相関量子系のダイナミクス:
電子系と large *N*超対称QCDの比較

Takashi Oka (U-Tokyo)

correlated electron system

N. Tsuji (Tokyo-U)
P. Werner (Fribourg)
M. Eckstein (Humburg)

Bethe ansatz: Oka PRB `12
Numerical:
Oka '03~, Aoki Tsuji, .. RMP `14 to appear

quantum spin

S. Takayoshi (NIMS)
M. Sato (Aoyama)

topological system

T. Kitagawa (Rakuten)
T. Mikami (U-Tokyo)

string theory/ hadron physics

K. Hashimoto (RIKEN iTHES, Osaka-U)
A. Sonoda (Osaka-U), N. Iizuka (RIKEN)

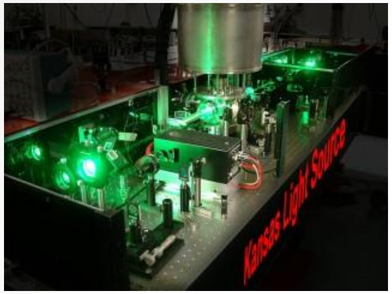
Gauge/gravity duality
Hashimoto Iizuka Oka PRD '11
Hashimoto Oka JHEP '13

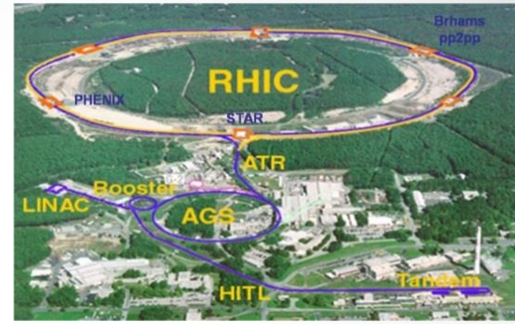
Planckian thermalization (scale invariant sys.)

$$\tau_{\rm th} = a\frac{\hbar}{k_B T_{\rm eff}}$$

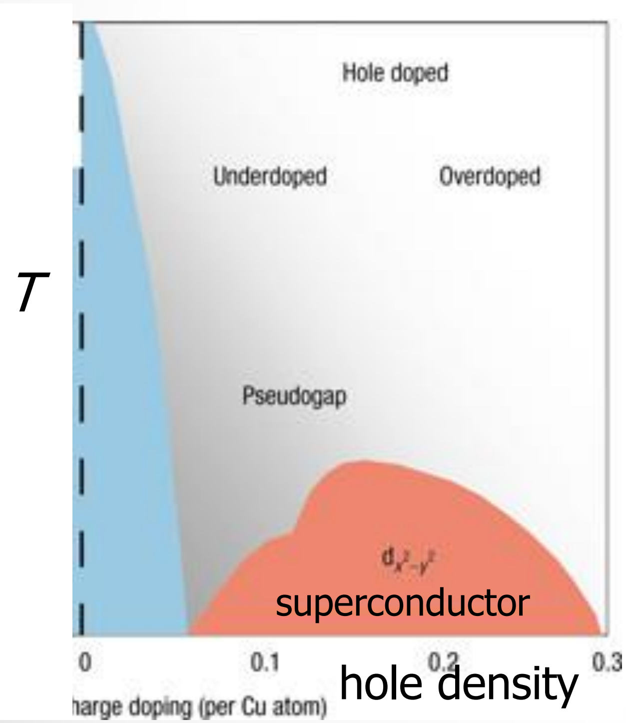
thermalization v.s. hydrodynamic regime

Strong field physics in Condensed matter and Nuclear physics

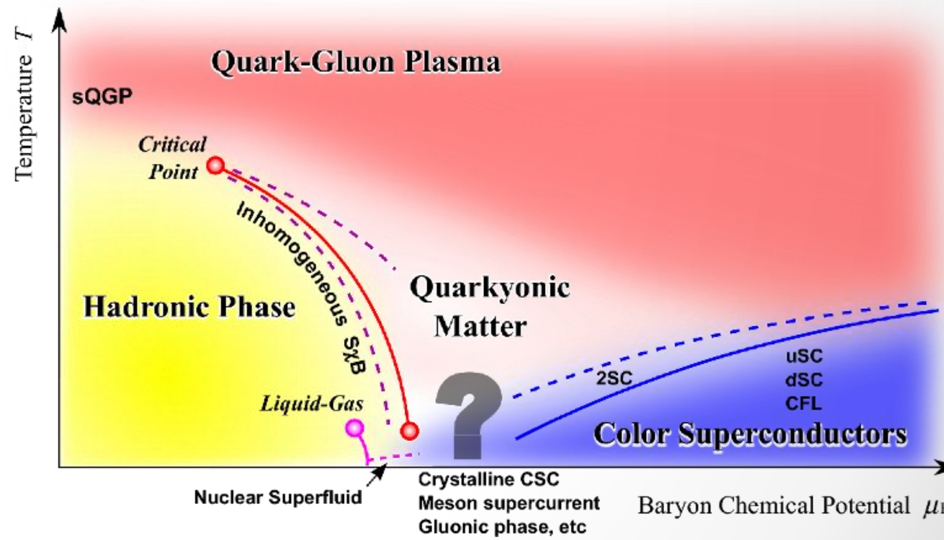




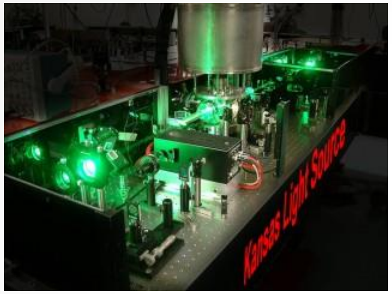
phase diagram of Hi Tc

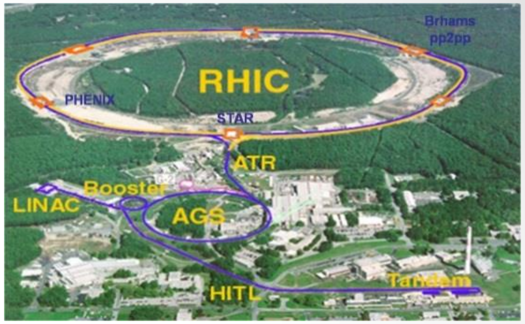


phase diagram of hadron
(Fukushima-Hatsuda)



