network.

Deterministic entangler of atoms

- This circuit generates N-atom GHZ states deterministically.
  - Initialize all atoms to |0>.
  - Input of single photon, $2^{-1/2}$(|H>+|V>). Elimination of which-path information by BS.
  - Final atomic state, $2^{-1/2}$(|0,0,…,0>+|1,1,…,1>)
- cluster state can also be generated.
- Extension to large N(=3,4,…) is not difficult.



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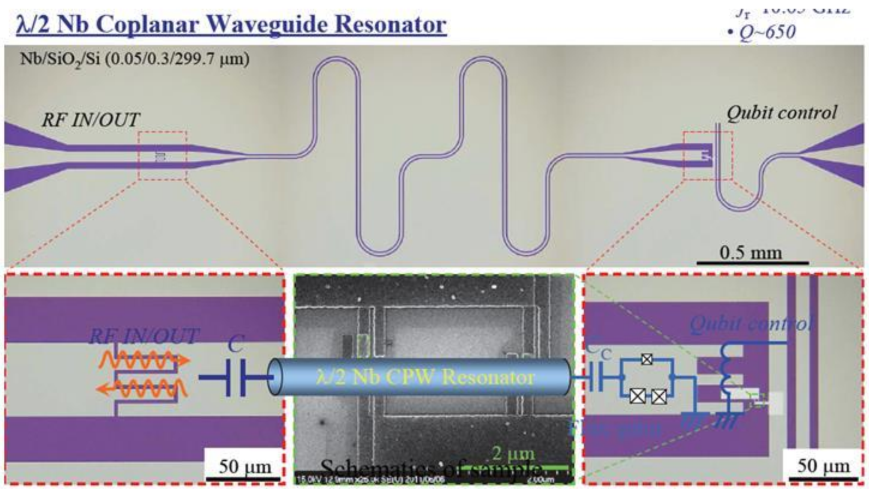
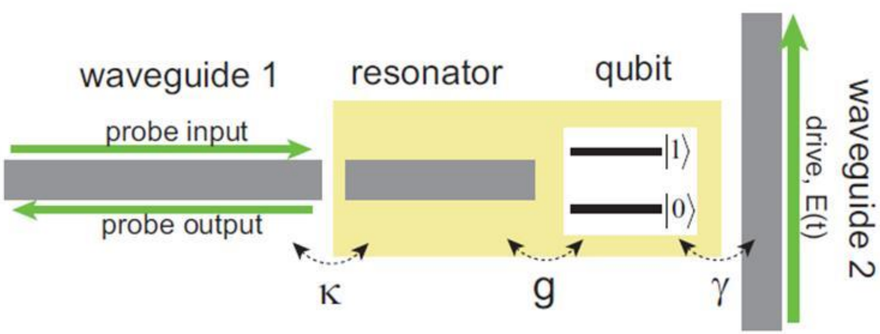
猪股邦宏（理化学研究所）
中村泰信（東大先端研）
山本剛（NEC）

We implement an impedance-matched Λ system in circuit QED .



1D field (semi-infinite) +
resonator +
qubit



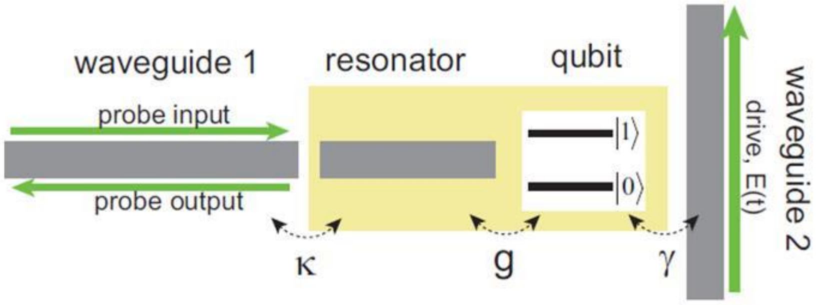
Drive field → qubit
Probe field → resonator

Inomata et al. Phys Rev B **86** 140508(R) (2012)

Hamiltonian

driven JC model

$$\mathcal{H}(t) = \mathcal{H}_{sys}(t) + \mathcal{H}_{damp},$$
$$\mathcal{H}_{sys}(t) = \omega_q \sigma^\dagger \sigma + \omega_r a^\dagger a + g(\sigma^\dagger a + a^\dagger \sigma) + \sqrt{\gamma}[E(t)\sigma^\dagger + E^*(t)\sigma],$$
$$\mathcal{H}_{damp} = \int dk \left[k b_k^\dagger b_k + \sqrt{\kappa/2\pi}(a^\dagger b_k + b_k^\dagger a) \right] + \int dk \left[k c_k^\dagger c_k + \sqrt{\gamma/2\pi}(\sigma^\dagger c_k + c_k^\dagger \sigma) \right]$$



