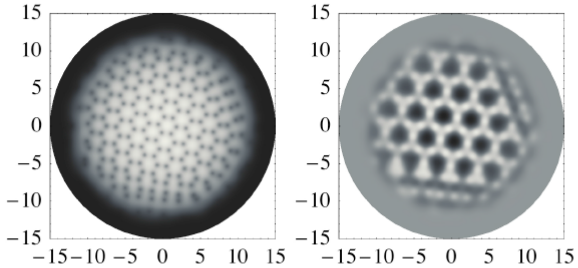
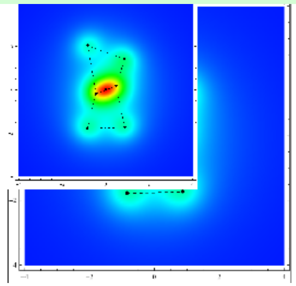
多成分凝縮系における渦構造
BECからQCDまで



Mar 11th 2014
@Gakushuin U.



新田宗土/Muneto Nitta
(慶應義塾大学/Keio U.)

Topological Quantum Phenomena in Condensed Matter with Broken Symmetries

Keio University
1858
CALAMVS GLADIO FORTIOR

References

BEC (Bose-Einstein condensates)

Lattice of vortex molecules with **Mattia Cipriani**

[1] Phys.Rev.Lett. 111 (2013) 170401[arXiv:1303.2592 [cond-mat.quant-gas]]

[2] Phys.Rev.A88 (2013) 013634 [arXiv:1304.4375 [cond-mat.quant-gas]]

Vortex graphs (or N-omers) with **M.Eto(衛藤稔)**

[3] Europhys.Lett. 103 (2013) 60006 [arXiv:1303.6048 [cond-mat.quant-gas]]

QCD (Quantum Chromodynamics)

[4] **Invited review:** Vortices and solitons in dense QCD,
with M.Eto, Y.Hirono(広野雄士), S.Yasui(安井繁宏)
Prog.Theor.Exp.Phys.:012D01,2014 [arXiv:1308.1535 [hep-ph]]

[5] **Lattice of non-Abelian vortices**
with M.Kobayashi(小林未知数)，E.Nakano(仲野英司)
arXiv:1311.2399 [hep-ph]

Plan of My Talk

§1 BEC and vortices
§2 2 comp BECs and vortex dimers
§3 Lattice of vortex dimers
§4 QCD and non-Abelian vortices
§5 Summary

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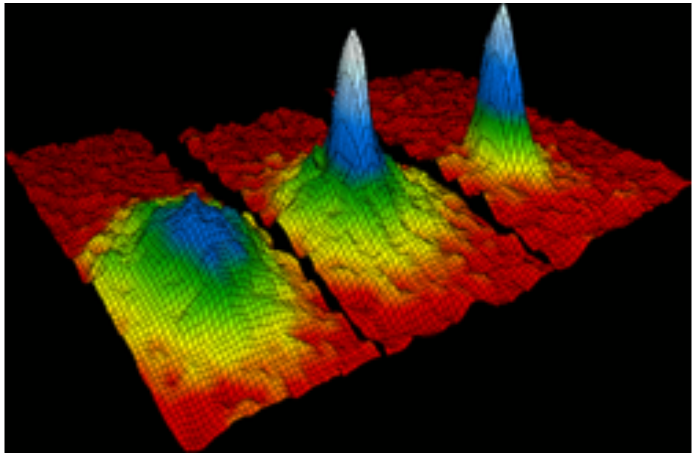
``Pure" BEC (99% is condensed)



Cold atomic gases

1995 cold atomic bose gas

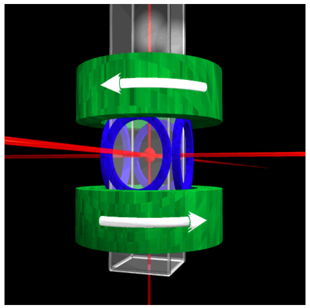
^{87}Rb, ^{23}Na, ^{7}Li

Cornell (Colorado), Ketterle(MIT)

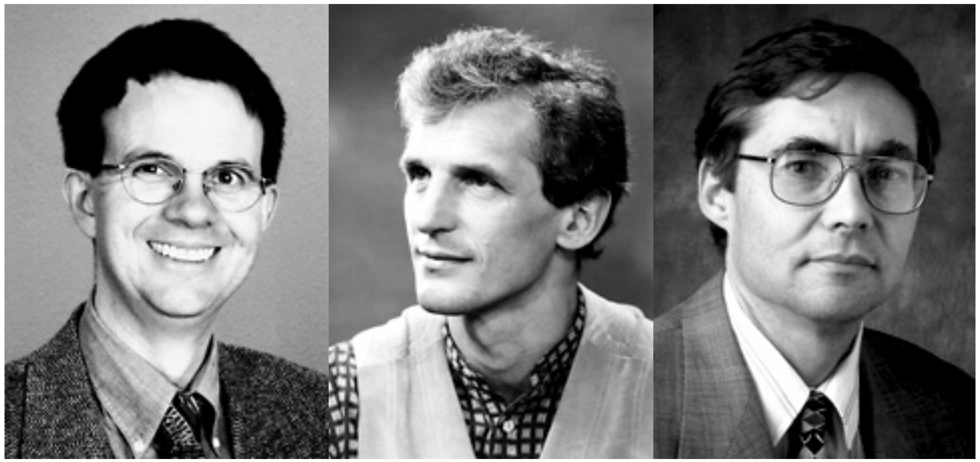
& Wieman (Colorado)

2003 cold atomic fermion gas

JILA(Colorado), MIT



doppler laser cooling
magneto-optical trap
evaporative cooling



Temperature ~ 10^{-6}, 10^{-7} K
Number ~ 10^{6}, Size ~ 10^{-3}cm

Scalar BEC, ^{4}He superfluid

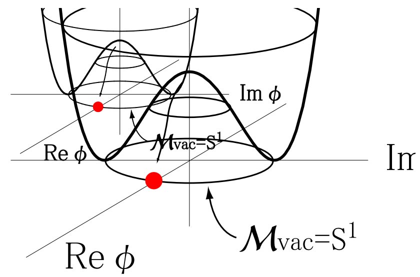
Gross-Pitaevskii (nonlinear Schrödinger) Equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2M}\nabla^2 + V_{\rm ext} - \mu + g|\psi|^2\right]\psi = \frac{\delta E}{\delta\psi^*} \qquad g \equiv \frac{4\pi\hbar^2 a_S}{M}$$

μ : chemical potential M : mass of atoms a_S : s-wave scattering length

trapping potential $V_{\rm ext} = \frac{1}{2}M\omega^2 r^2$

$$U(\psi) = -\mu|\psi|^2 + \frac{g}{2}|\psi|^4$$



Gross-Pitaevskii energy functional

$$E[\psi] = \int d^3\mathbf{r}\left\{\frac{\hbar^2}{2M}|\nabla\psi|^2 + (V-\mu)|\psi|^2 + \frac{g}{2}|\psi|^4\right\}$$

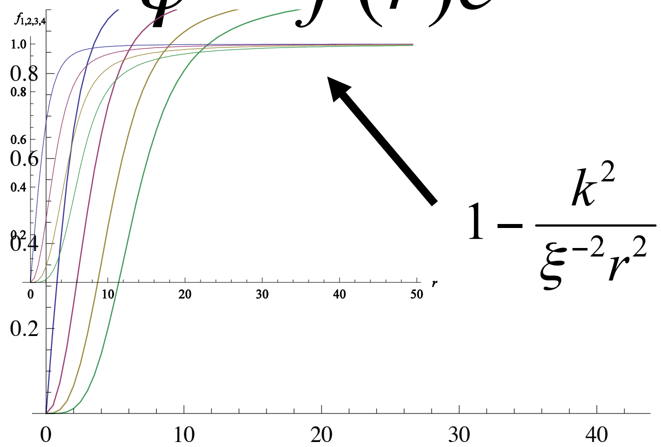
***Note*: derived by Bogoliubov theory**
weakly interactive Bose gas
with point interaction $V(r) = g\delta(r)$

Quantization $k \in \pi_1[U(1)] \cong \mathbf{Z}$ $\oint d\mathbf{r} \cdot \mathbf{v}_{\text{eff}} = \frac{h}{M} k$

$$\mathbf{v}_{\text{eff}} = \frac{1}{2i} \frac{\Psi^* \nabla \Psi - \Psi \nabla \Psi^*}{\Psi^* \Psi}$$

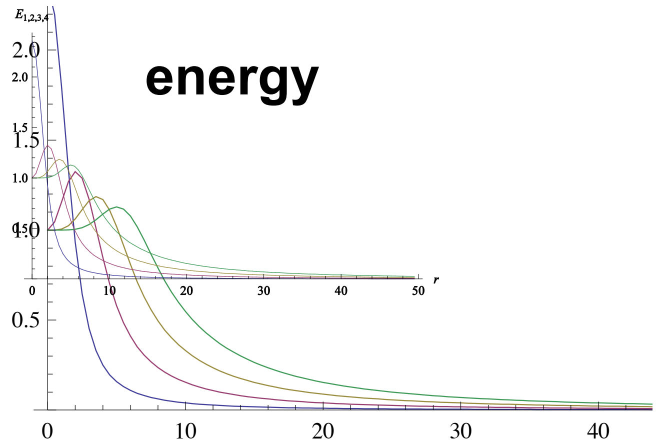
Superfluid verlocity

$$\psi = f(r) e^{ik\theta} \qquad V_{\text{ext}} = 0$$



$$1 - \frac{k^2}{\xi^{-2} r^2}$$

energy



tension $T = 2\pi v^2 k^2 \log \Lambda$ **sytem size** Λ

Intervortex force $F = \frac{4\pi v^2}{R}$ **distance** R

Rotation

Rotating frame $\nabla \to \nabla - i\frac{M}{\hbar}\mathbf{\Omega}\times\mathbf{r}$

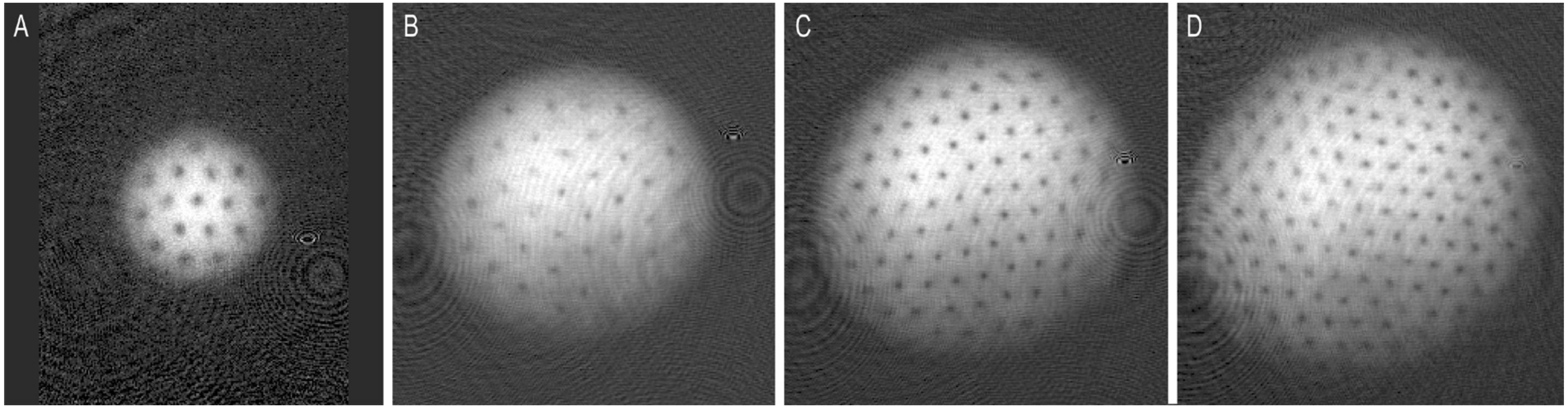
$$E[\psi] = \int d^3\mathbf{r}\left\{\frac{\hbar^2}{2M}\left|\left(\nabla - i\frac{M}{\hbar}\mathbf{\Omega}\times\mathbf{r}\right)\psi\right|^2 + (V_{\rm eff} - \mu)|\psi|^2 + \frac{g}{2}|\psi|^4\right\}$$

$$V_{\rm eff} = V_{\rm ext} - \frac{M}{2}\Omega^2 r^2$$

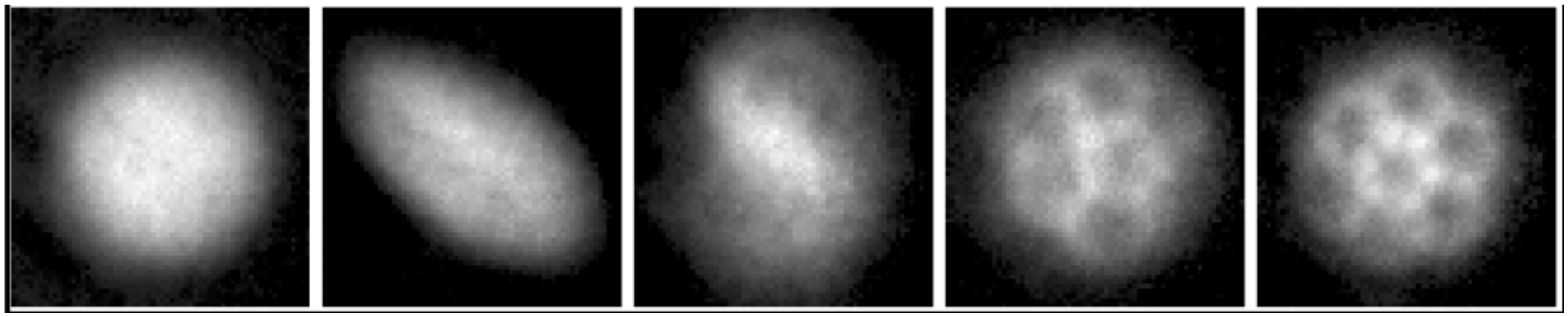
Superconductors under magnetic fields

＝ generaction of a vortex lattice

Vortex lattice in BEC (expretiment), 2001



MIT [Abo-Shaer et.al, Science 292 (2001) 476]