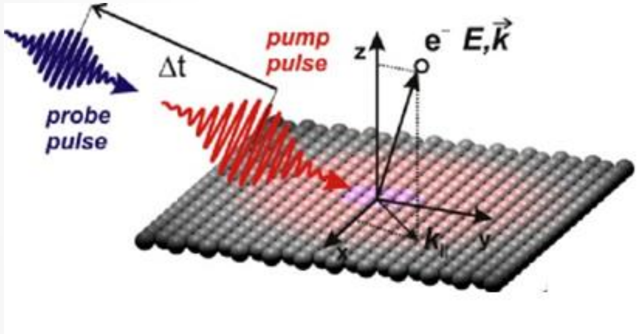
Strong field physics in Condensed matter and Nuclear physics

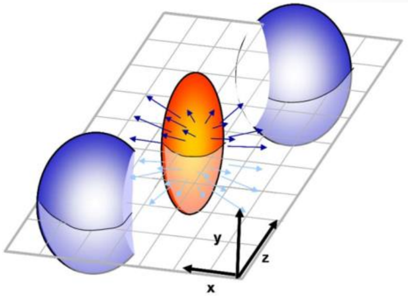




pump probe exp.

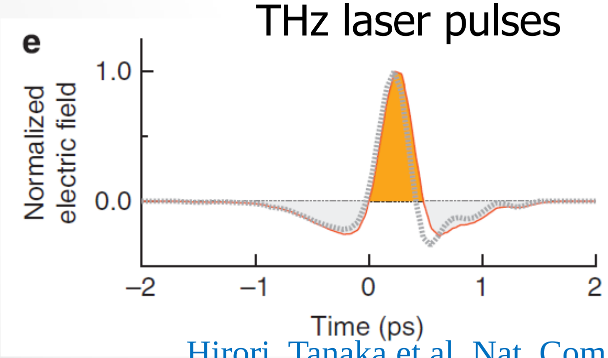
ion collision



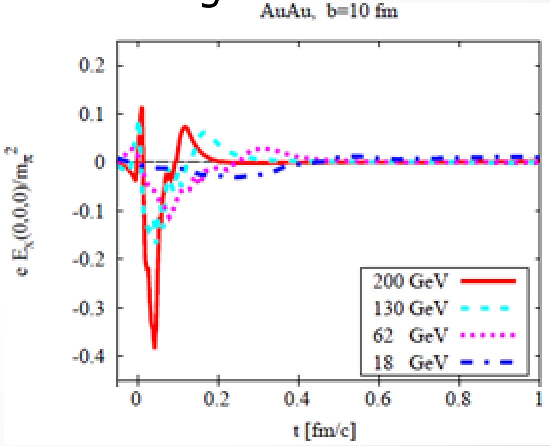


THz laser pulses

strong E and B fields

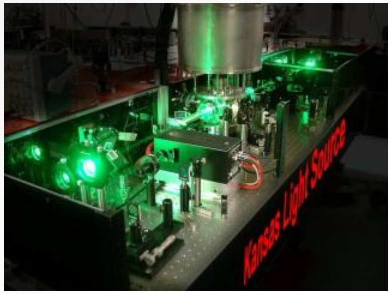


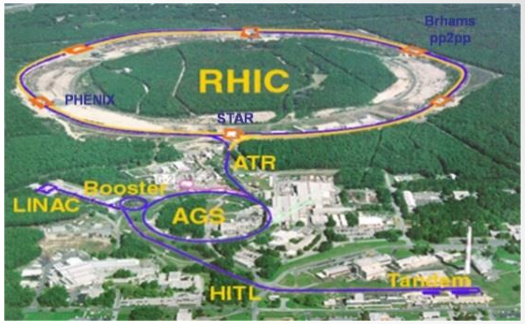
Hirori, Tanaka et al. Nat. Com. 2011



Takashi Oka

Strong field physics in Condensed matter and Nuclear physics





Interesting problems

Laser induced metallization

Quark gluon plasma (QGP)

Excitation

- doublon-hole production
- lattice distortion/phonon oscillation

- Schwinger mechanism
  (quark-antiquark pair production)
- Deconfinement transition of gluons