Parameters

$\omega_q/2\pi = 5\ GHz$

$\omega_r/2\pi = 10\ GHz$

$g/2\pi = 500\ MHz$

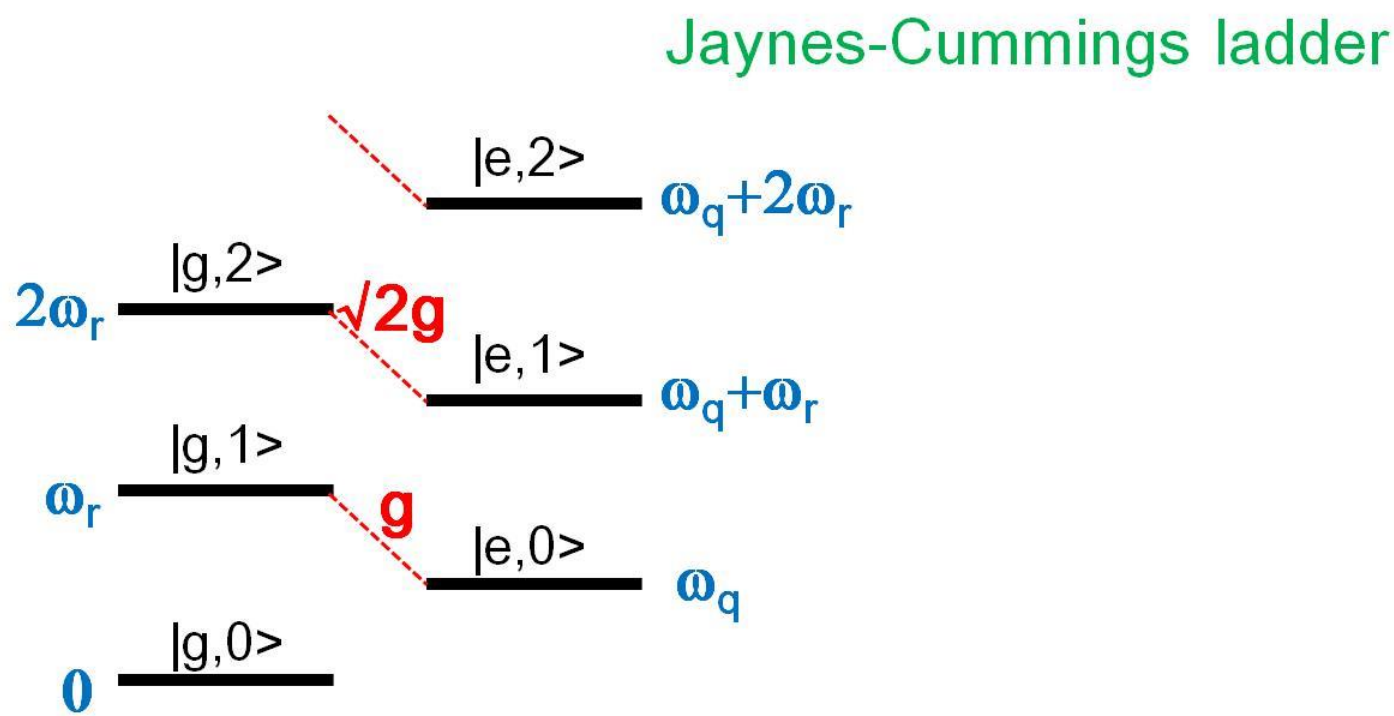
$\kappa/2\pi = 20\ MHz$

$\gamma/2\pi = 1\ MHz$

dispersive regime (detuning Δ>>g)

good one-dimensionality (κ>>γ)

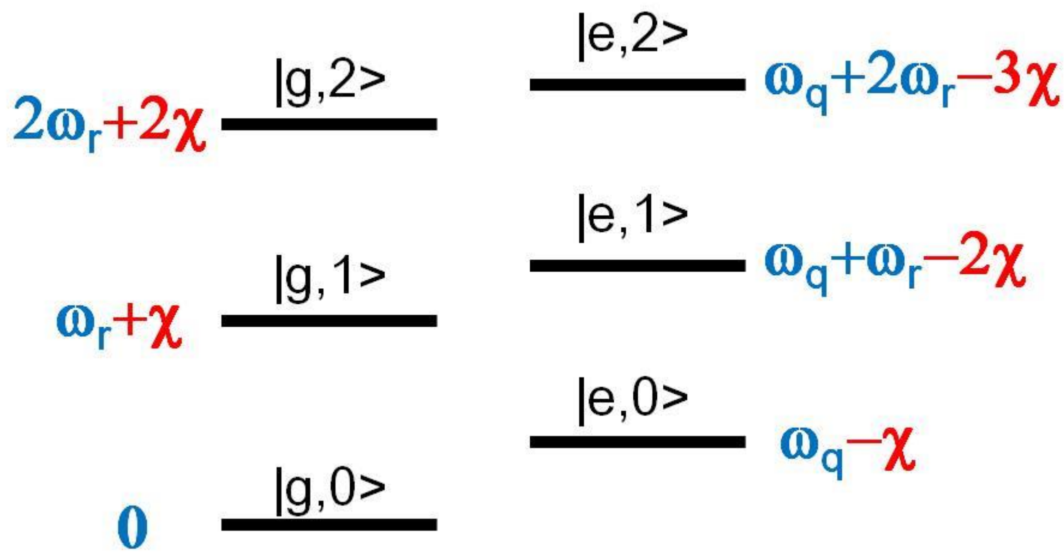
Level structure of qubit + resonator (dispersive regime)



Coupling g does not mix the states,
but induces dispersive level shifts ($\chi = g^2/\Delta$)

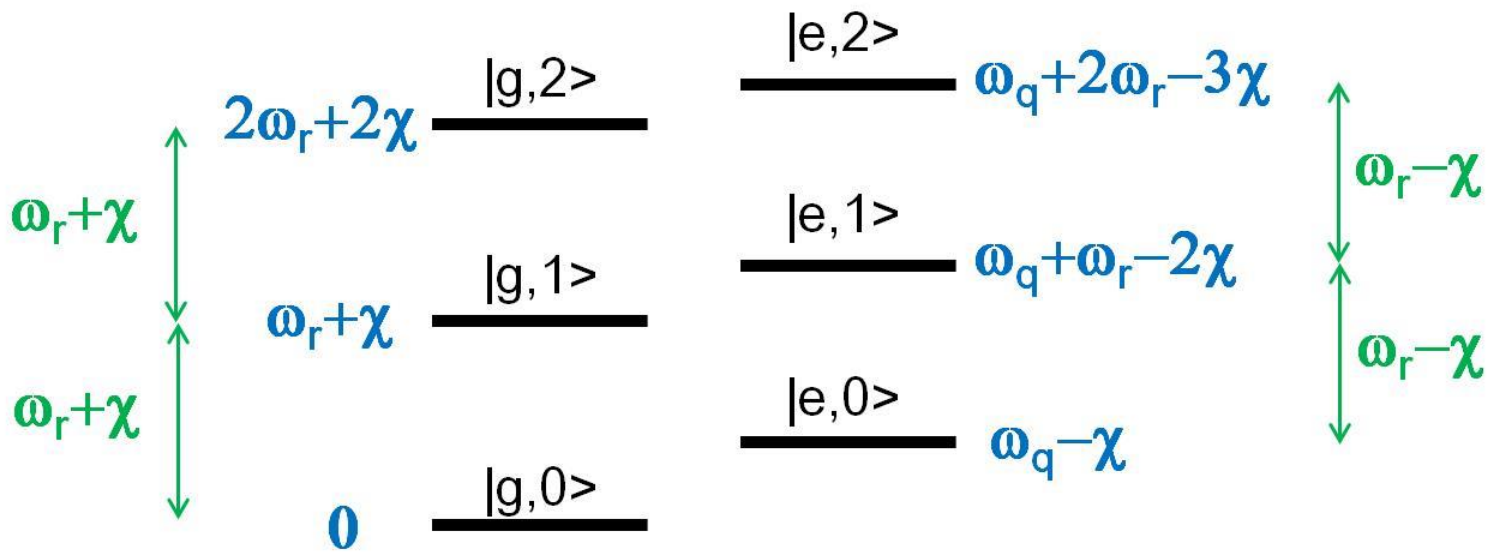
Level structure of qubit + resonator (dispersive regime)