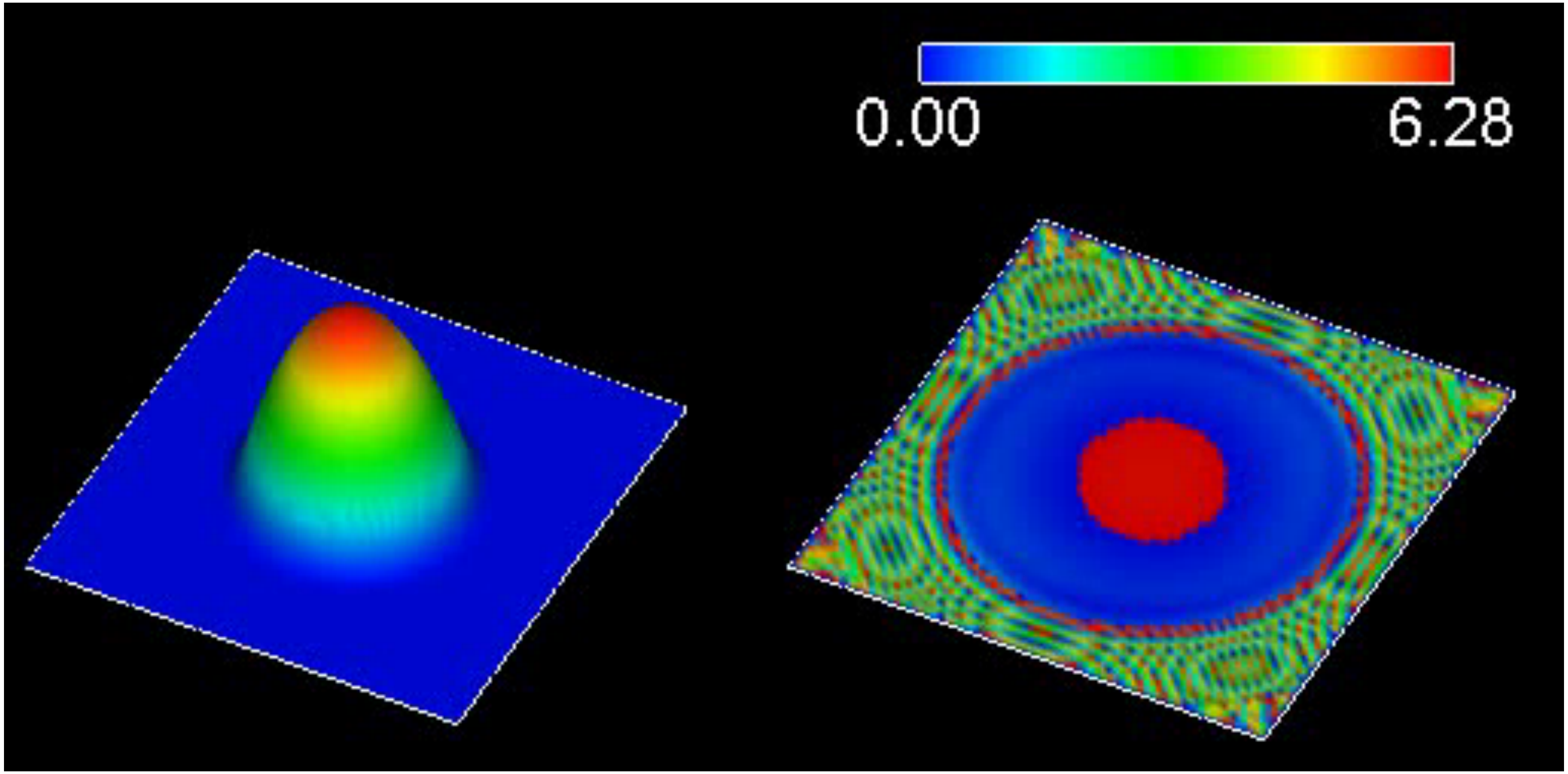


K. W. Madison et al. PRL 86, 4443 (2001)

Vortex lattice in BEC (simulation)

K.Kasamatsu
(Kinki U.)

amplitude phase



Plan of My Talk

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§5 Summary

Miscible 2 component BEC

$$\left(\psi_1, \psi_2\right)$$

Gross-Pitaevskii energy functional

$$E[\psi] = \int d^3\mathbf{r} \left\{ \sum_i \left(\frac{\hbar^2}{2m_i} |\nabla \psi_i|^2 + (V_{\text{ext}} - \mu_i) |\psi_i|^2 \right) + \sum_{i,j} \frac{g_{ij}}{2} |\psi_i|^2 |\psi_j|^2 \right\}$$

Experiments

Rb, 2 comp BEC (of the same atom with different hyperfine $m_1 = m_2$)

① $|1, -1\rangle$, $|2, 1\rangle$: Matthews, Anderson, Haljan, Hall, Wieman, and Cornell, Phys. Rev. Lett., **83**, 2498 (1999).

② $|2, 1\rangle$, $|2, 2\rangle$: Maddaloni, Modugno, Fort, Minardi and Inguscio, Phys. Rev. Lett. **85**, 2413 (2000)

In the following

$$\mu_1 = \mu_2 \equiv \mu,$$

$$g_{11} = g_{22} \equiv g, \quad g_{12} > 0$$

miscible

$$g > g_{12}$$

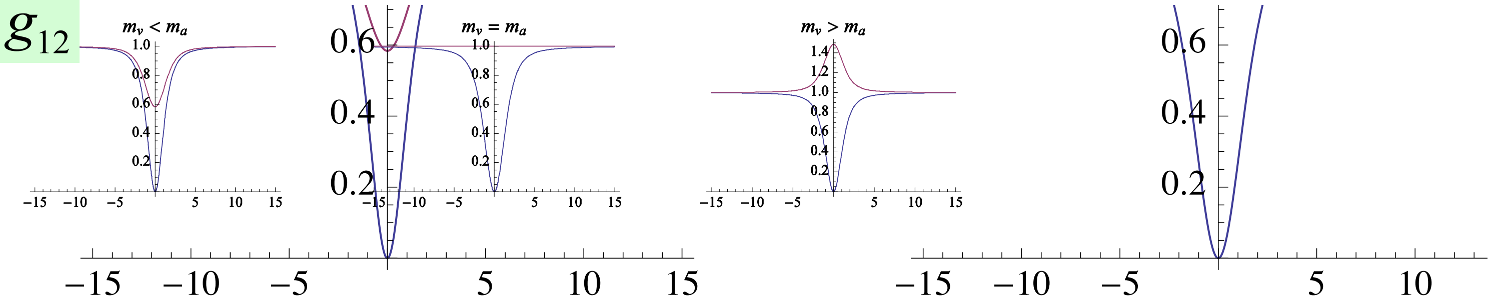
phase separation

$$g < g_{12}$$

Fractional vortex

Eto-Kasamatsu-MN-Takeuchi-Tsubota
Phys.Rev. A83 (2011) 063603 [arXiv:1103.6144]

$g > g_{12}$



g_{12}<0 **attraction**

g_{12}=0 **non-interactive**

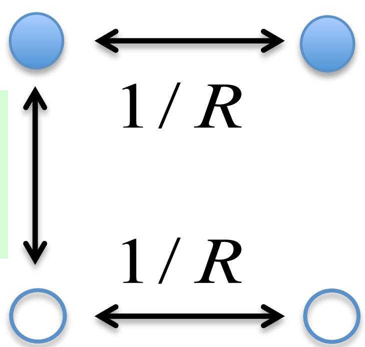
g_{12}>0 **repulsion**

Invervortex force
(@distance ***R***)

(1,0) $(e^{i\theta},1)$

$g_{12}\dfrac{\log R}{R^3}$

(0,1) $(1,e^{i\theta})$



$g_{12} > 0$

Integer vortex Φ_0 **(1,1)**

A pair of fractional vortices

(1,0) **circulation**

$$\Phi_{(1,0)} = \frac{v_1^2}{v_1^2 + v_2^2}\Phi_0$$

(0,1)

$$\Phi_{(0,1)} = \frac{v_2^2}{v_1^2 + v_2^2}\Phi_0$$

$$v_i = |\Psi_i|$$

$$\Phi_0 = \frac{h}{m}$$

Quantization of circulation (1,0) fractional

$$\Phi_{(1,0)} = \int d^2x\, \boldsymbol{\omega} = \oint_{S^1_\infty} d\mathbf{r}\cdot\mathbf{v} = \frac{v_1^2}{v_1^2 + v_2^2}\Phi_0$$

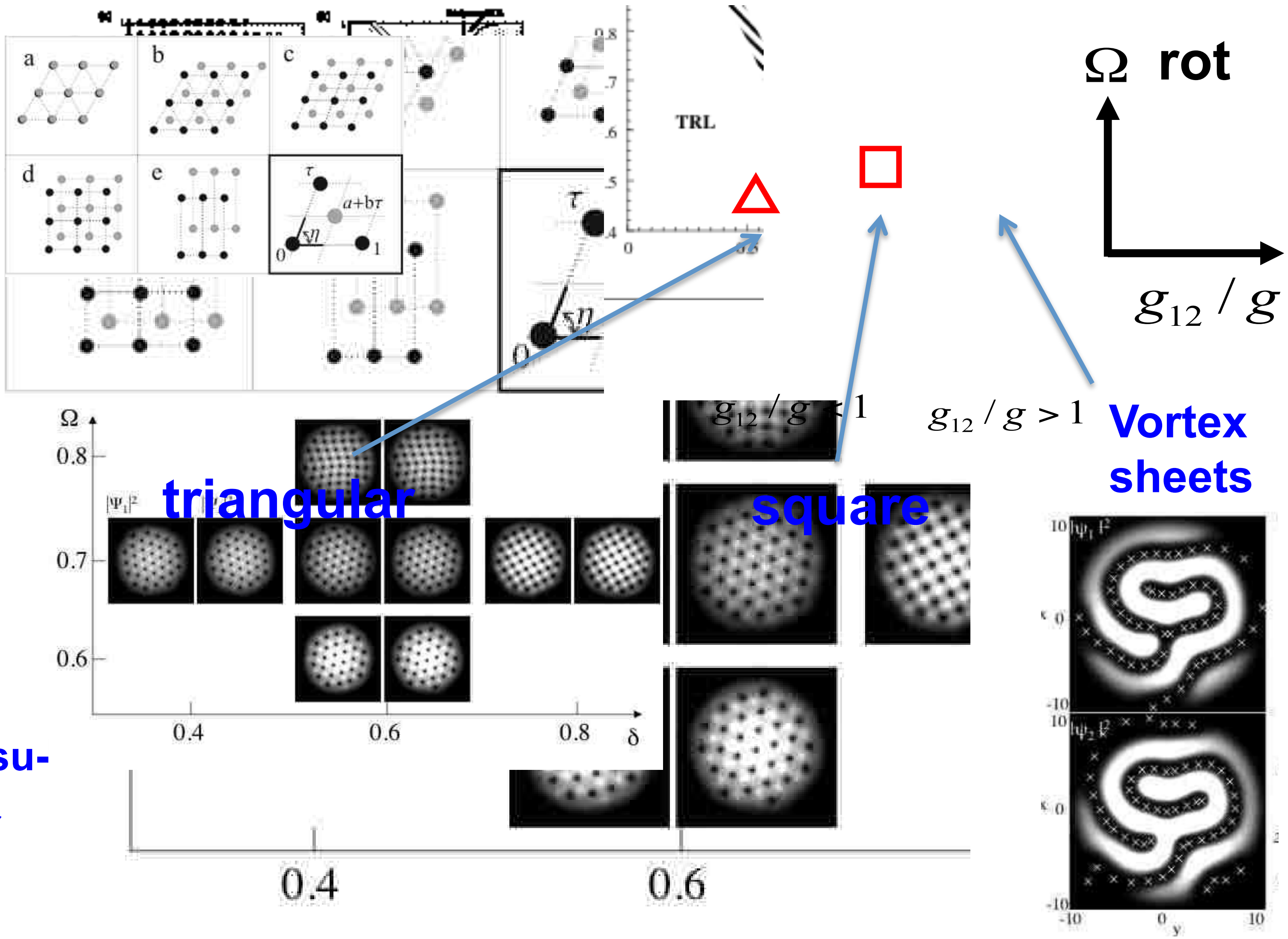
$$\mathbf{v} = \frac{v_1^2}{v_1^2 + v_2^2}\frac{\nabla\theta_1}{2\pi}\Phi_0$$

Tanaka('01), Babaev('02) for 2gap superconductors

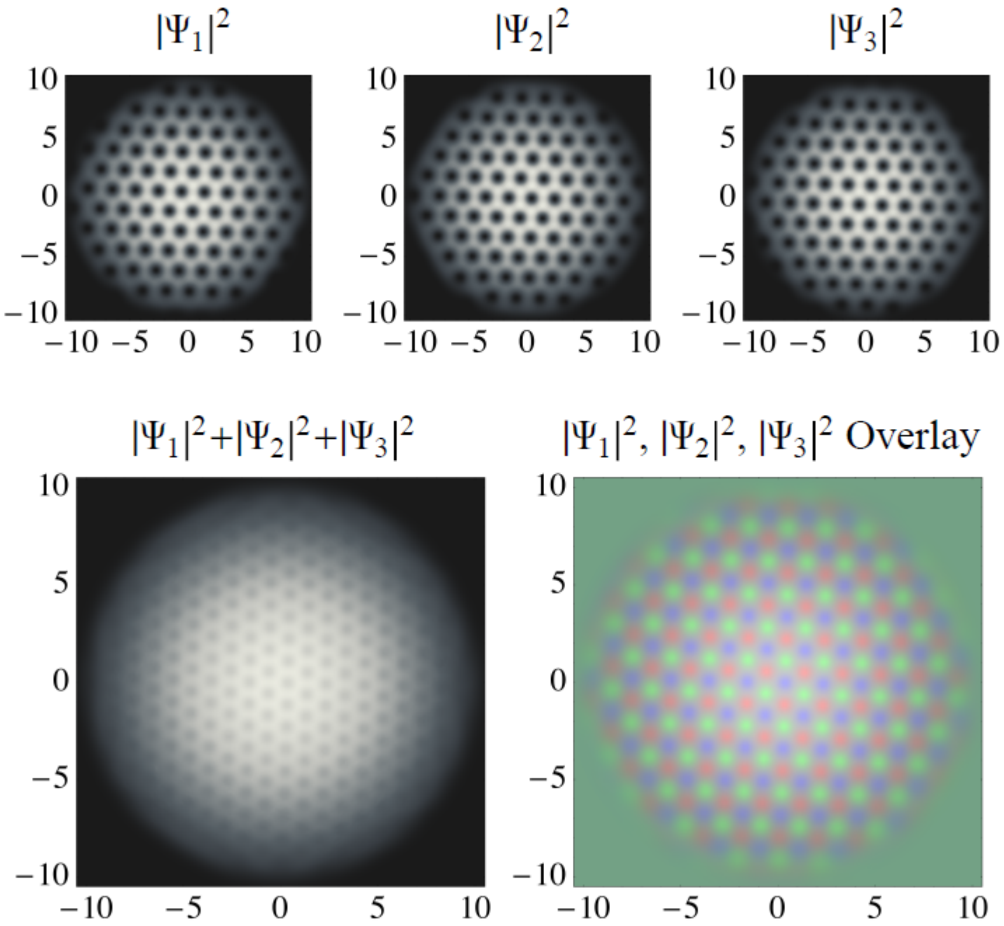
Vortex Phase diagram

Mueller &Ho('02)

Kasamatsu-Tsubota& Ueda('03)



Vortex lattices of miscible 3 component BECs *without* Rabi



Cipriani & MN
Phys.Rev.A88
(2013) 013634
arXiv:1304.4375
[cond-mat.quant-gas]

Always Abrikosov

 # comp = # edges of triangle

Simulating QCD
(color superconductor)

Miscible 2 component BEC + Rabi

Gross-Pitaevskii energy functional

$$E[\psi] = \int d^3\mathbf{r} \left\{ \sum_i \left(\frac{\hbar^2}{2m} |\nabla \psi_i|^2 + (V_{\text{ext}} - \mu_i)|\psi_i|^2 \right) + \sum_{i,j} \frac{g_{ij}}{2} |\psi_i|^2 |\psi_j|^2 - \sum_{i,j} \omega_{ij} \psi_i^* \psi_j + \text{c.c} \right\}$$

internal coherent coupling
(Rabi oscillation)
Josephson coupling=supercond

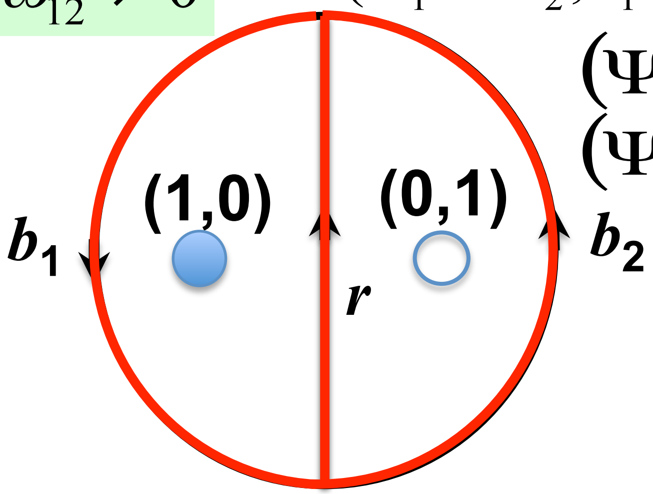
$$-\psi_1^* \psi_2 + \text{c.c} = -2|\psi_1||\psi_2|\cos(\theta_1 - \theta_2)$$