Relaxation

Gapped: exponentionally slow relaxation

$$\tau_{\rm relax} \sim e^{\alpha \Delta_{\rm gap}/W}$$

Gapless: fast relaxation

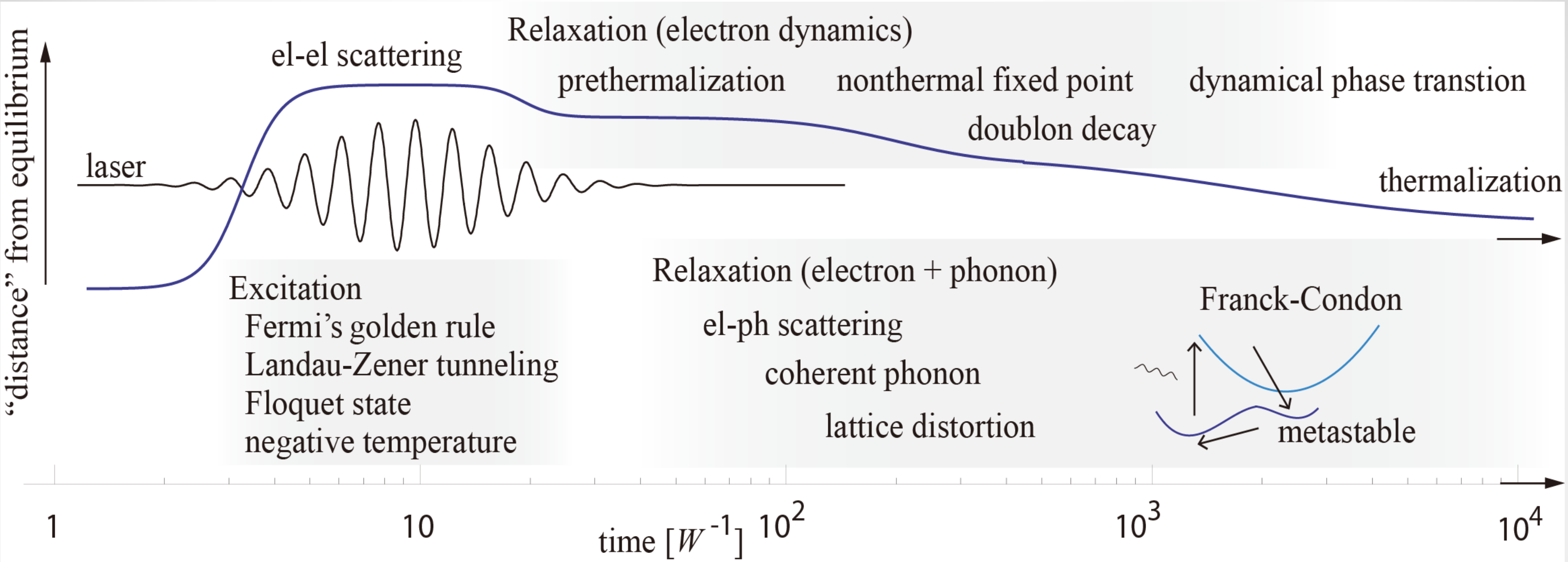
$$\tau_{\rm relax} \sim 1/{\rm Pol.}(W_1, W_2, \ldots)$$

W_i: parameters
(coupling const., temperature,..)

Above threshold: fast thermalization

→ hydrodynamic system

"temperature", "current",…



Aoki, Tsuji, Eckstein, Kollar, Oka, Werner RMP to appear

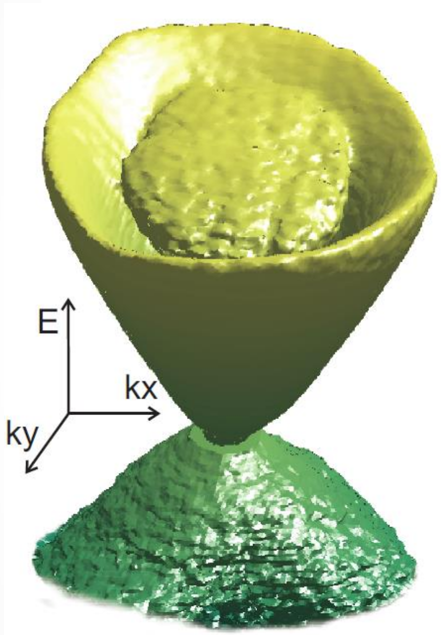
「強相関系の非平衡物理」日本物理学会誌，２０１２年４月号

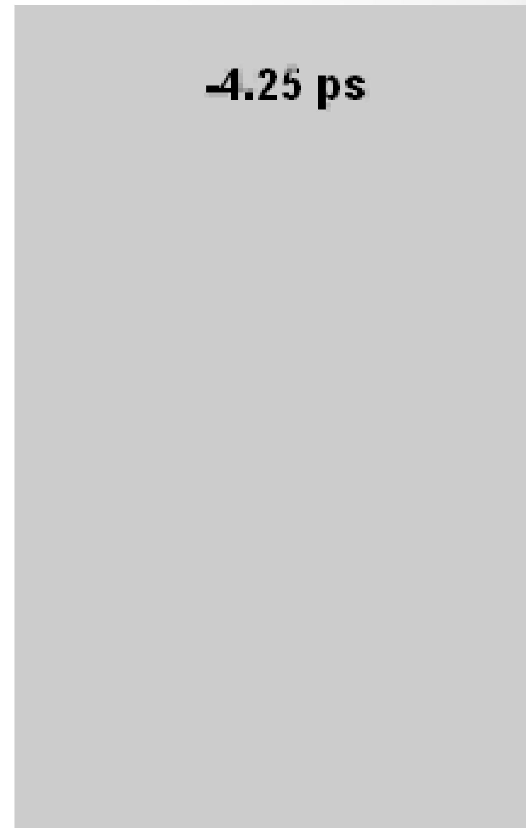
Pump-probe technique

Time resolved ARPES (angle resolved photo emission spectroscopy)

data from N. Gedik (MIT)