Jaynes-Cummings ladder



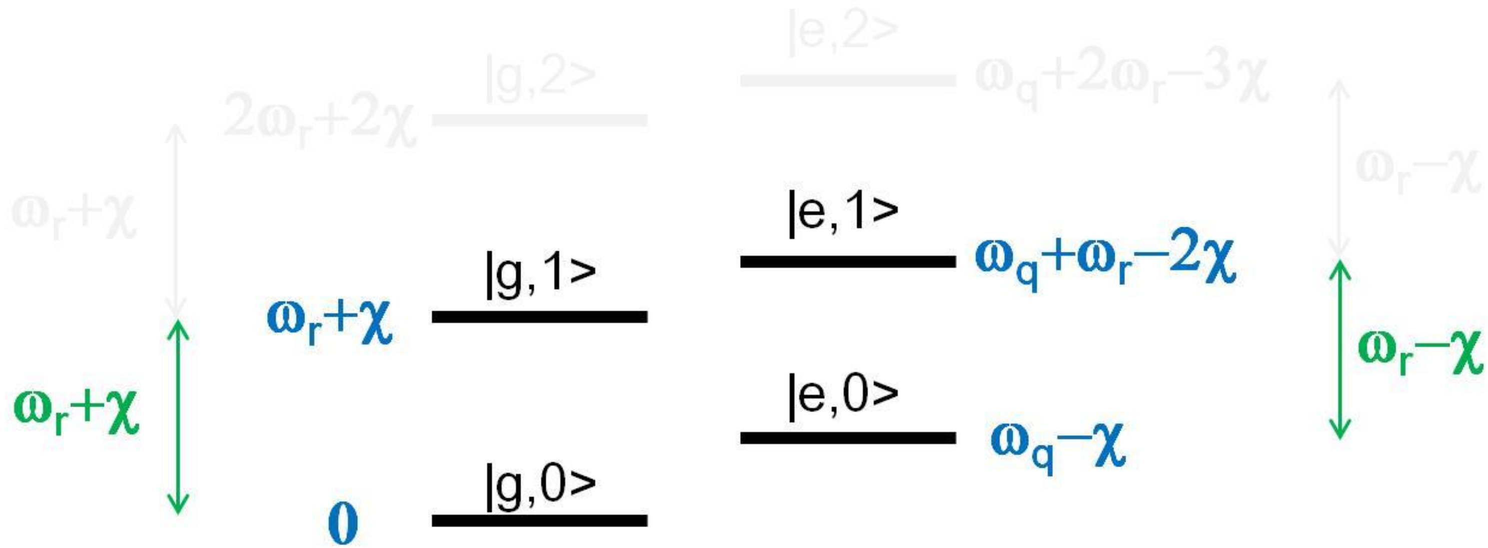
Coupling g does not mix the states,
but induces dispersive level shifts ($\chi=g^2/\Delta$)

Level structure of qubit + cavity (dispersive regime)



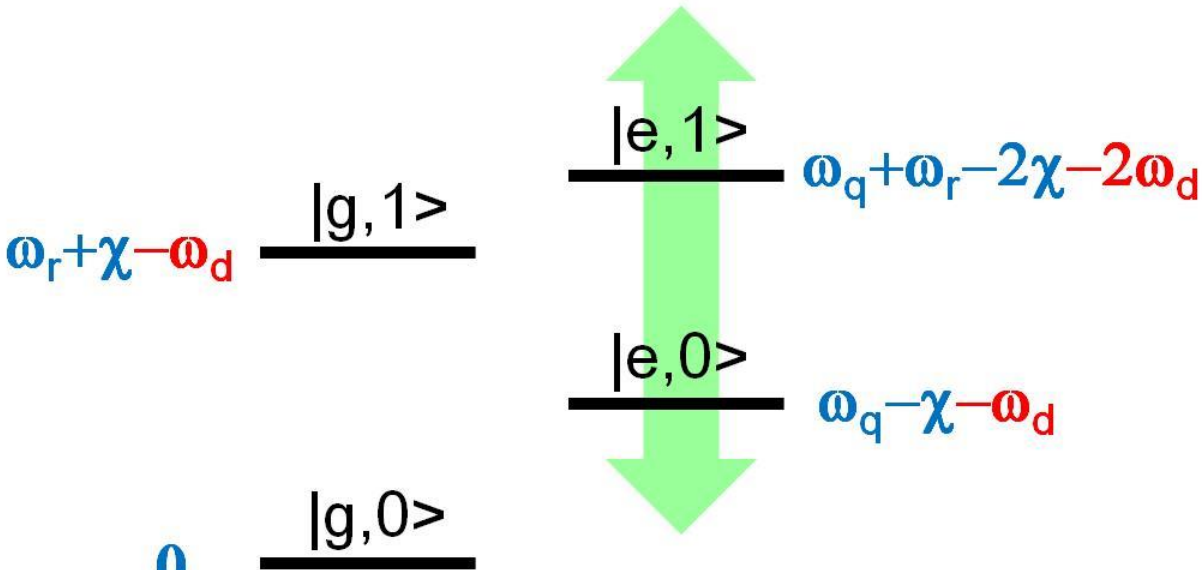
Resonator frequency depends on the qubit state
(difference of 2χ)

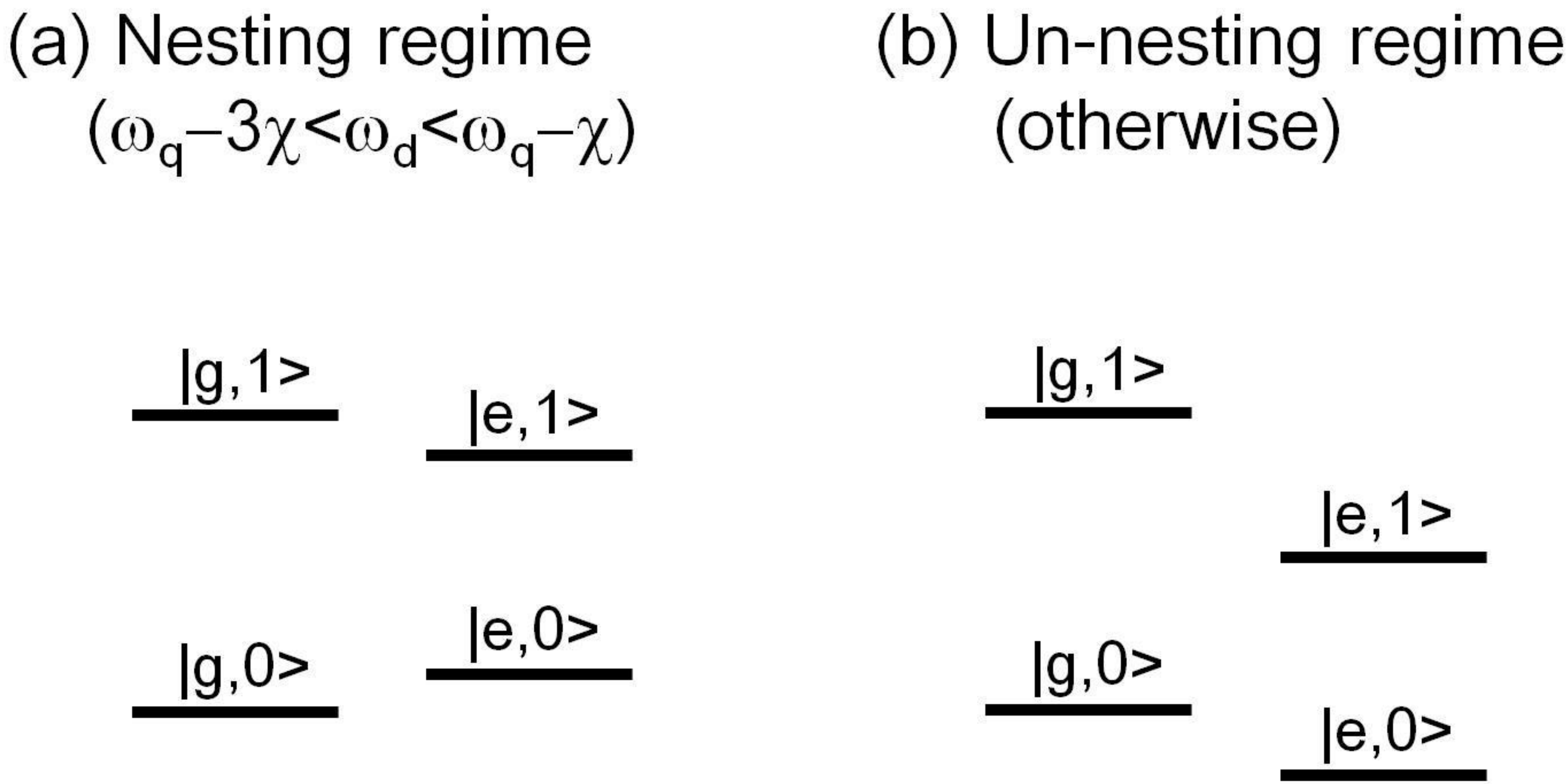
Level structure of qubit + cavity (dispersive regime)