$\omega_{12} > 0$ $\theta_1 = \theta_2$ **Phases coincide**

$\omega_{12} < 0$ $\theta_1 = \theta_2 + \pi$ π **Phase**

$\omega_{12} \neq 0$

$(m_1 = m_2, v_1 = v_2)$



$$(\Psi_1, \Psi_2) \sim (e^{i\theta_1}, 1) = e^{i\theta_1/2}(1,1)e^{i\theta_1\sigma_3/2}$$

$$(\Psi_1, \Psi_2) \sim (1, e^{i\theta_2}) = e^{i\theta_2/2}(1,1)e^{-i\theta_2\sigma_3/2}$$

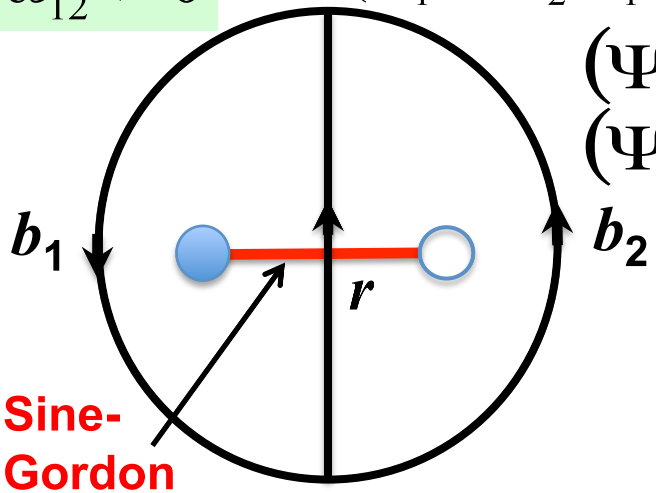
$$U(1)_{\text{gauge}} \quad U(1)_{\text{relative}}$$

$$(1,0) = \frac{1}{2}(1,1) + \frac{1}{2}(1,-1) : b_1 + r$$

$$(0,1) = \frac{1}{2}(1,1) - \frac{1}{2}(1,-1) : b_2 - r$$

$\omega_{12} \neq 0$

$(m_1 = m_2, v_1 = v_2)$



Sine-Gordon kink

$$(\Psi_1, \Psi_2) \sim (e^{i\theta_1}, 1) = e^{i\theta_1/2}(1,1)e^{i\theta_1\sigma_3/2}$$

$$(\Psi_1, \Psi_2) \sim (1, e^{i\theta_2}) = e^{i\theta_2/2}(1,1)e^{-i\theta_2\sigma_3/2}$$

$$U(1)_{\text{gauge}} \quad U(1)_{\text{relative}}$$

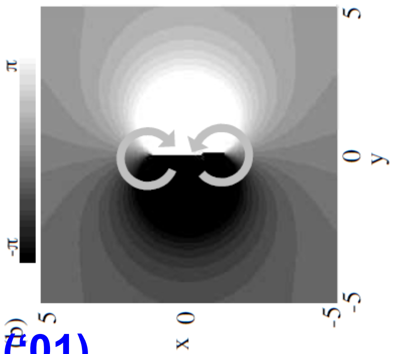
$$(1,0) = \frac{1}{2}(1,1) + \frac{1}{2}(1,-1): \quad b_1 + r$$

$$(0,1) = \frac{1}{2}(1,1) - \frac{1}{2}(1,-1): \quad b_2 - r$$

$$-\omega\left(\psi_1^* \psi_2 + \text{c.c}\right)$$

$$= -2|\psi_1||\psi_2|\omega\cos(\theta_1 - \theta_2)$$

Kasamatsu Tsubota & Ueda, PRL93('04)



Kink: Tanaka('01),
Molecule: Babaev('02), Goryo *et.al* ('07)
for 2 gap superconductors

Along the path r,
$\Delta\theta = \theta_1 - \theta_2$ changes 2π

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Miscible 2 component BEC + Rabi

Gross-Pitaevskii energy functional

$$E[\psi] = \int d^3\mathbf{r} \left\{ \sum_i \left(\frac{\hbar^2}{2m_i} |\nabla \psi_i|^2 + (V_{\text{ext}} - \mu_i)|\psi_i|^2 \right) + \sum_{i,j} \frac{g_{ij}}{2} |\psi_i|^2 |\psi_j|^2 - \sum_{i,j} \omega_{ij} \psi_i^* \psi_j + \text{c.c} \right\}$$

$\neq 0$

internal coherent coupling (Rabi oscillation)

Josephson coupling=supercond

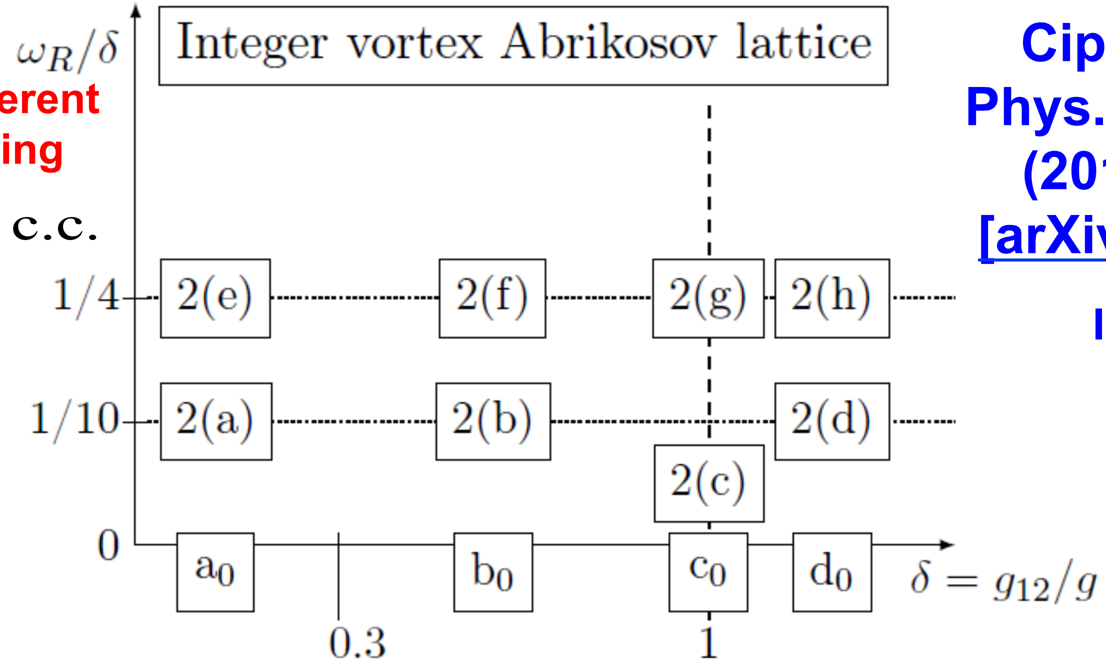
We rotate the system. $\nabla \rightarrow \nabla - i\mathbf{\Omega} \times \mathbf{r}$

We introduce the trapping potential. V_{ext}

Vortex lattices of miscible 2 component BECs with Rabi

Internal coherent (Rabi) coupling

$\omega \Psi_1^* \Psi_2 + \text{c.c.}$

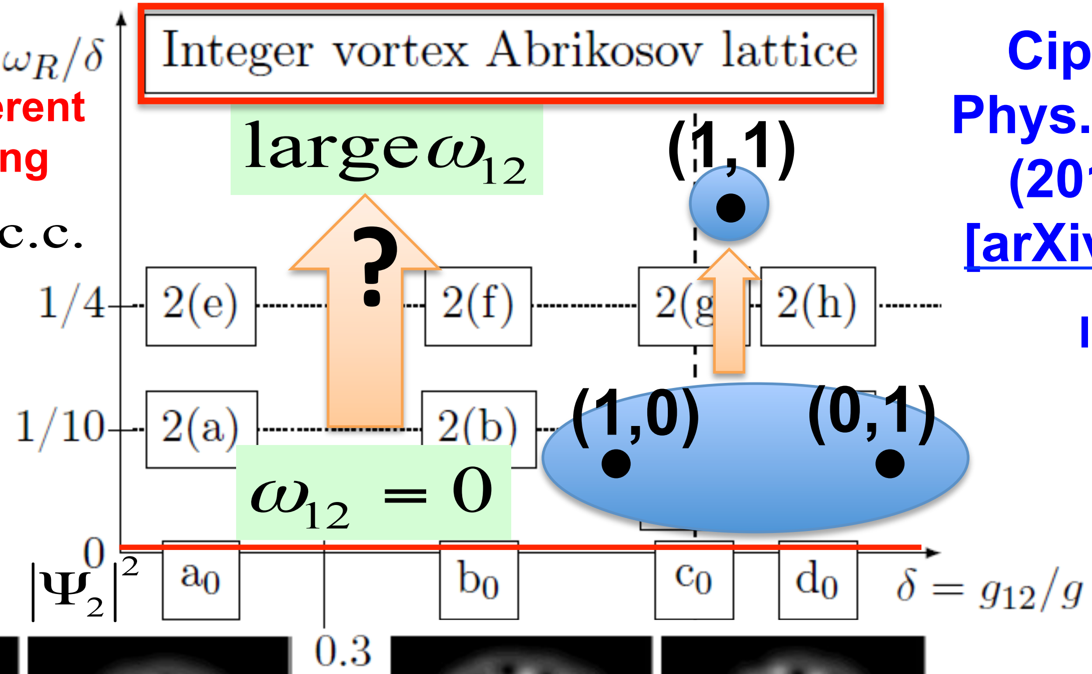


Cipriani & MN Phys.Rev.Lett. 111 (2013) 170401 [arXiv:1303.2592]

Inter-component coupling

$g_{12}|\Psi_1|^2|\Psi_2|^2$

Vortex lattices of miscible 2 component BECs with Rabi



Internal coherent (Rabi) coupling

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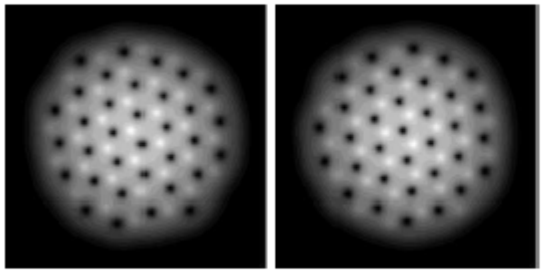
Cipriani & MN Phys.Rev.Lett. 111 (2013) 170401 [arXiv:1303.2592]

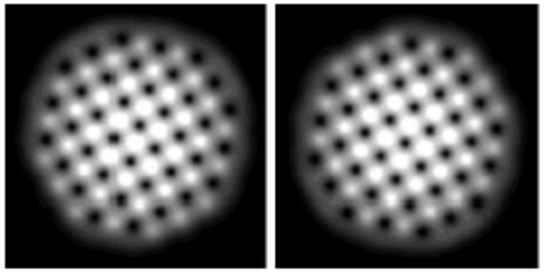
Inter-component coupling

$g_{12}|\Psi_1|^2|\Psi_2|^2$

$|\Psi_1|^2$ $|\Psi_2|^2$



Triangular lattice

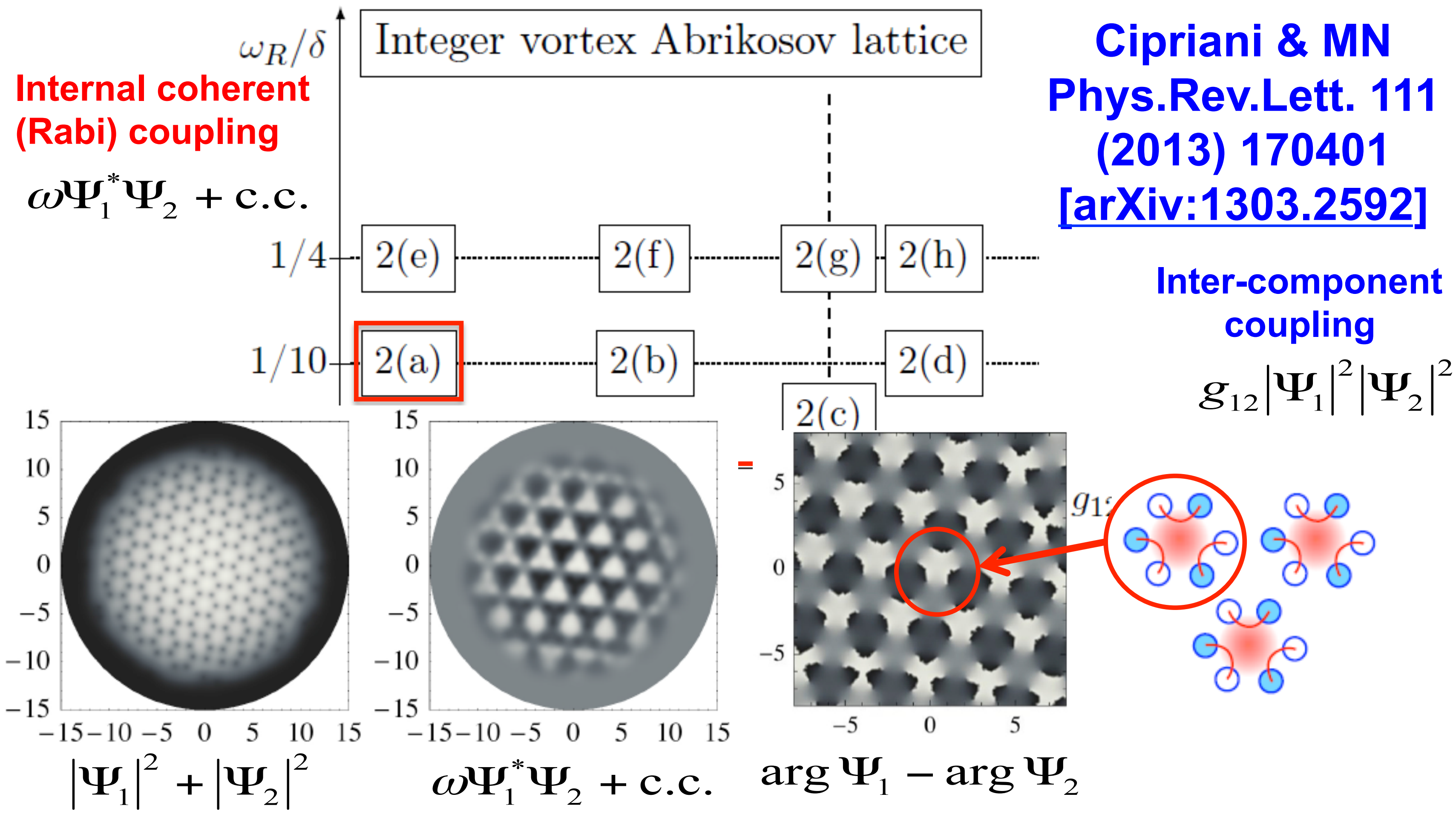


Square lattice

Kasamatsu, Tsubota & Ueda, PRL91 ('03)

Vortex lattices of miscible 2 component BECs with Rabi

Cipriani & MN
Phys.Rev.Lett. 111 (2013) 170401
[arXiv:1303.2592]