$I(\mathrm{E}, \mathrm{k}_x, \mathrm{k}_{y,}\ \mathrm{t}<0)$



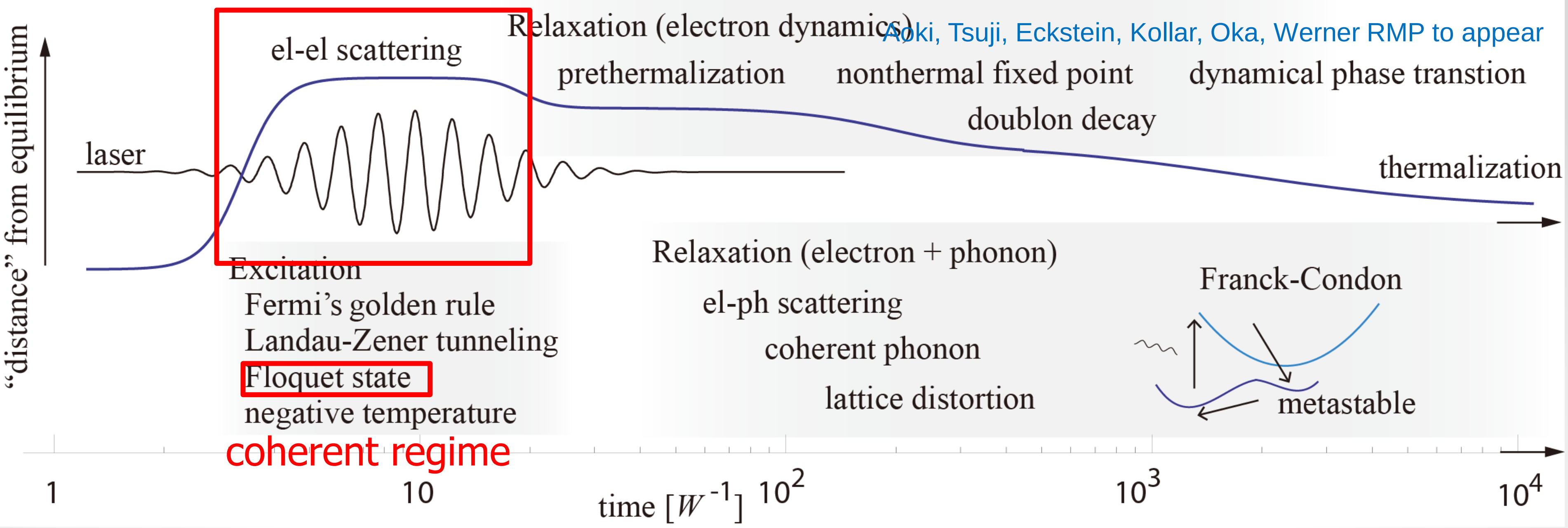


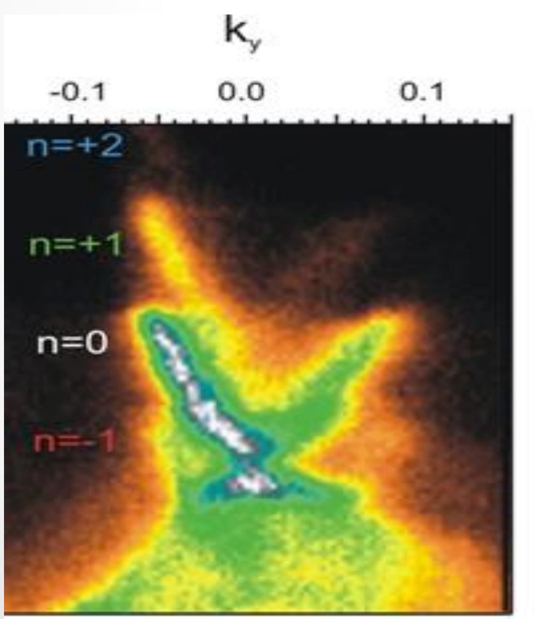
Wang et al. … N. Gedik ***Phys. Rev. Lett. 109***, 127401 (2012)