We use the lowest 4 levels to generate Λ system

Effects of qubit drive (1):
In rotating frame at ω_d (drive freq),
we can control relative height of two columns by ω_d.



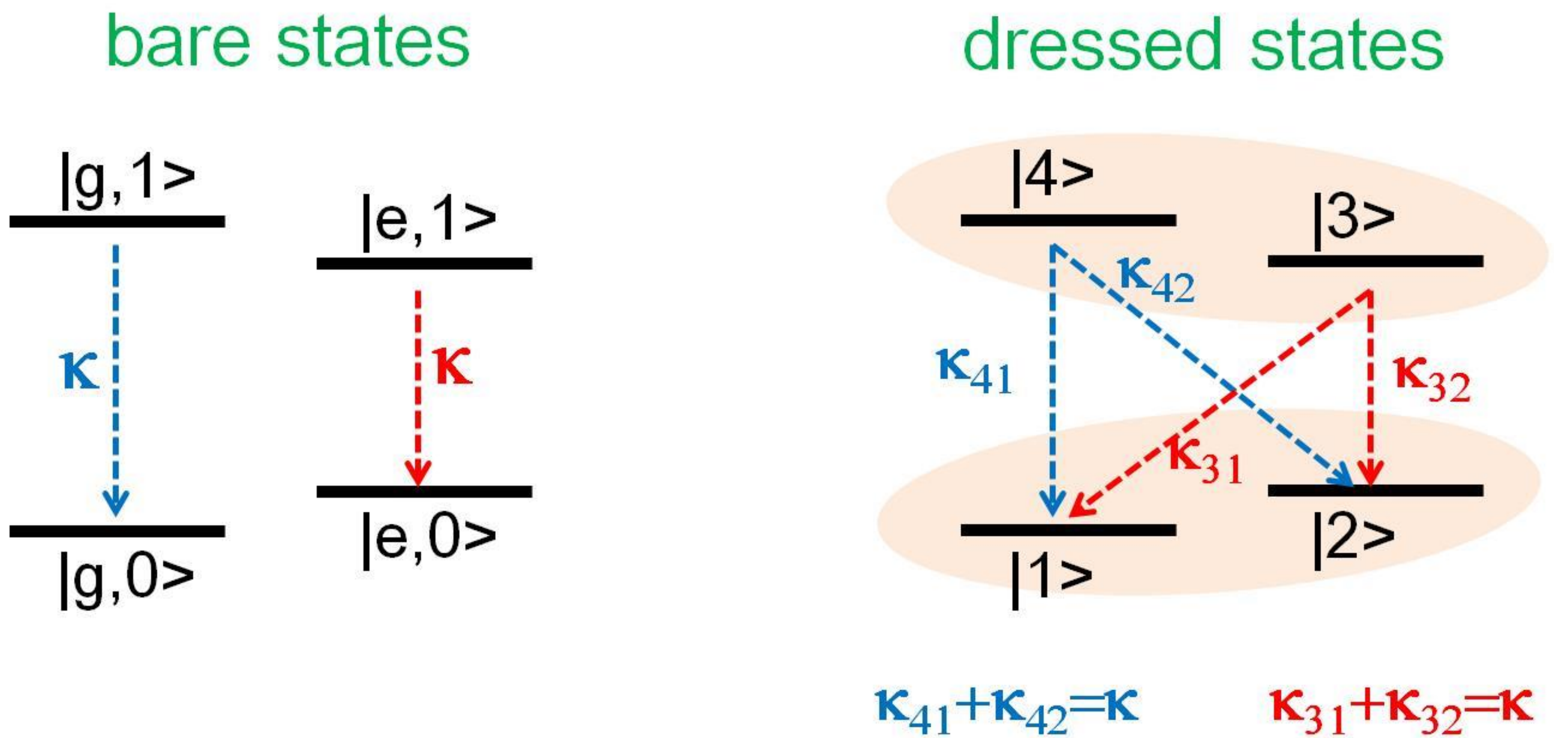

(a) Nesting regime
(ωq–3χ<ωd<ωq–χ)
(b) Un-nesting regime
(otherwise)
|g,1>
|e,1>
|e,0>
|g,0>
|g,1>
|e,1>
|g,0>
|e,0>

Effects of qubit drive (2):