Vortex lattices of miscible 2 component BECs with Rabi

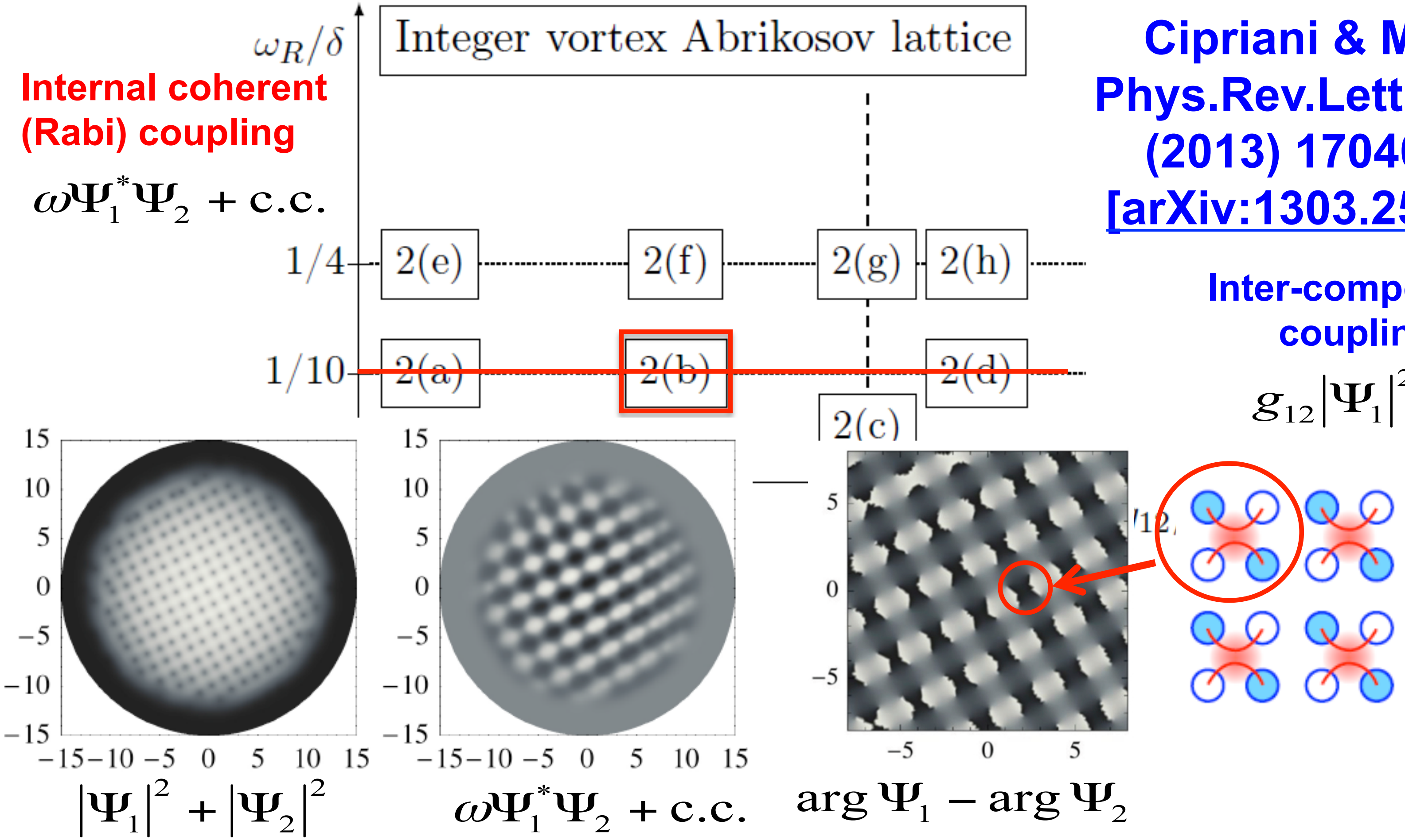
Cipriani & MN
Phys.Rev.Lett. 111 (2013) 170401
[arXiv:1303.2592]

Internal coherent (Rabi) coupling

$\omega \Psi_1^* \Psi_2 + \text{c.c.}$

Inter-component coupling

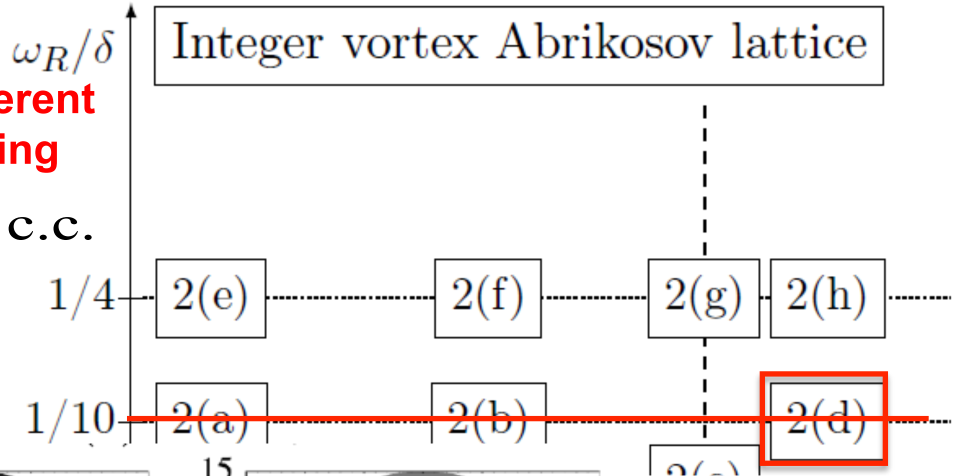
$g_{12} |\Psi_1|^2 |\Psi_2|^2$



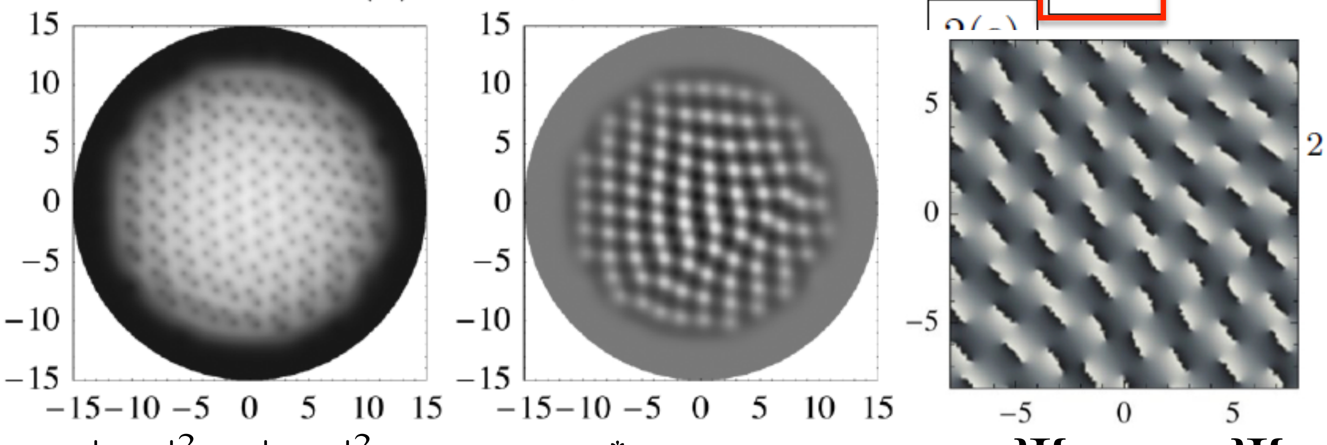
Vortex lattices of miscible **2** component BECs with Rabi

Internal coherent (Rabi) coupling

$\omega \Psi_1^* \Psi_2 + \text{c.c.}$

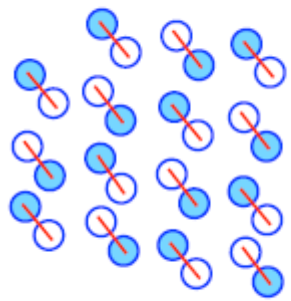


Cipriani & MN
Phys.Rev.Lett. 111 (2013) 170401
[arXiv:1303.2592]

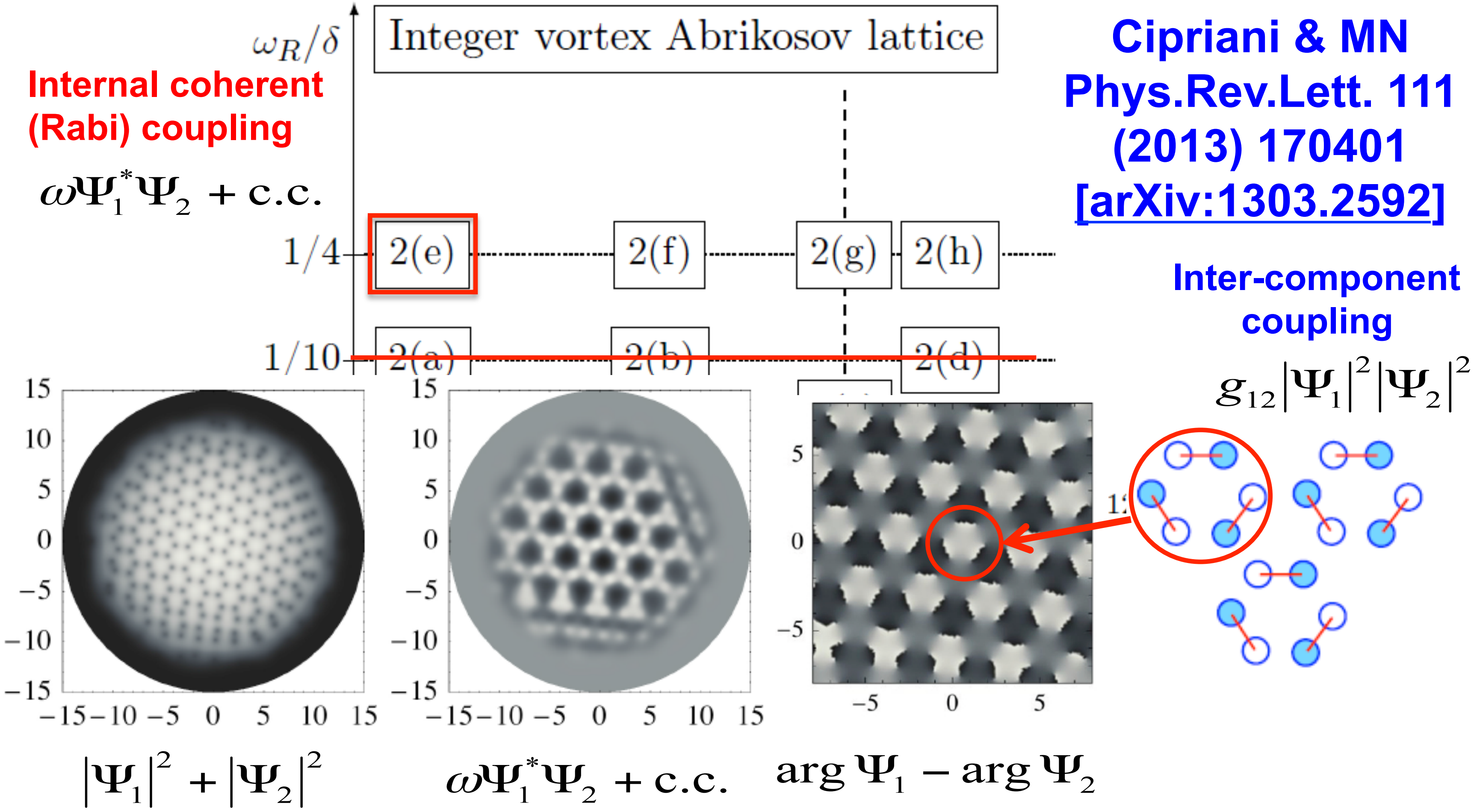


$|\Psi_1|^2 + |\Psi_2|^2$ $\quad \omega \Psi_1^* \Psi_2 + \text{c.c.}$ $\quad \arg \Psi_1 - \arg \Psi_2$