Gedik@MIT group

Difference Movie

Aoki, Tsuji, Eckstein, Kollar, Oka, Werner RMP to appear

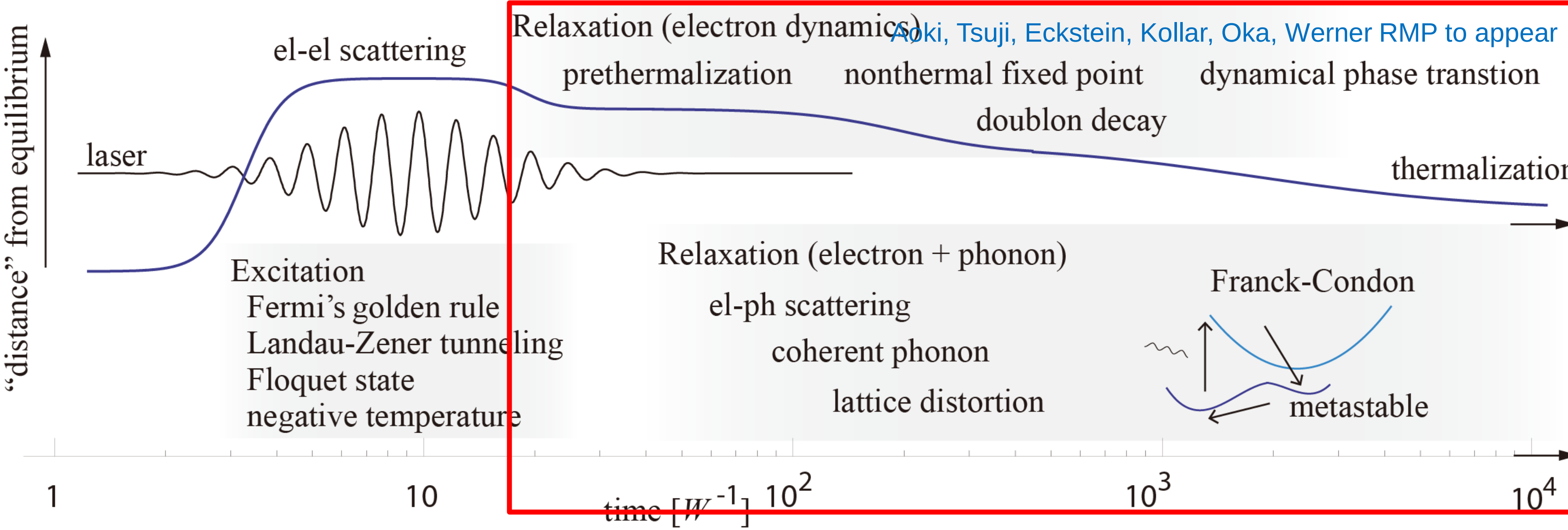




Wang *et al.* … Science '13

Floquet state = electron + *n*-photon

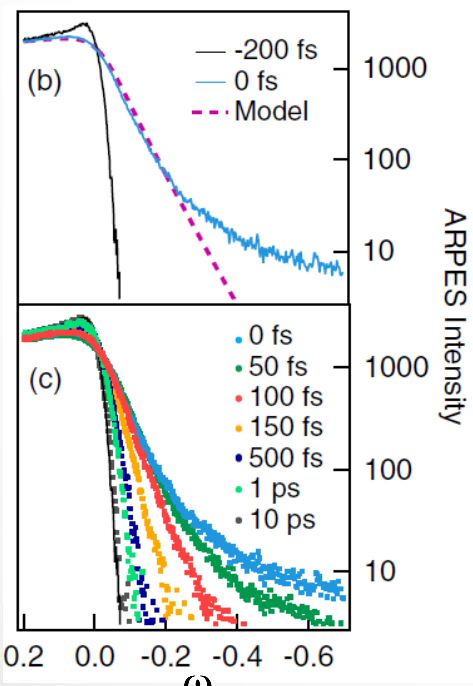
Topology can be changed! Oka Aoki '09



$Bi_2SrCaCu_2O_{8+\delta}$

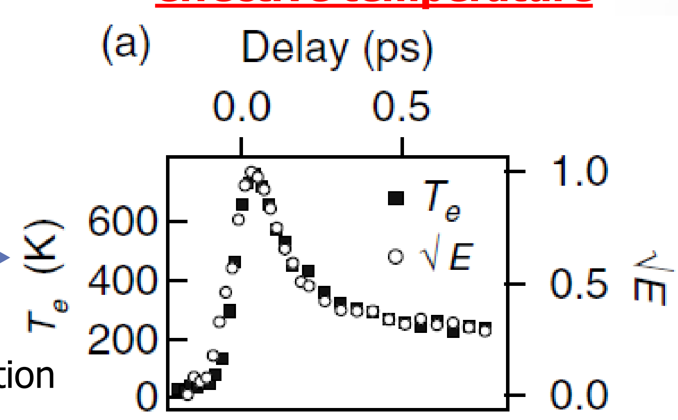
fermion distribution