Drive field mixes the upper/lower states

|g,0>, |e,0> → |1>, |2>

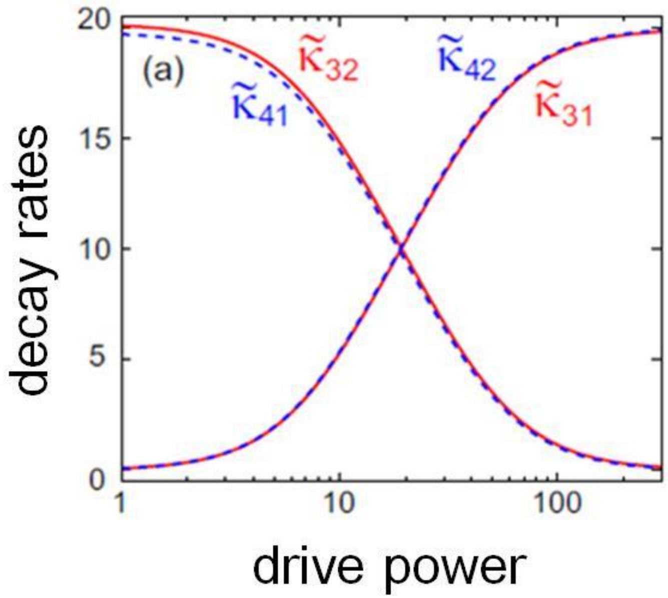
|g,1>, |e,1> → |3>, |4>

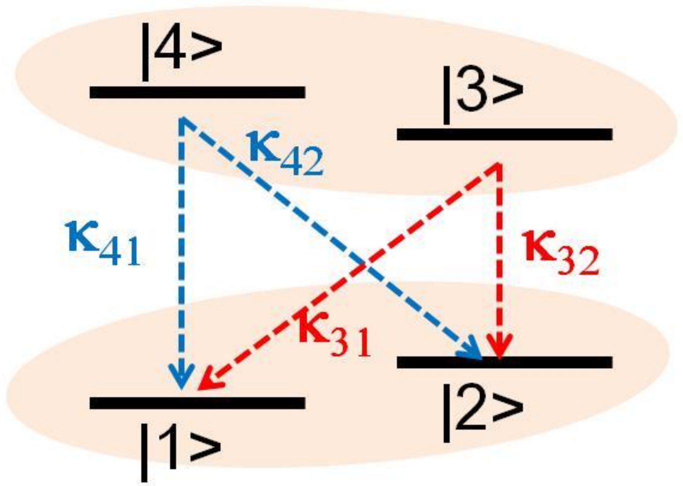


Only vertical decay is allowed in bare states.

Oblique decay is also allowed in dressed states.

Decay rates (nesting regime)

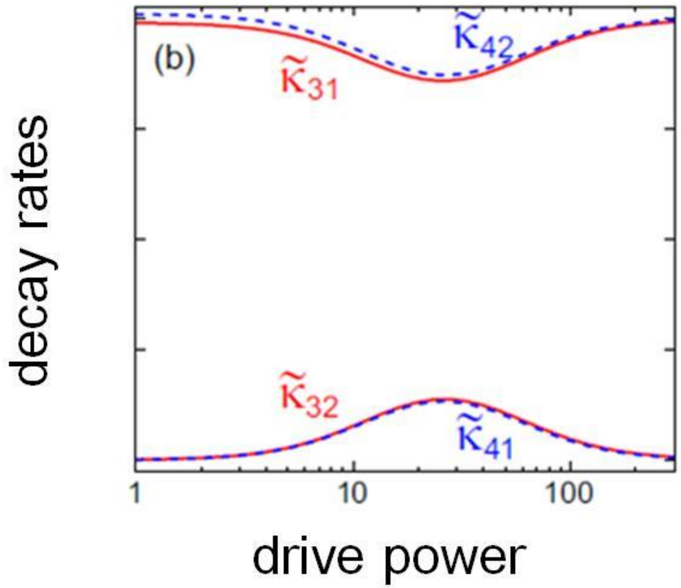


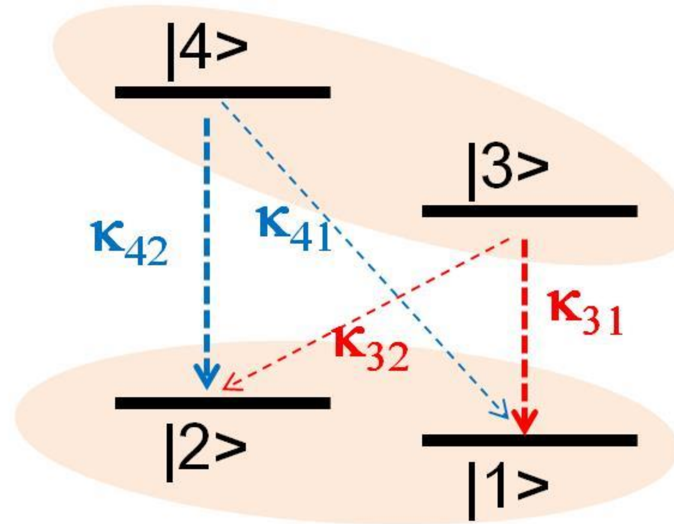


Vertical decay is dominant for weak drive.

Decay rates become identical ($\kappa_{41}= \kappa_{42}$, $\kappa_{31}= \kappa_{32}$) at proper drive power

Decay rates (un-nesting regime)





Vertical decay is dominant for any drive power

Selection rules

	Weak drive	Strong drive
Nesting regime	$\|3\rangle \to \|2\rangle$ $\|4\rangle \to \|1\rangle$	$\|3\rangle \to \|1\rangle$ $\|4\rangle \to \|2\rangle$
Un-nesting regime	$\|3\rangle \to \|1\rangle$ $\|4\rangle \to \|2\rangle$	

opposite

same

For weak drive, decay occurs vertically.

For strong drive, decay occurs 3→1 and 4→2 by parity selection rule.

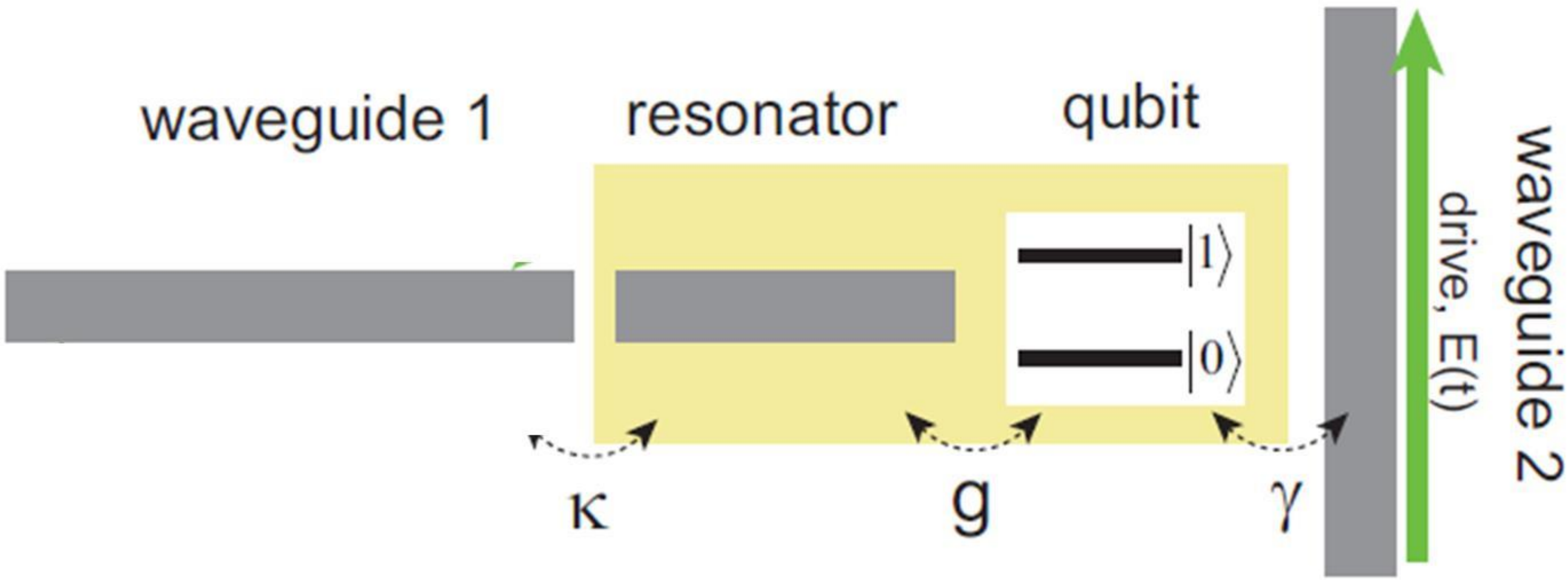
$$|\tilde{4}\rangle = |0,1\rangle + |1,1\rangle$$