Phase separation

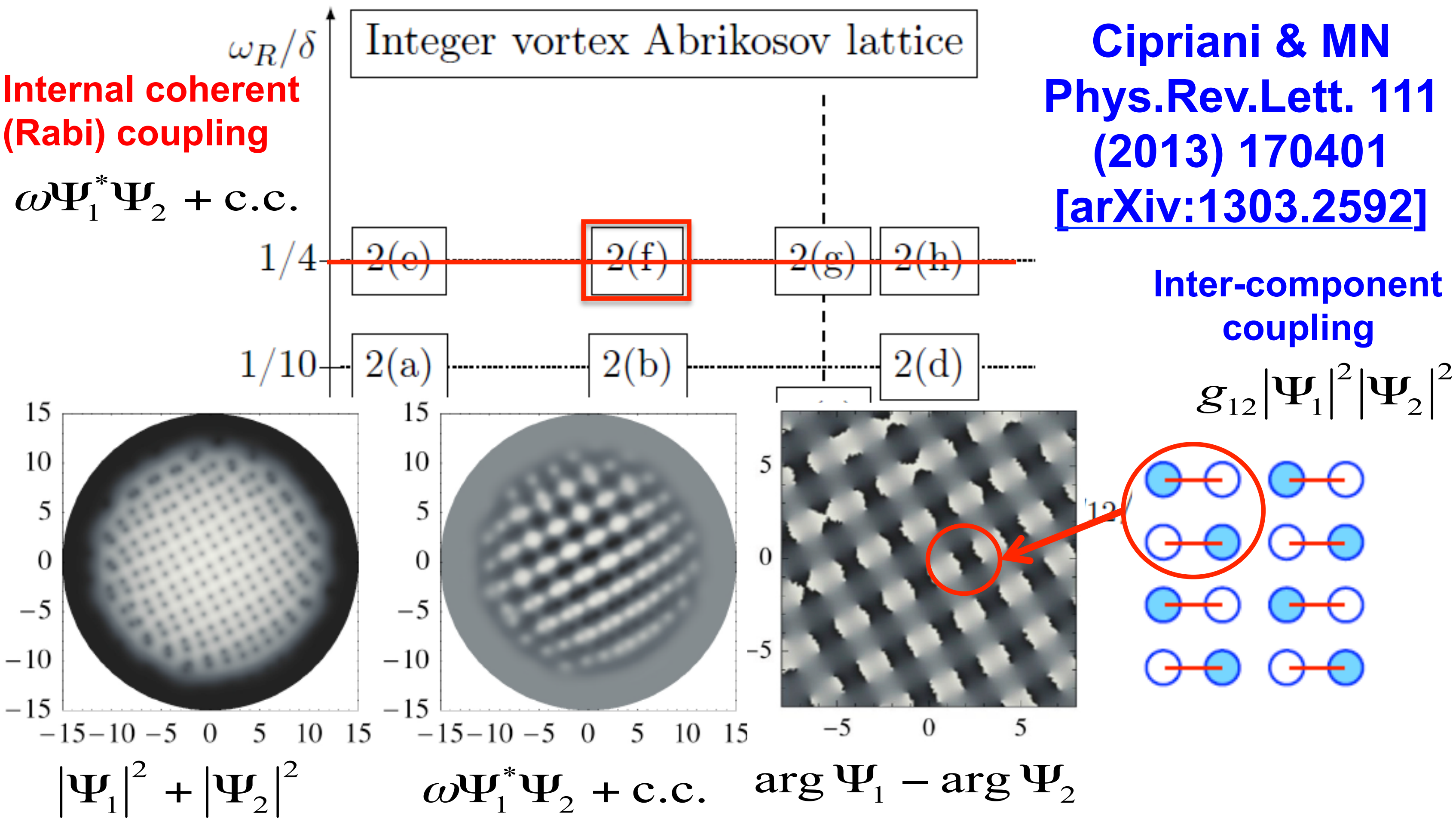


Vortex lattices of miscible 2 component BECs with Rabi

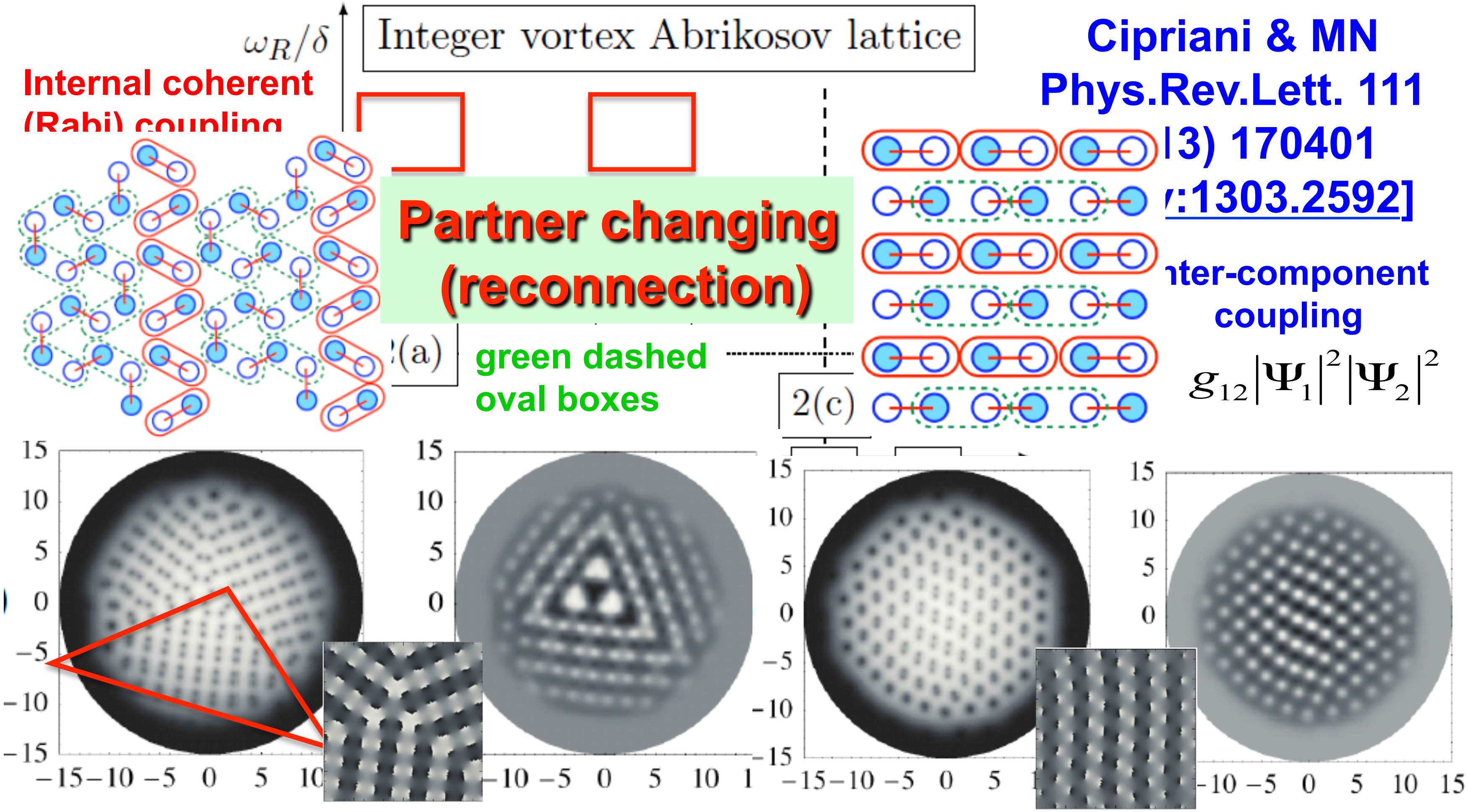


Vortex lattices of miscible 2 component BECs with Rabi

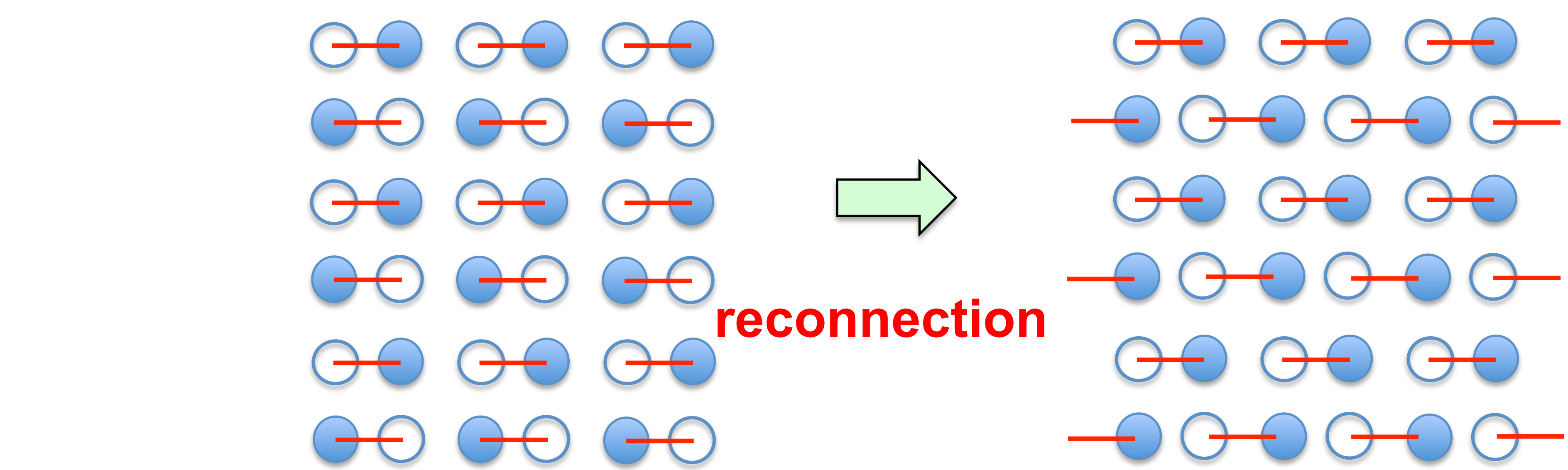
Cipriani & MN Phys.Rev.Lett. 111 (2013) 170401 [arXiv:1303.2592]



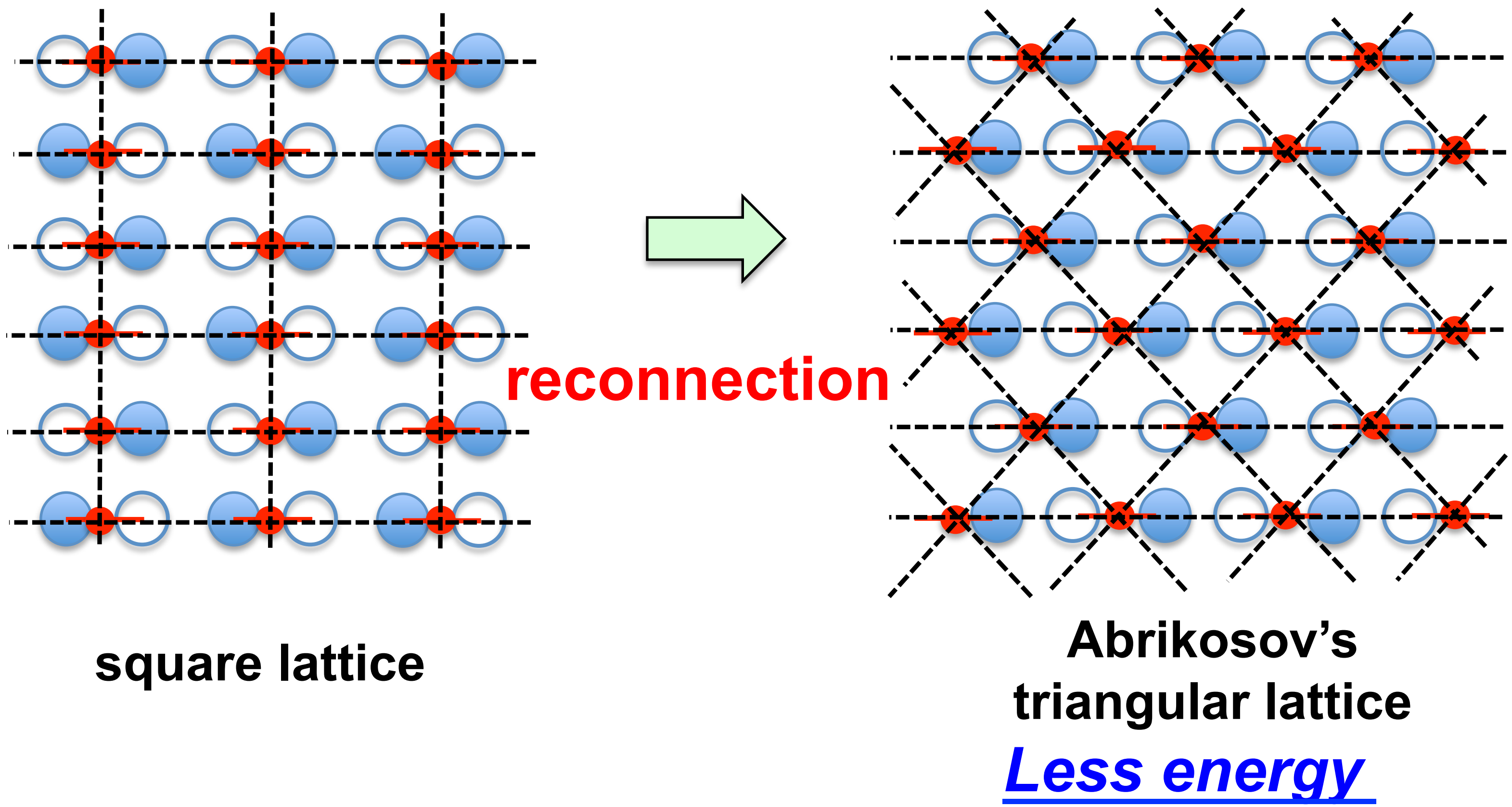
Vortex lattices of miscible 2 component BECs with Rabi