Hi Tc cuprate

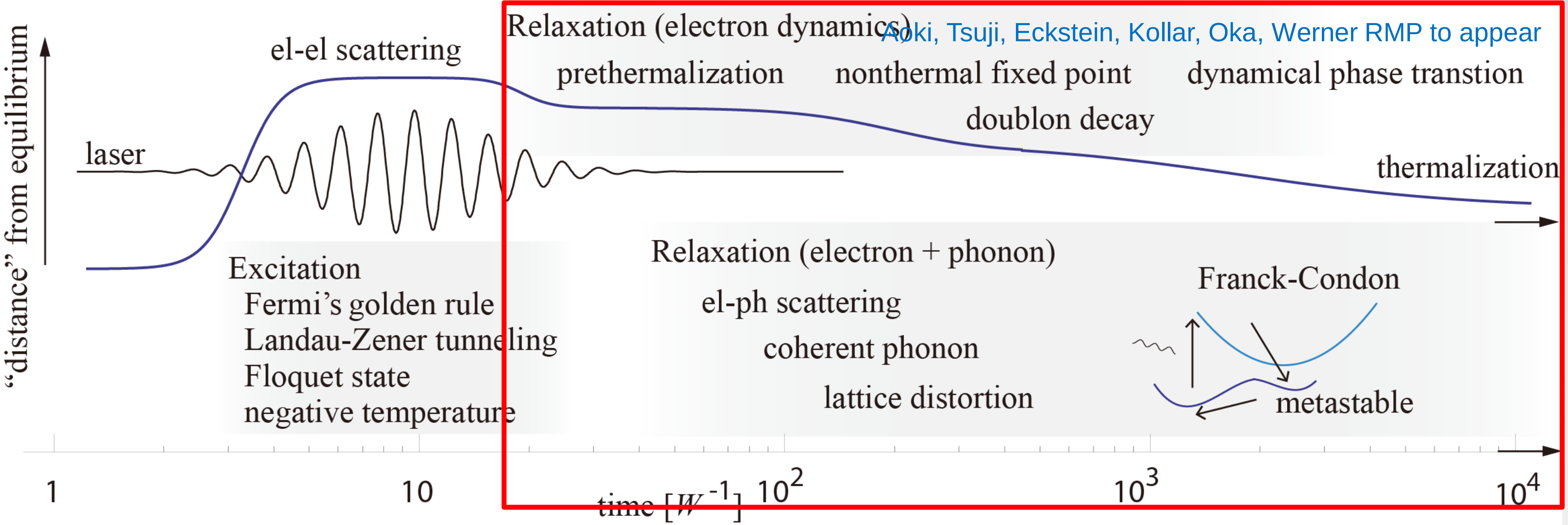


fit with Fermi-function

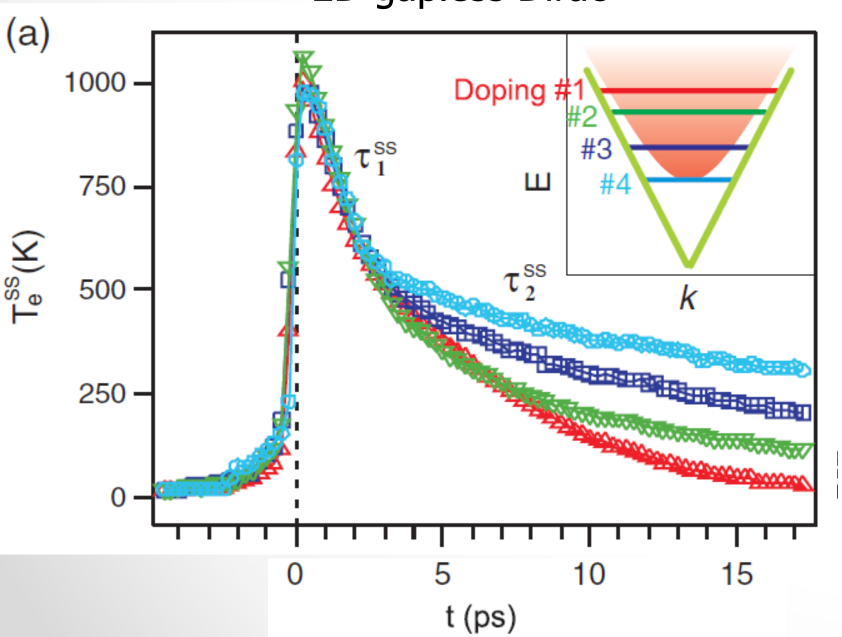
effective temperature



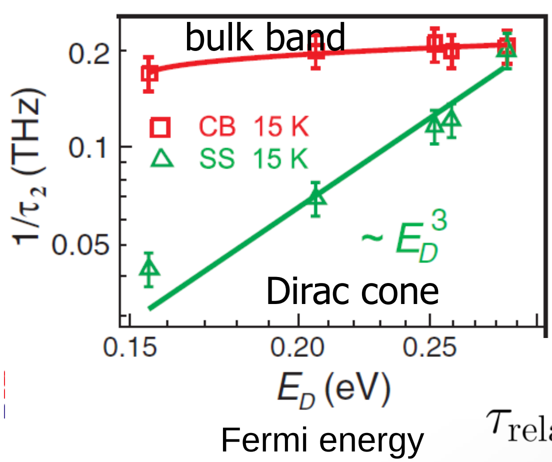
Perfetti *et al.* PRL '07



2D gapless Dirac



Wang *et al.* Gedik group PRL 2012



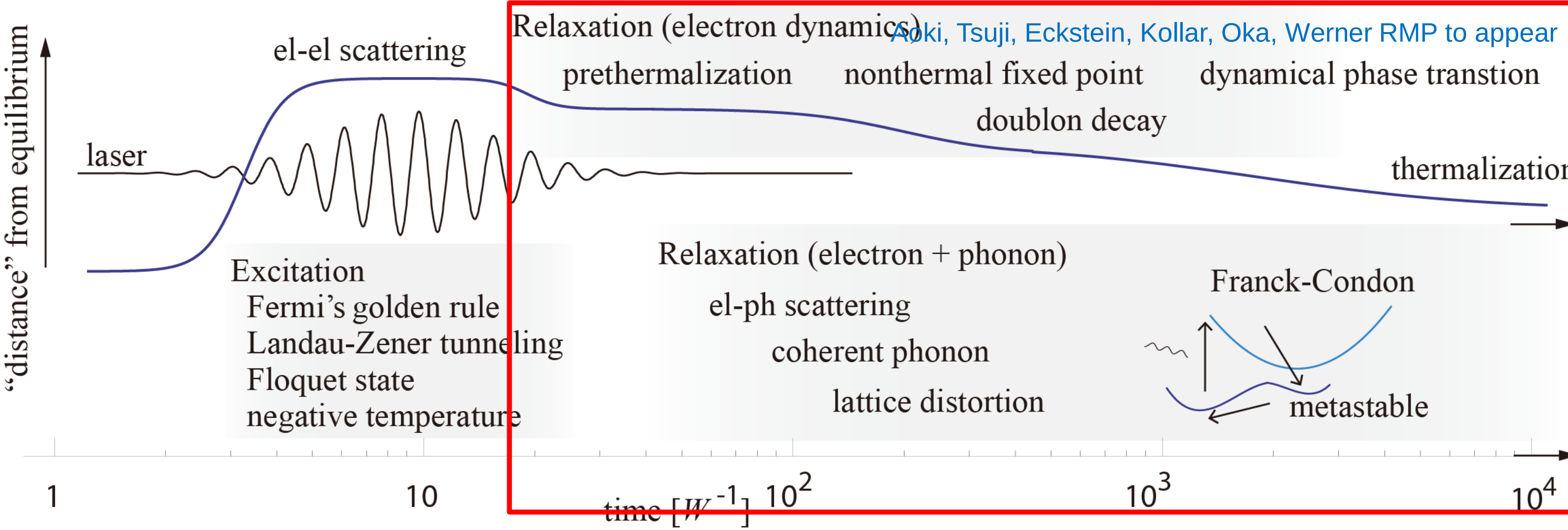
$$\tau_{\text{relax}} \sim 1/E^3_{\text{Fermi.}}$$