$$|\tilde{3}\rangle = |0,1\rangle - |1,1\rangle$$

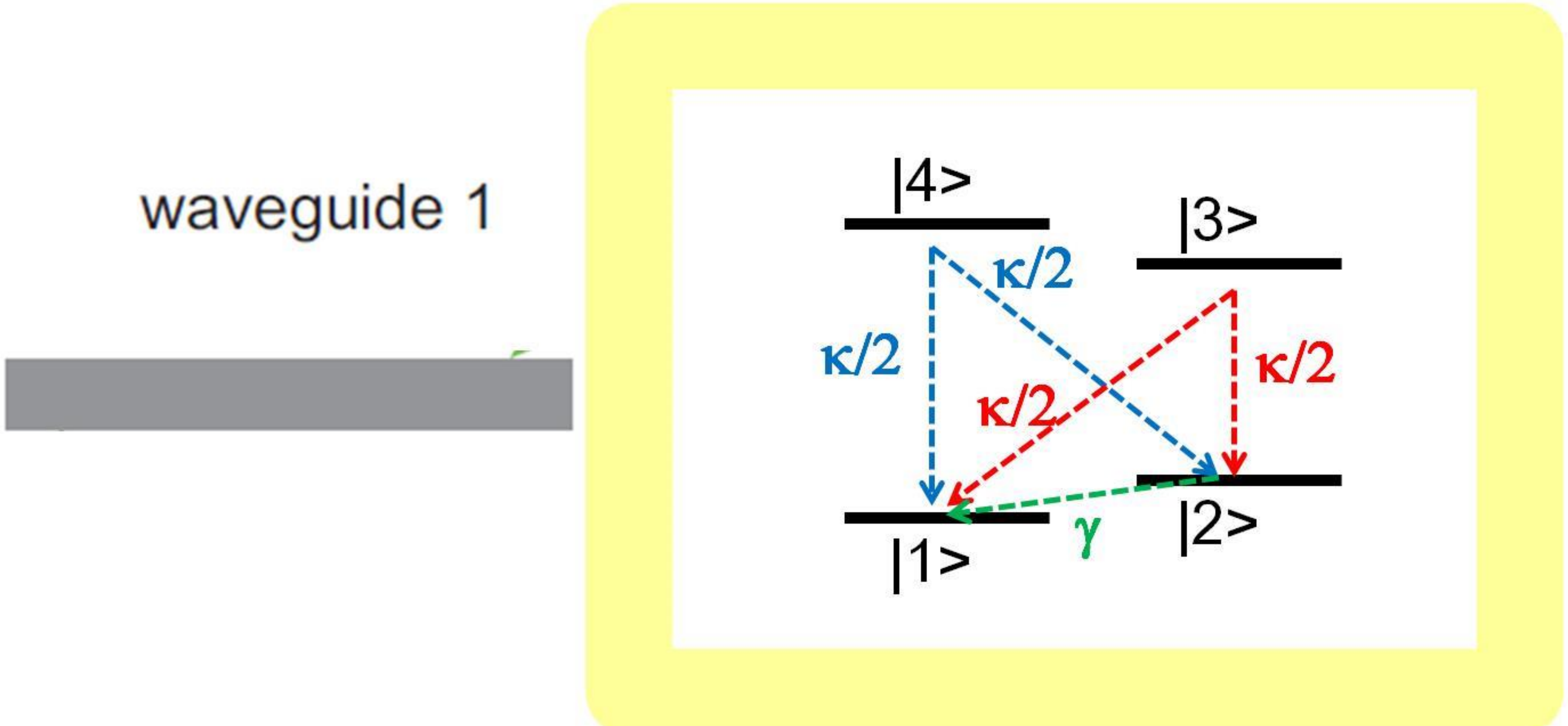
$$|\tilde{2}\rangle = |0,0\rangle + |1,0\rangle$$

$$|\tilde{1}\rangle = |0,0\rangle - |1,0\rangle$$

Under a proper drive frequency & power, qubit + cavity system functions as impedance-matched Λ system.



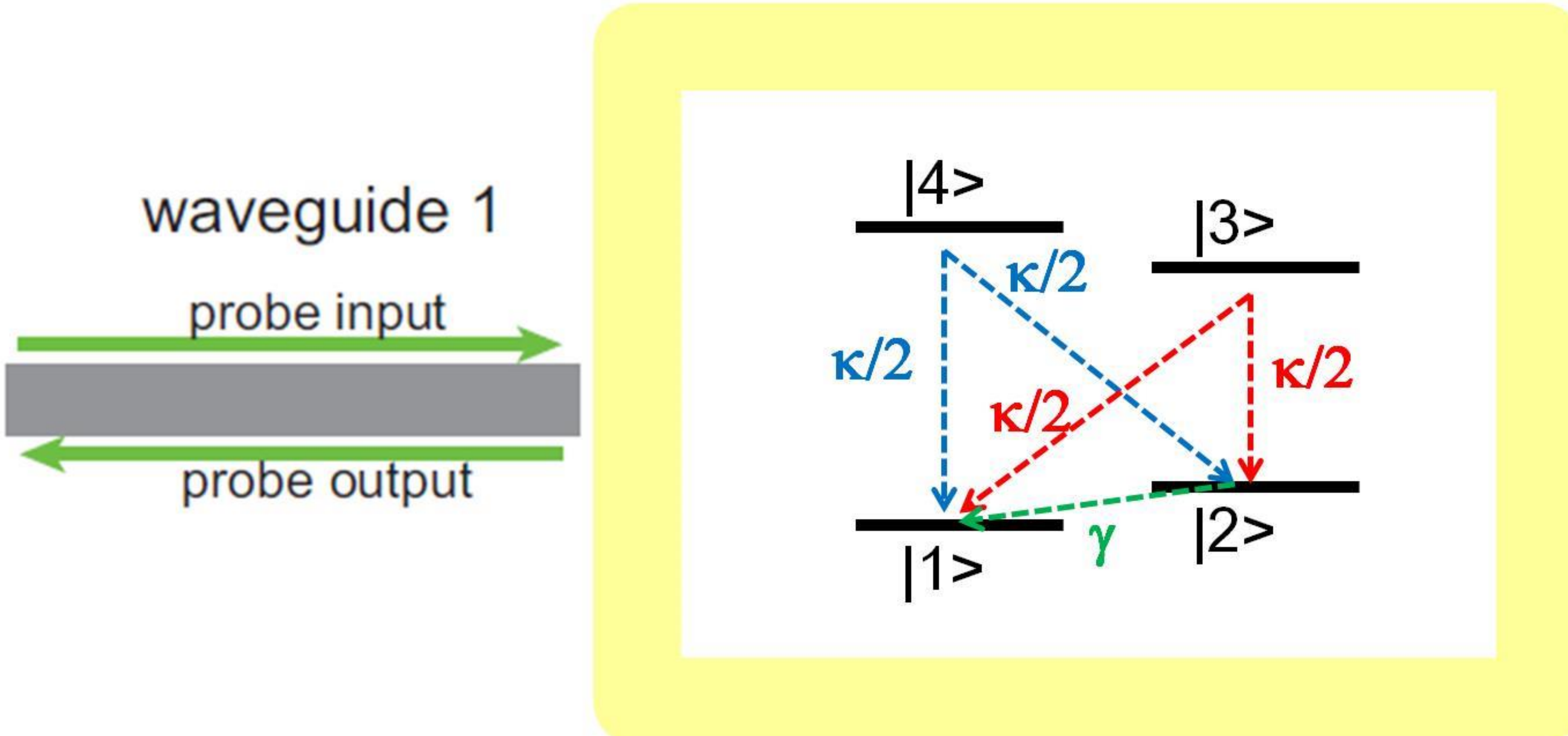
Under a proper drive frequency & power, qubit + cavity system functions as impedance-matched Λ system.