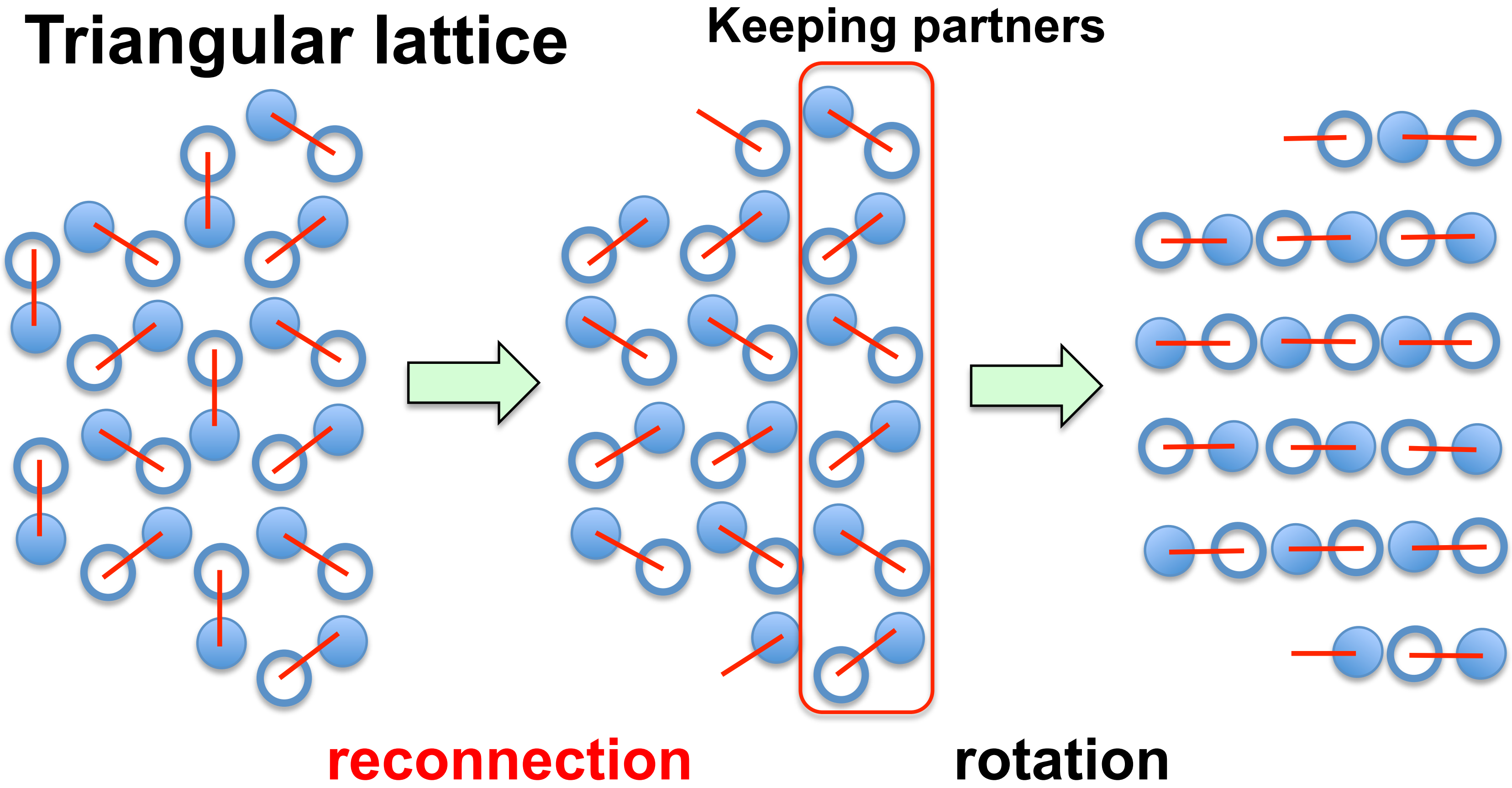


Square lattice

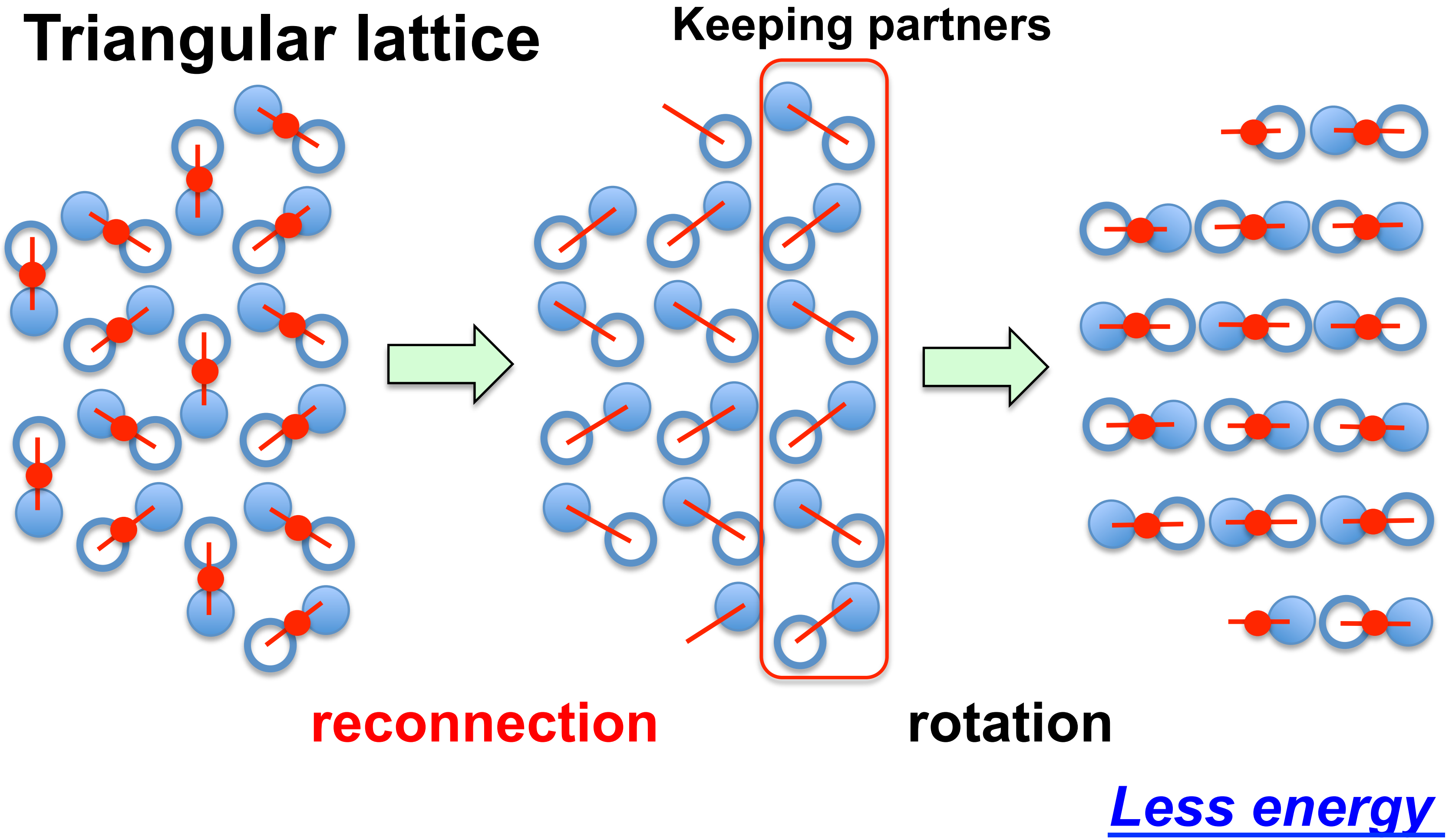


Square lattice





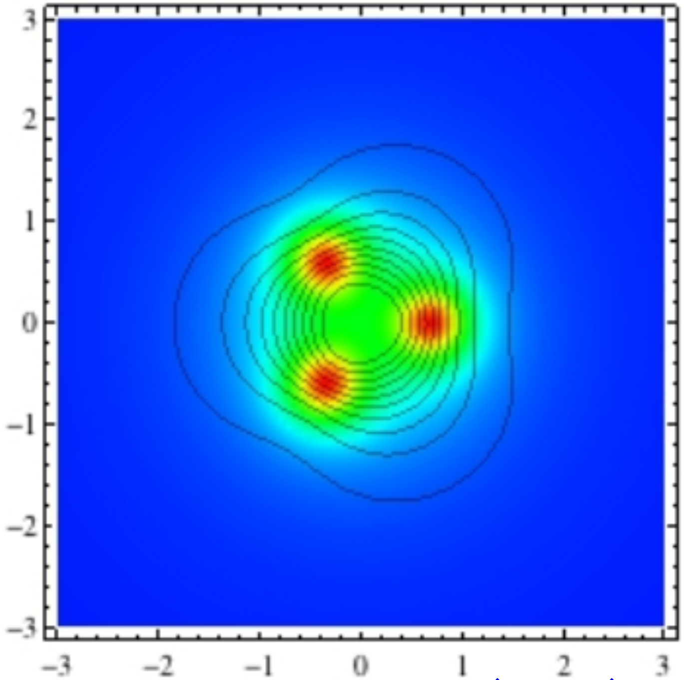
Triangular lattice
Keeping partners
reconnection
rotation


Triangular lattice
Keeping partners
reconnection
rotation
Less energy

BEC **Vortex trimer**

Three component BEC

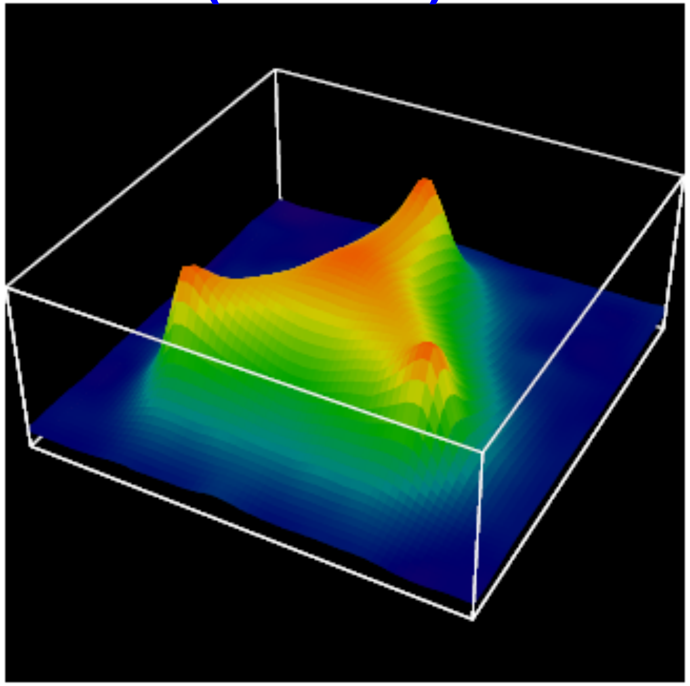
+ internal coherent couplings



Eto-MN, Phys.Rev.A85(2012)053645
[arXiv:1201.0343[cond-mat.quant-gas]]

Baryon = q-q-q QCD

Y-junction of fluxes
(not Δ)



Ichie-Suganuma *et.al* ('03)

4 component BEC

Eto & MN, Europhys.Lett. 103 (2013) 60006
[arXiv:1303.6048 cond-mat.quant-gas]

Gross-Pitaevskii energy functional

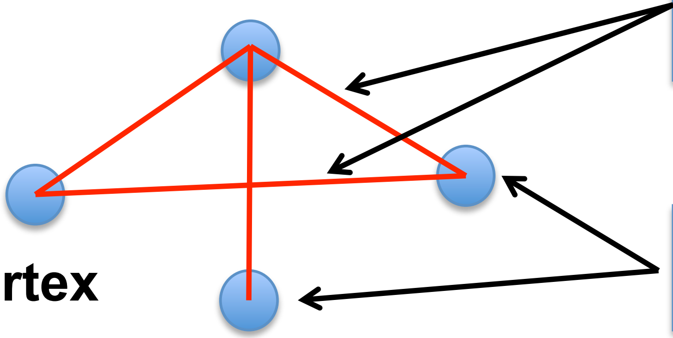
$$E[\psi] = \int d^3\mathbf{r} \left\{ \sum_i \left(\frac{\hbar^2}{2m_i} |\nabla \psi_i|^2 + (V_{\rm ext} - \mu_i)|\psi_i|^2 \right) + \sum_{i,j} \frac{g_{ij}}{2} |\psi_i|^2 |\psi_j|^2 - \sum_{i,j} \omega_{ij} \psi_i^* \psi_j + {\rm c.c} \right\}$$

$V_{\rm ext}$ =0

internal coherent coupling
(Rabi oscillation)

Josephson coupling=supercond

Graph theory (Mathematics)



(3,2,2,1)

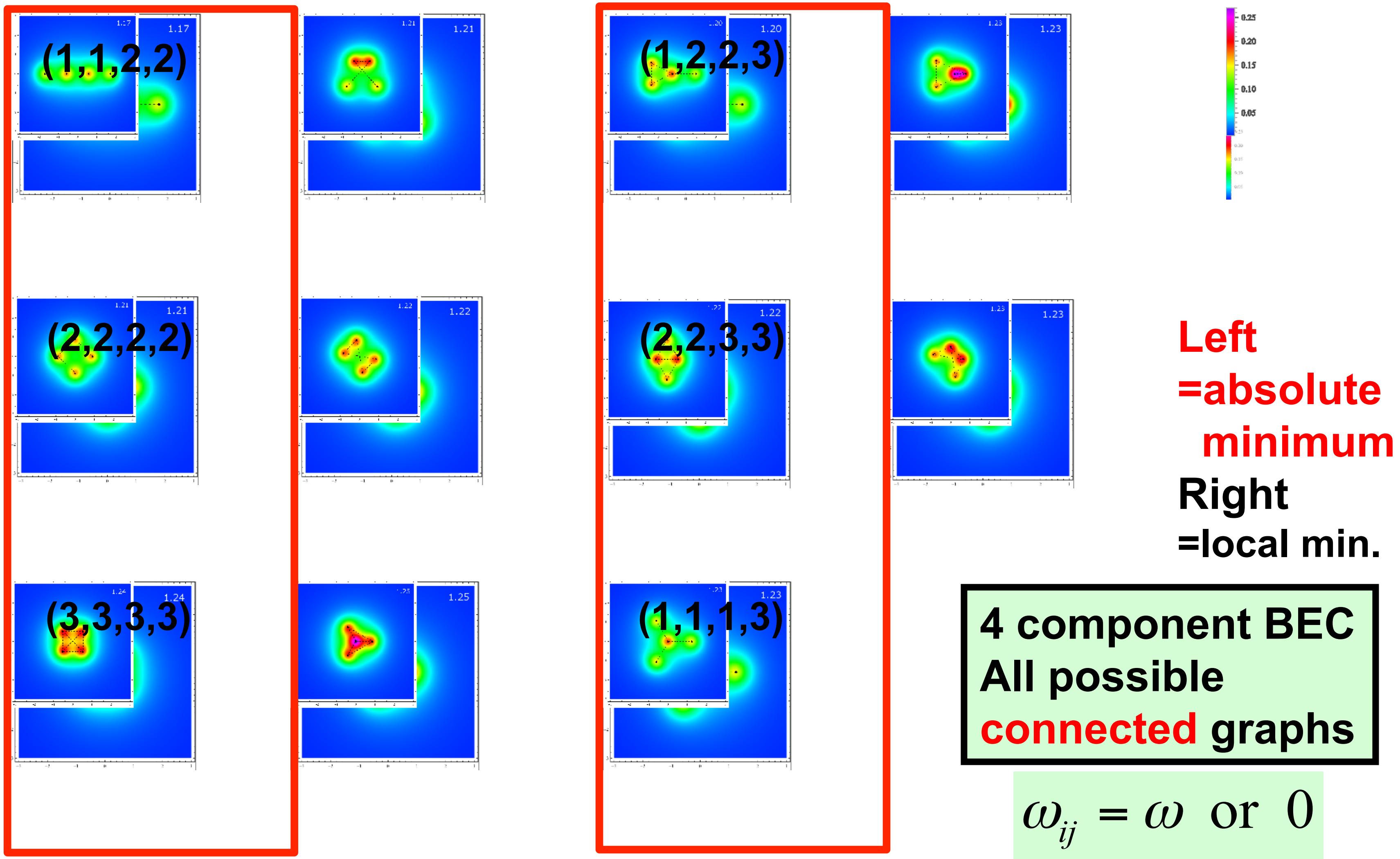
↑

edges
@ each vertex

Edges = coherent couplings

$\omega_{ij} \neq 0$

Vertices = Vortices



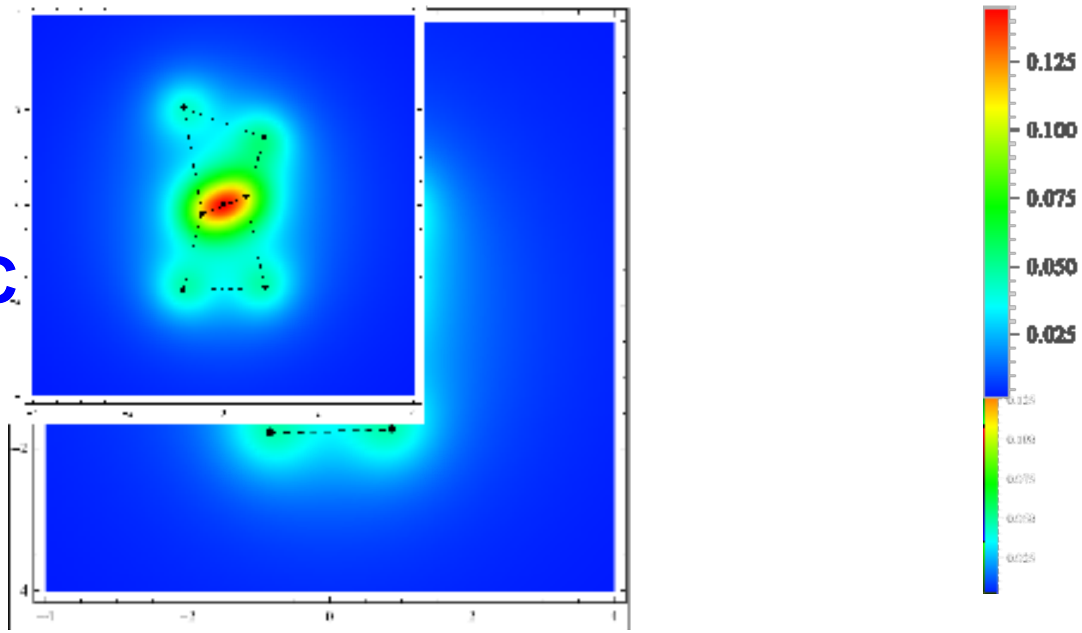
4 component BEC
All possible
connected graphs

$$\omega_{ij} = \omega \text{ or } 0$$

One can manipulate
the **shape** of graphs
as one likes,
by changing ω_{ij} .

Eto & MN, Europhys.Lett. 103 (2013) 60006
[arXiv:1303.6048 cond-mat.quant-gas]

7 component BEC



Plan of My Talk

§1 BEC and vortices

§2 2 comp BECs and vortex dimers

§3 Lattice of vortex dimers

§4 QCD and non-Abelian vortices

§5 Summary

Quantum Chromo Dynamics (QCD)

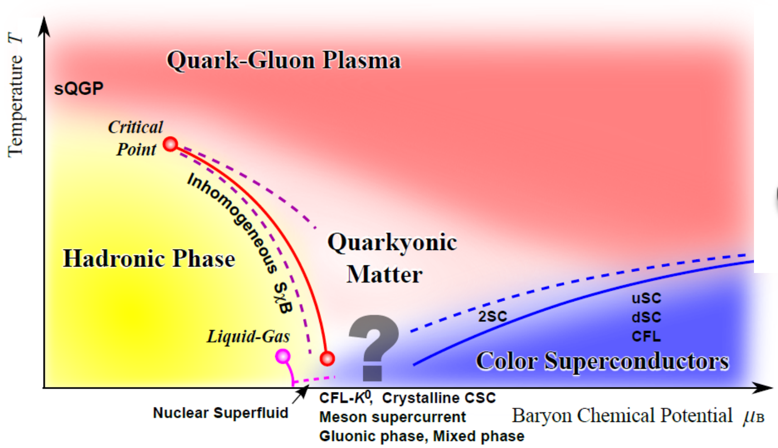
Quark matter

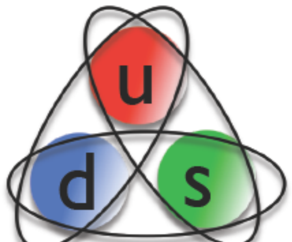
Color-flavor locked (CFL) phase

quarks

q^i_α i = u,d,s flavor(global) SU(3)
α = r,g,b color(gauge) SU(3)

"Color superconductor"





@ high density

Bailin-Love('79),
Iwasaki-Iwado('95)
Alford-Rajagopal-Wilczek('98)

$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ij\,k} q^j_\beta q^k_\gamma \sim \mathbf{1}_{\alpha i}$$

3x3 matrix

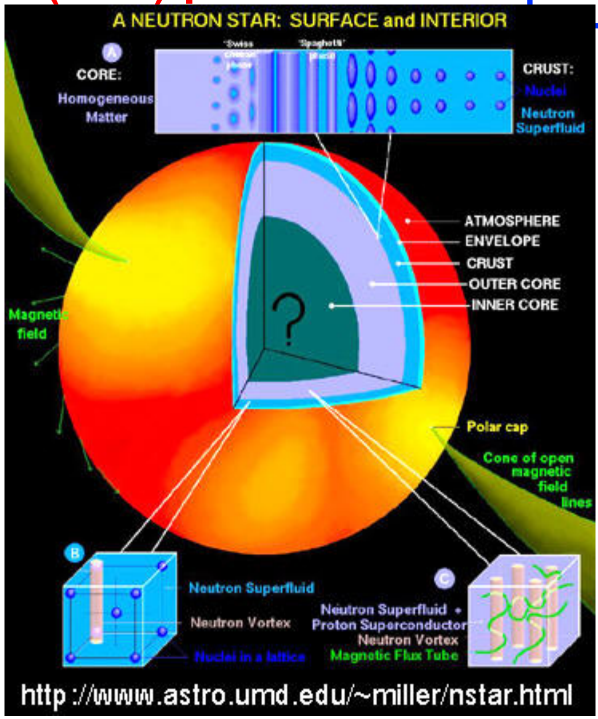
from Fukushima & Hatsuda
Rept.Prog.Phys. 74 (2011) 014001