Polynomial relaxation

$$\tau_{\text{relax}} \sim 1/\text{Pol.}(W_1, W_2, \ldots)$$

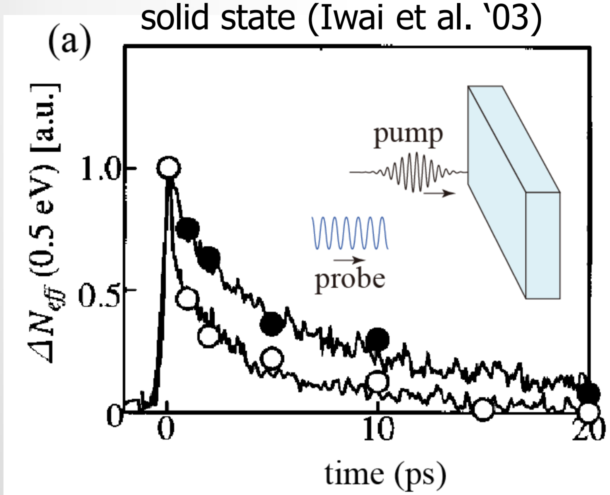
*2D Dirac has no other *E*-scale

*ちなみに理論は分かっていない

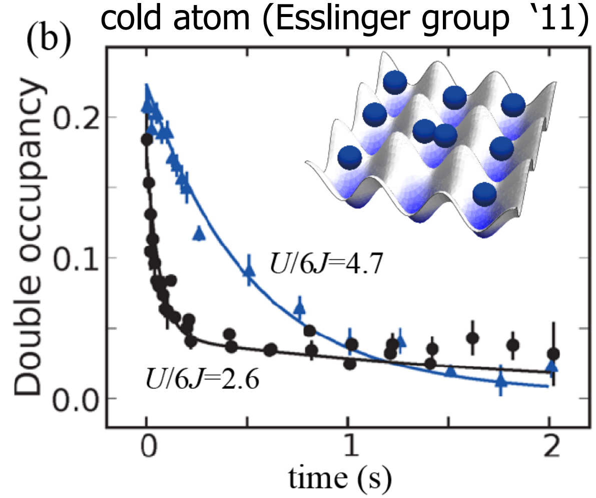


Mott insulator

solid state (Iwai et al. '03)



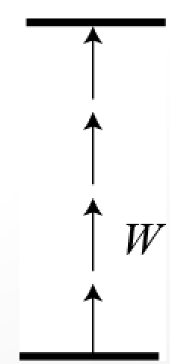
cold atom (Esslinger group '11)



Exponential relaxation

$$d + h \longrightarrow N \text{ magnon}$$

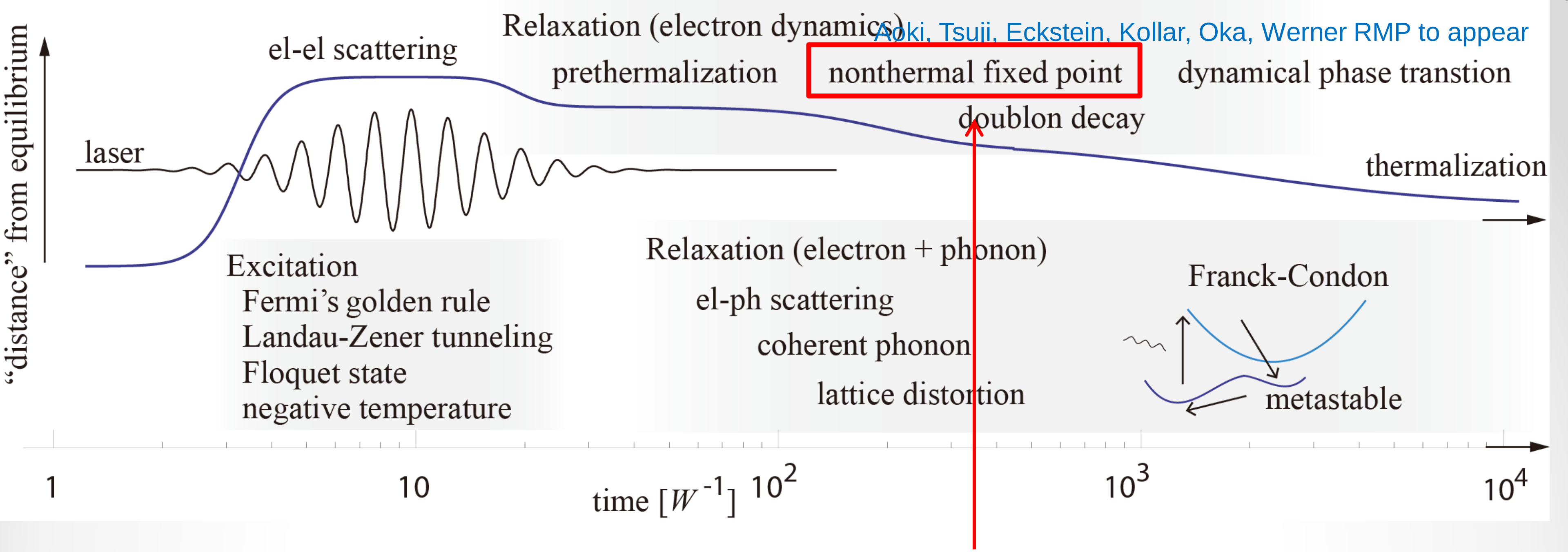
$$\tau_{\text{relax}}^{-1} \sim g^N = e^{\Delta_{\text{gap}}/W \log(g)}$$



$$N = \Delta_{\text{gap}}/W$$

Theory:
Sensarma et al., Eckstein et al., Tsuji et al. (SC phase)

prethermalization



Big challenge

Is a transient ordered phase possible?