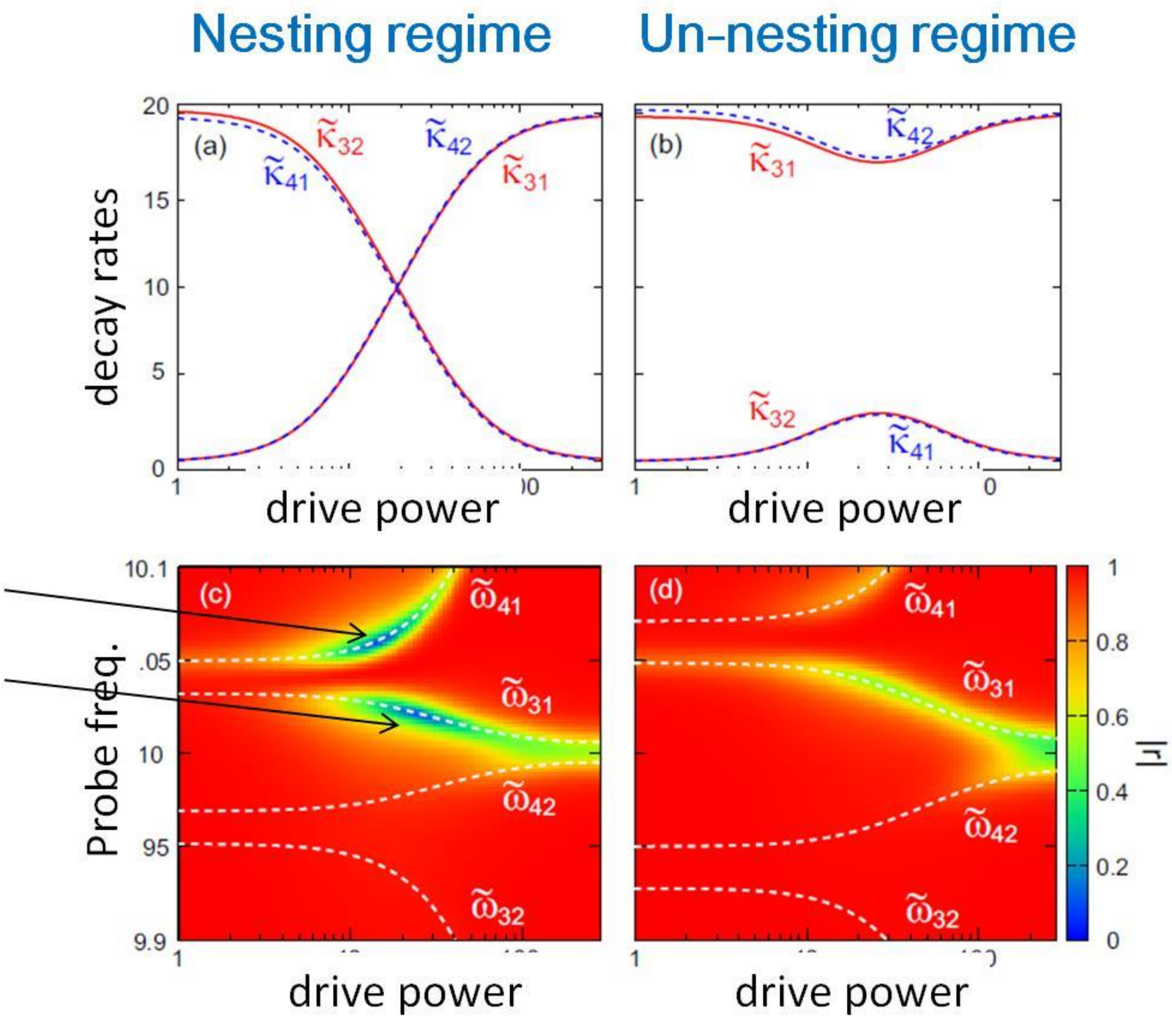
|2>→|1> decay occurs through the qubit decay

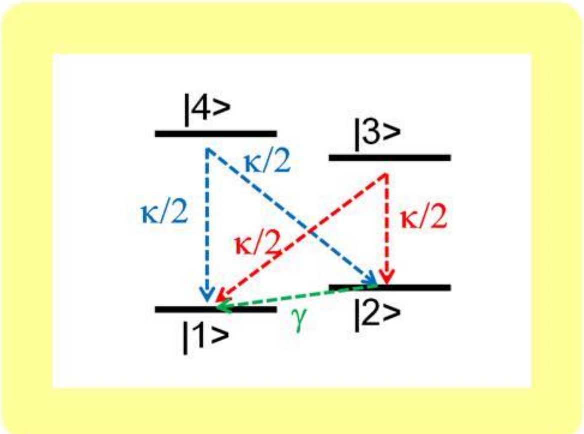
Δ system

We apply a weak probe wave to observe the optical response of this Λ system.