Color superconductivity
as well as *superfluidity*

Quantum Chromo Dynamics (QCD)

Quark matter

Color-flavor locked (CFL) phase



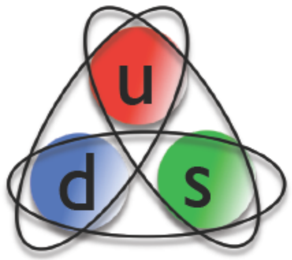
quarks

q_{α}^{i} $\quad i$ = u,d,s flavor(global) SU(3)

α = r,g,b color(gauge) SU(3)

"Color superconductor"

@ high density



Bailin-Love('79),
Iwasaki-Iwado('95)
Alford-Rajagopal-Wilczek('98)

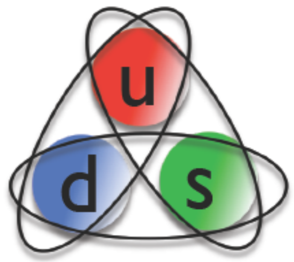
$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_{\beta}^{j} q_{\gamma}^{k} \sim \mathbf{1}_{\alpha i}$$

3x3 matrix

neutron stars

Color superconductivity as well as ***superfluidity***

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^\beta q_k^\gamma$$

$$\alpha = 1,2,3 \ (\mathrm{r,g,b}) \quad i = 1,2,3 \ (\mathrm{u,d,s})$$

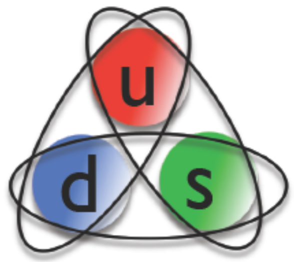
$$\Phi_{\alpha i} = \begin{pmatrix} d_{[g}s_{b]} & s_{[g}u_{b]} & u_{[g}d_{b]} \\ d_{[b}s_{r]} & s_{[b}u_{r]} & u_{[b}d_{r]} \\ d_{[r}s_{g]} & s_{[r}u_{g]} & u_{[r}d_{g]} \end{pmatrix} \begin{matrix} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{matrix}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

$$G = U(1)_{\mathrm{B}} \times SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{F}}$$

$$\Phi_{\alpha i} \to e^{i\alpha} g_{\mathrm{color}} \Phi_{\alpha i} g_{\mathrm{flavor}}$$

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^\beta q_k^\gamma$$

$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

Ground state

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{pmatrix} \begin{matrix} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{matrix}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

color-flavor locked (CFL)

$$G = U(1)_\mathrm{B} \times SU(3)_\mathrm{C} \times SU(3)_\mathrm{F}$$

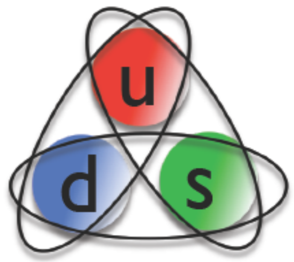
$$\rightarrow H = SU(3)_{\mathrm{C+F}} \quad g_{\mathrm{color}} = g_{\mathrm{flavor}}^{-1}$$

~~$U(1)_\mathrm{B}$~~ superfluidity

~~$SU(3)_\mathrm{C}$~~ color superconductivity

Color superconductor

$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^\beta q_k^\gamma$$



Integer quantized superfluid vortex
(Abelian)

$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

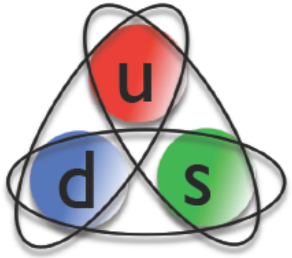
$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix} \begin{matrix} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{matrix}$$

$$\bar{u} = ds \qquad \bar{d} = sb \qquad \bar{s} = ud$$

Iida & Baym, Forbes & Zhitnitsky('02)

Color superconductor

$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^\beta q_k^\gamma$$



$$\alpha = 1,2,3 \ (\mathrm{r,g,b}) \quad i = 1,2,3 \ (\mathrm{u,d,s})$$

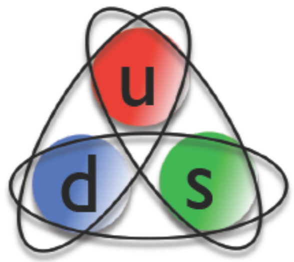
1/3 quantized vortex

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix} \begin{matrix} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{matrix}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

Balachandran, Digal & Matsuura (BDM) ('05)
Nakano, MN & Matsuura ('07), Eto & MN ('09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$= \exp\left(\frac{i\theta}{3}\right)\exp\frac{i\theta}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} \Delta_1(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

1/3 quantized

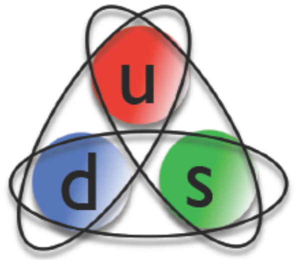
SU(3) color gauge tr

→ **color flux tube**

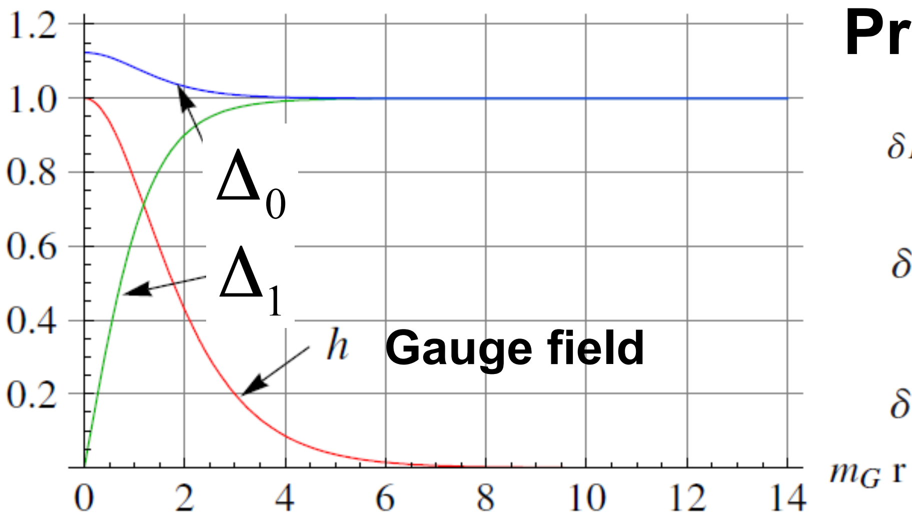
Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$



Profiles

$F \equiv \Delta_1 + 2\Delta_0,$ **trace**

$G \equiv \Delta_1 - \Delta_0,$ **traceless**

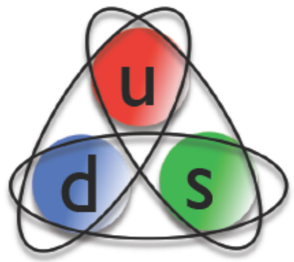
$$\delta F = q_\phi \sqrt{\frac{\pi}{2m_\phi r}} e^{-m_\phi r} + \left(-\frac{1}{3m_\phi^2 r^2} + \mathcal{O}\left(\frac{1}{(m_\phi r)^4}\right)\right)$$

$$\delta G \simeq q_\chi K_{1/3}(m_\chi r) \simeq q_\chi \sqrt{\frac{\pi}{2m_\chi r}} e^{-m_\chi r},$$

$$\delta h \simeq q_G m_G r K_1(m_G r) \simeq q_G \sqrt{\frac{\pi m_G r}{2}} e^{-m_G r}$$

Eto & MN ('09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$= \exp\left(\frac{i\theta}{3}\right) \exp\frac{i\theta}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

1/3 quantized

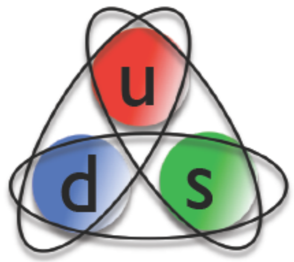
SU(3) color gauge tr

→ color flux tube

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix}$$

$$= \exp\left(\frac{i\theta}{3}\right) \exp\frac{i\theta}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) \end{pmatrix}$$

1/3 quantized

SU(3) color gauge tr

→ color flux tube

Superfluid vortex

Non-Abelian vortex

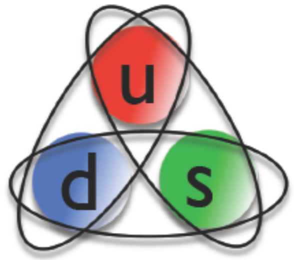
Non-Abelian vortices

Color Fluxes

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$



Abelian vortex

No flux

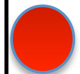
$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

Which are energetically favored?

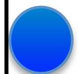
Non-Abelian vortices

Color Fluxes

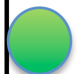
$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

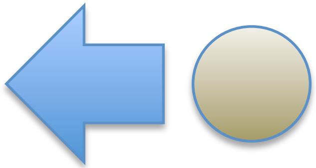


$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

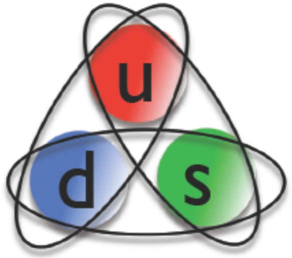


$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$





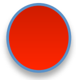
Split

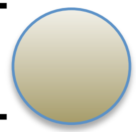


Abelian vortex

No flux

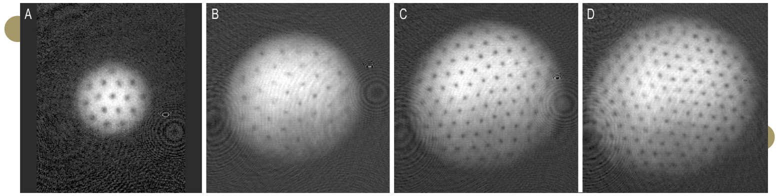
$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

$E[$  $]$

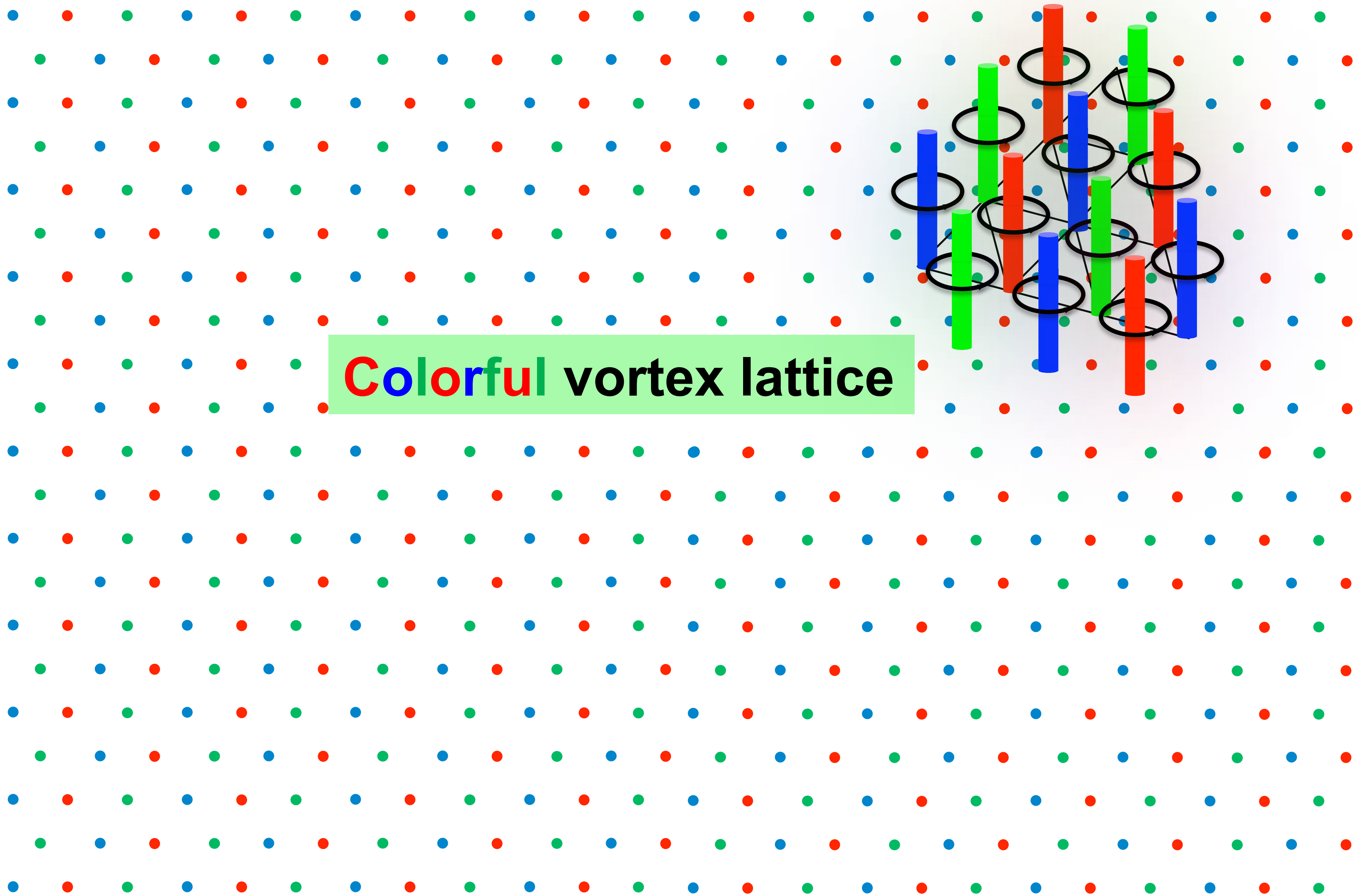
$= E[$  $] = \frac{1}{9} E[$  $]$

$= E[$  $]$

Nakano, MN & Matsuura ('07)



Abrikosov vortex lattice


Colorful vortex lattice

Colorful vortex lattice

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$H = SU(3)_{\mathrm{C+F}}$$
$$\downarrow$$
$$K = [SU(2) \times U(1)]_{\mathrm{C+F}}$$

@ vortex core

Nambu-Goldstone modes localized around the vortex

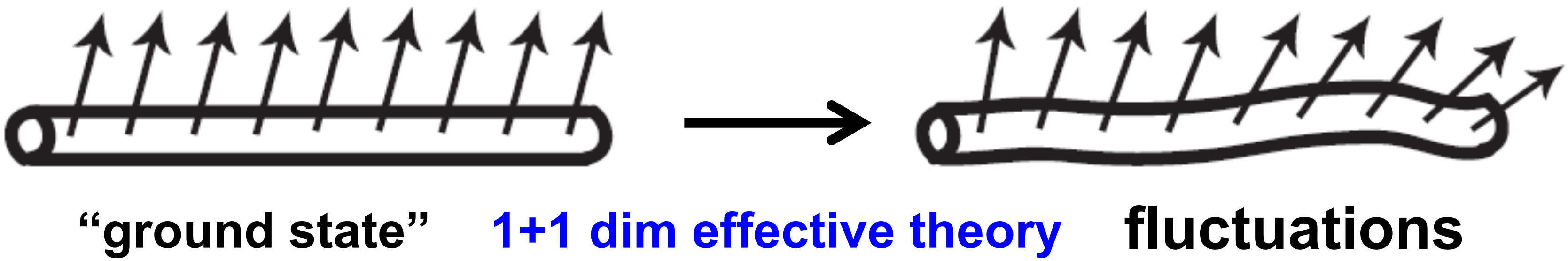
$$\mathbf{C} \times \frac{H}{K} = \mathbf{C} \times \frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbf{C} \times \mathbf{C}P^2$$