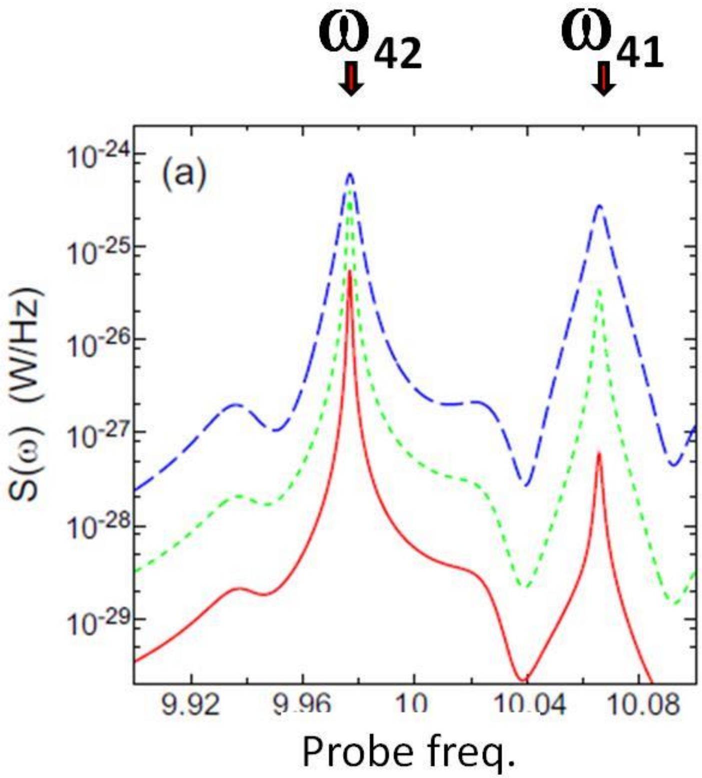
Reflectivity

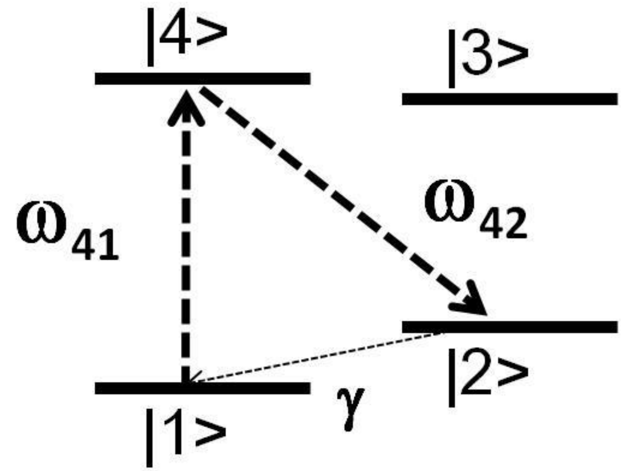




Impedance matching (no reflection) occurs, when
(i) two decay rates are equal and (ii) probe is tuned to ω_{41} or ω_{31}

Power spectrum of reflected wave

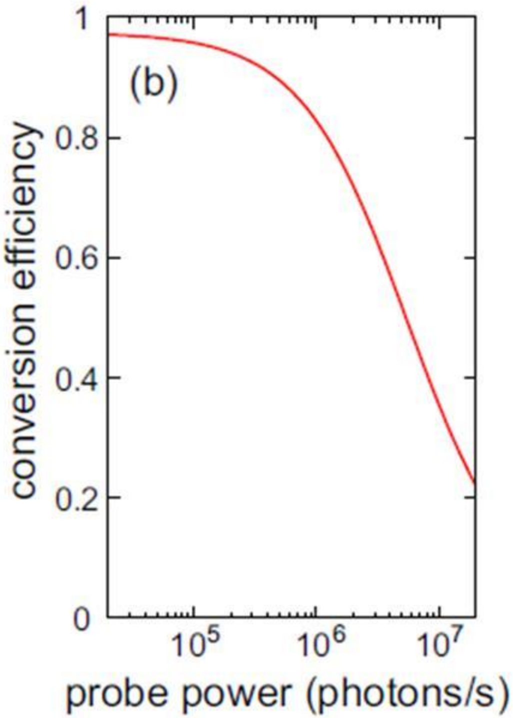


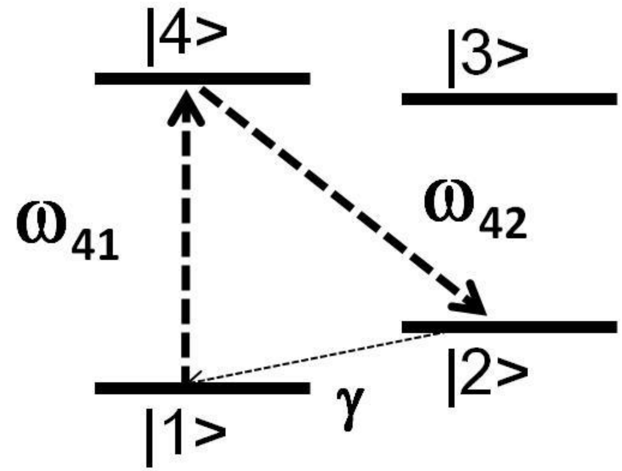


Input frequency is ω_{41}. Output frequency is mostly ω_{42}.

Nearly complete down-conversion by one reflection

Conversion efficiency



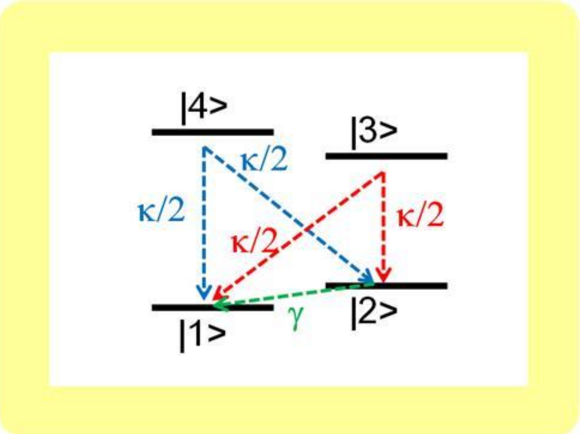


Conversion efficiency is nearly unity for weak input, and is lowered for stronger input (saturation)

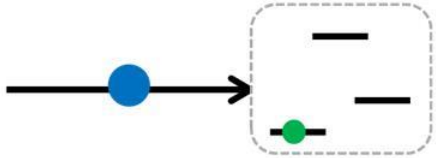
Bottleneck process is 2→1 (qubit decay).

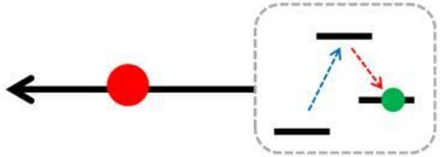
CONCLUSION

Driven cavity-QED system behaves as impedance-matched Λ system.



PRL **111** 153601 (2013)





Deterministic frequency conversion by single reflection is observed.

- disappearance of reflected wave (-25dB)
- power spectrum (75%)
- applicable to single microwave photon detector

SUMMARY

(1) Charm of 1D optical systems

Mode matching, destructive interference

(2) Single-photon response of impedance-matched Λ system

Deterministic Raman transition, SWAP, root SWAP

PRA **82** 010301(R) (2010)
APS Physics Synopsis (July 19, 2010)

(3) Implementation by circuit QED: theory & experiment

Dressed state engineering, imp matching (−25dB),
deterministic frequency conversion (75%)

PRL **111** 153601 (2013)
NJP **15** 115010 (2013)

(4) Summary