Kelvon | Color magnon

Continuous family of solutions exists

Eto, Nakano & MN ('09)

= Gapless modes propagating along the vortex line



$$\Phi_{\alpha i} = \left(\begin{array}{c|cc} \Delta_1(r)e^{i\theta} & 0 & 0 \\ \hline 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{array}\right)$$

$$H = SU(3)_{\mathrm{C+F}}$$
$$\downarrow$$
$$K = [SU(2) \times U(1)]_{\mathrm{C+F}}$$

@ vortex core

Nambu-Goldstone modes localized around the vortex

$$\mathbf{C} \times \frac{H}{K} = \mathbf{C} \times \frac{SU(3)_{\mathrm{C+F}}}{SU(2) \times U(1)} = \mathbf{C} \times \mathbf{C}P^2$$

Kelvon: Type-II; Color magnon: Type-I

Continuous family of solutions exists

Eto, Nakano & MN('09)

= Gapless modes propagating along the vortex line

$$L_{\mathrm{Kelvon}} = X\dot{Y} - Y\dot{X} - T\left(X'^2 + Y'^2\right)$$ Type-II

$$\phi = \left(\phi^1, \phi^2, \phi^3\right) \mathbf{C}^3$$

$$\mathcal{L}_{\mathbb{C}P^2} = C \sum_{\alpha=0,3} K_\alpha[\partial^\alpha \phi^\dagger \partial_\alpha \phi + (\phi^\dagger \partial^\alpha \phi)(\phi^\dagger \partial_\alpha \phi)],$$ Type-I

homogenous coordinates $\phi^\dagger \phi = 1$

Interaction between vortices

Long-range vortex-interaction by exchanging **phonons**

$$E_{\rm int} = -4\pi v^2 \log R$$

Nakano,MN &Matsuua ('07)

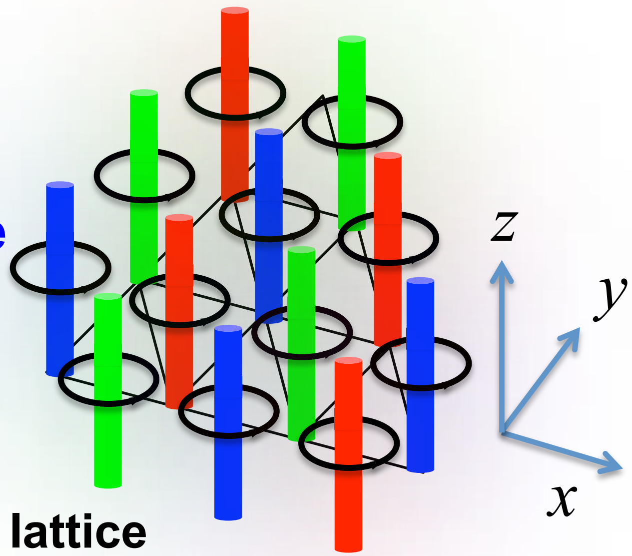
$$F = -\frac{\partial E_{\rm int}}{\partial R} = \frac{4\pi v^2}{R}$$

Same with superfluid vortices

This does not see colors

Positions are locked as Abrikosov's triangular lattice

cf) lattice oscillation = Tkachenko mode

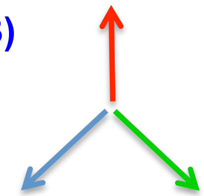


CP^2 "color" spin on a triangular lattice

cf) CP^1=S^2: Heisenberg spin on a triangular lattice

Short-range vortex-interaction

Eto,Hirono,Yasui & MN ('13)



by exchanging **gluons** (massive gauge)

$$E_{\text{int,gluon}} \propto G(\phi_1,\phi_2)\exp(-m_g R)$$

repulsion among colors

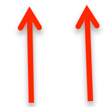
$$G(\phi_1,\phi_2) \equiv \phi_1^\dagger T^a \phi_1 \ \phi_2^\dagger T^a \phi_2$$

T^a SU(3)

Short-range vortex-interaction

by exchanging **adj scalar** (gap)

Auzzi-Eto-Vinci('07)

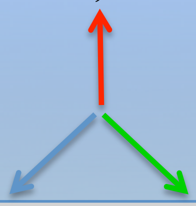


$$E_{\text{int,adj}} \propto -G(\phi_1,\phi_2)\exp(-m_{\text{adj}} R)$$

attraction among colors

Kobayashi,Nakano & MN,
arXiv:1311.2399

$$E_{\text{int,total}} = E_{\text{int,gluon}} + E_{\text{int,adj}}$$

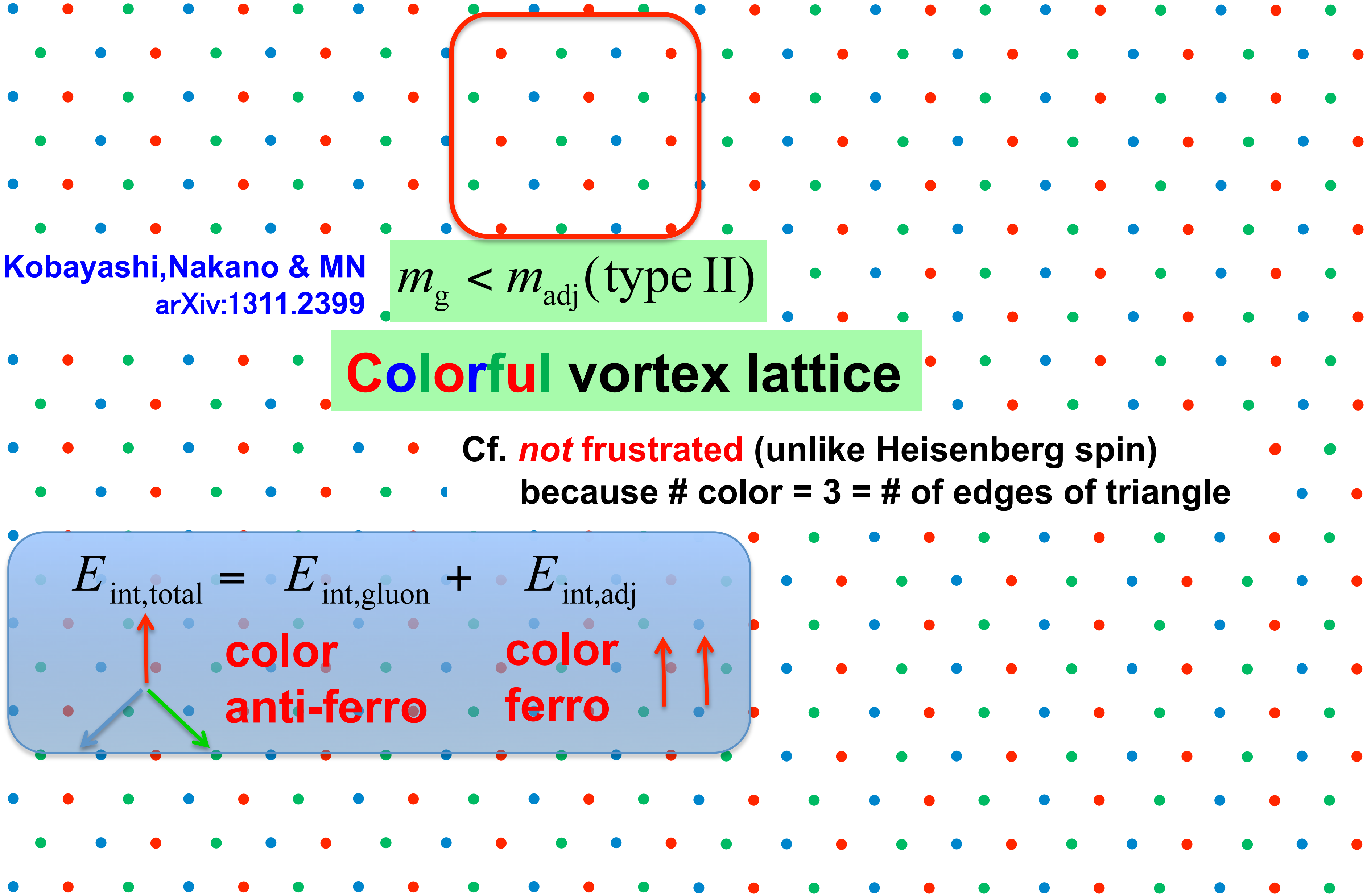


color
anti-ferro

color
ferro

$$H = \int dz \sum_{\langle i,j\rangle,A} \Big[-J_{xy} S_{i,A} S_{j,A} + K_3 \{ |\partial_z \phi_i|^2 + (\phi_i^\dagger \partial_z \phi_i)^2 \} \Big]$$

$$S_{i,A} := \phi_i^\dagger T_A \phi_i, \quad J_{xy} := \Delta^2 G(L)$$


Kobayashi,Nakano & MN
arXiv:1311.2399
$m_g < m_{adj}$ (type II)
Colorful vortex lattice
Cf. not frustrated (unlike Heisenberg spin)
because # color = 3 = # of edges of triangle
$E_{int,total} = E_{int,gluon} + E_{int,adj}$
color anti-ferro
color ferro

$$E_{\text{int,total}} = E_{\text{int,gluon}} + E_{\text{int,adj}}$$

color anti-ferro

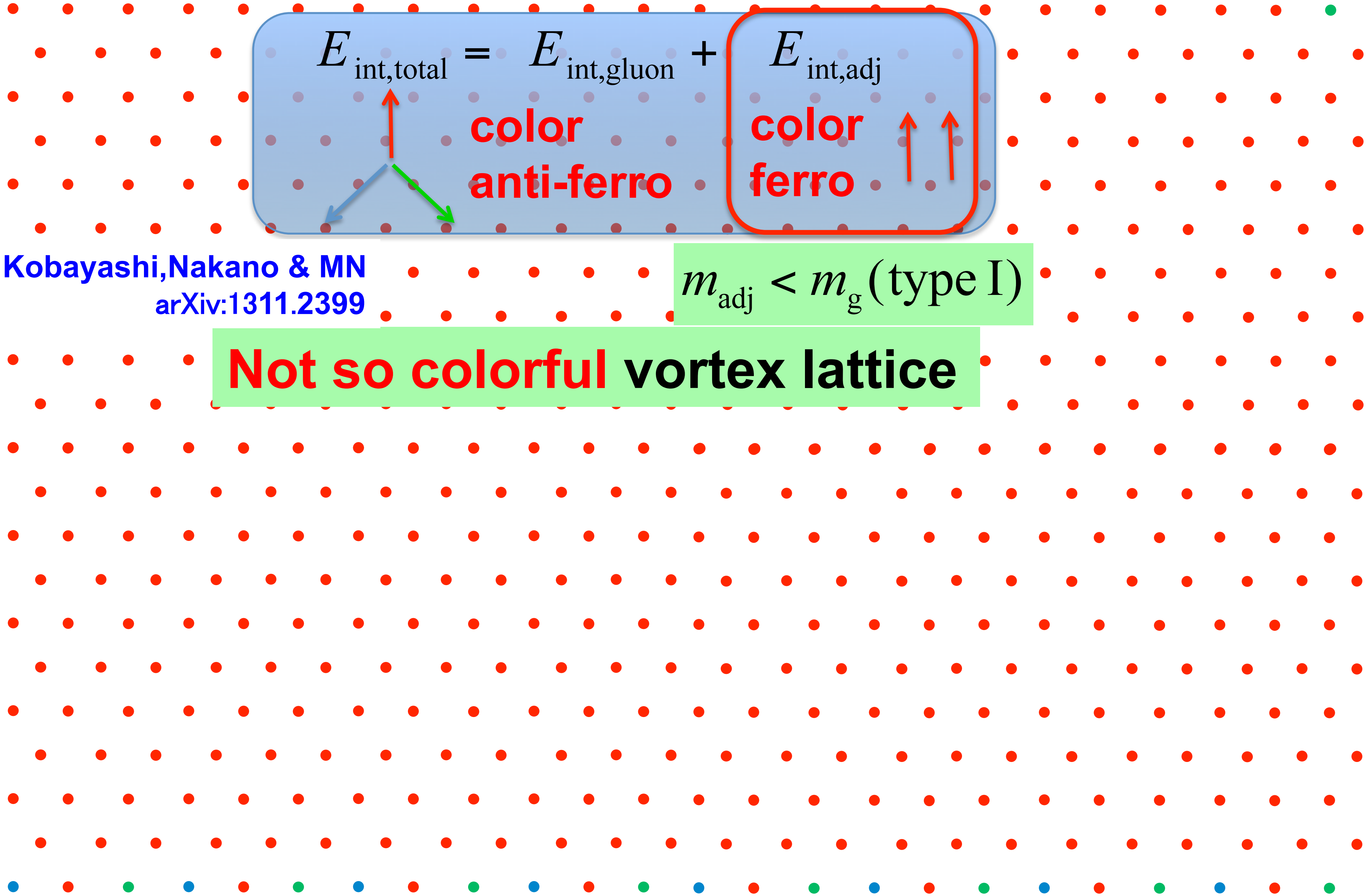
color ferro

Kobayashi, Nakano & MN

arXiv:1311.2399

$m_{\text{adj}} < m_{\text{g}}$ (type I)

Not so colorful vortex lattice



$$E_{\text{int,total}} = E_{\text{int,gluon}} + E_{\text{int,adj}}$$

color anti-ferro

color ferro

Kobayashi,Nakano & MN
arXiv:1311.2399

$m_{\text{adj}} < m_{\text{g}}$ (type I)

Not so colorful vortex lattice

Dense QCD $m_{\text{adj}} << m_{\text{g}}$ **= a color ferromagnet**

$$H = \int dz \sum_{\langle i,j\rangle,A} \Big[-J_{xy} S_{i,A} S_{j,A} + K_3 \{ |\partial_z \phi_i|^2 + (\phi_i^\dagger \partial_z \phi_i)^2 \} \Big]$$

$$S_{i,A} := \phi_i^\dagger T_A \phi_i, \quad J_{xy} := \Delta^2 G(L)$$

→ **Continuum limit of a lattice**

$$\mathcal{L}_{\text{eff}} = \sum_{\mu=0}^{3} \tilde{K}_\mu \left[\partial^\mu \phi^\dagger \partial_\mu \phi + (\phi^\dagger \partial^\mu \phi)(\phi^\dagger \partial_\mu \phi) \right]$$

Anisotropic CP^2 model

Order-disorder transition temp $T_c^{\text{order}} \sim \dfrac{J_{xy}}{k_{\max}} + K_3 k_{\max}$

Plan of My Talk

§1 BEC and vortices

§2 2 comp BECs and vortex dimers

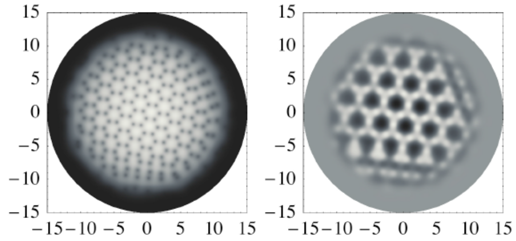
§3 Lattice of vortex dimers

§4 QCD and non-Abelian vortices

§5 Summary

Summary

Vortex Lattices under rotation



BEC: Lattices of vortex molecules

triangular / square lattices,

Rabi (Josephson) interaction, reconnection

QCD: Lattices of non-Abelian vortices

superfluid vortex: Abrikosov's triangular lattice

color flux tube

(i) type-I color super $m_{\rm adj} < m_{\rm g}$

color ferro = not so colorful lattice

(ii) type-II color super $m_{\rm g} < m_{\rm adj}$

color anti-ferro = colorful lattice