Ferromagnet Yes (exp: Takubo et al. PRL '05)

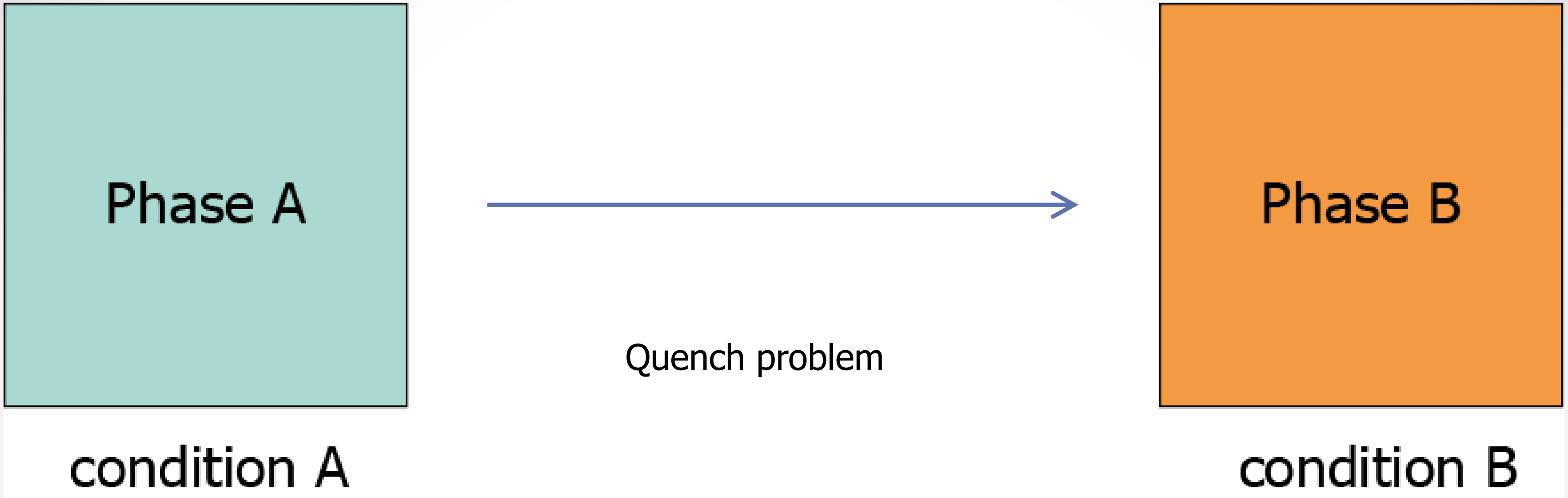
transient SC (Tsuji et al. PRL '13)

Haldane-AKLT phase (Takayoshi-Sato-Oka '14)

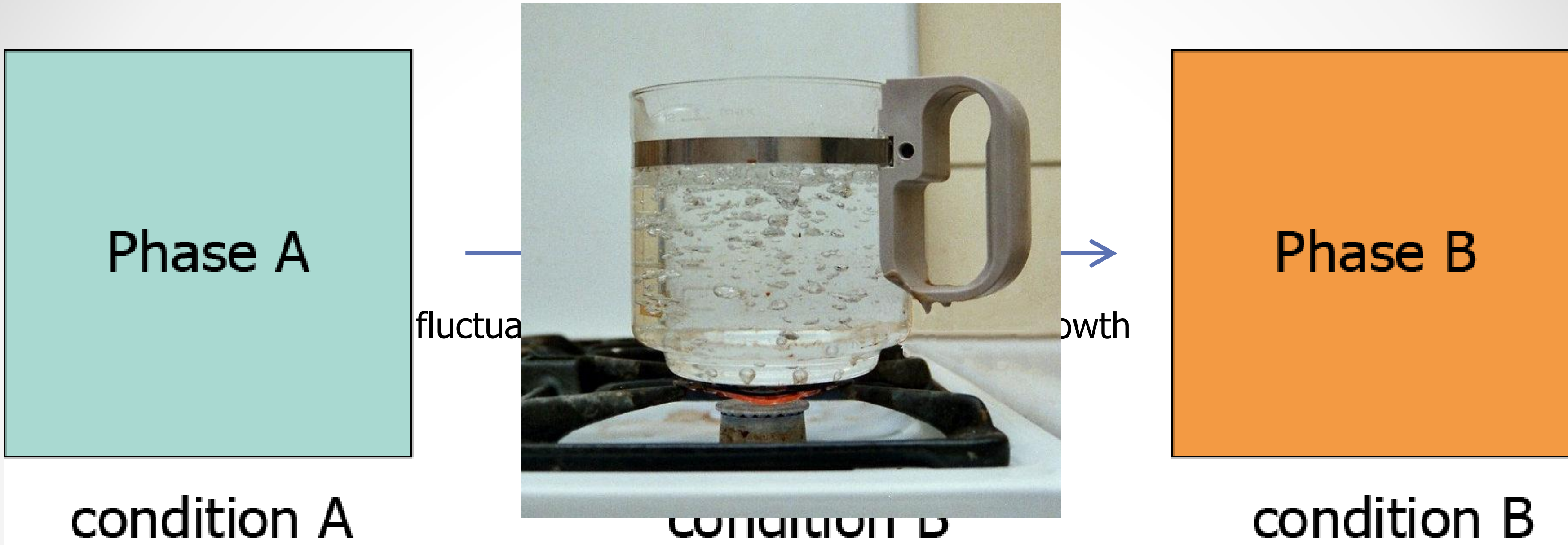
Must redo the theory of phase transitions

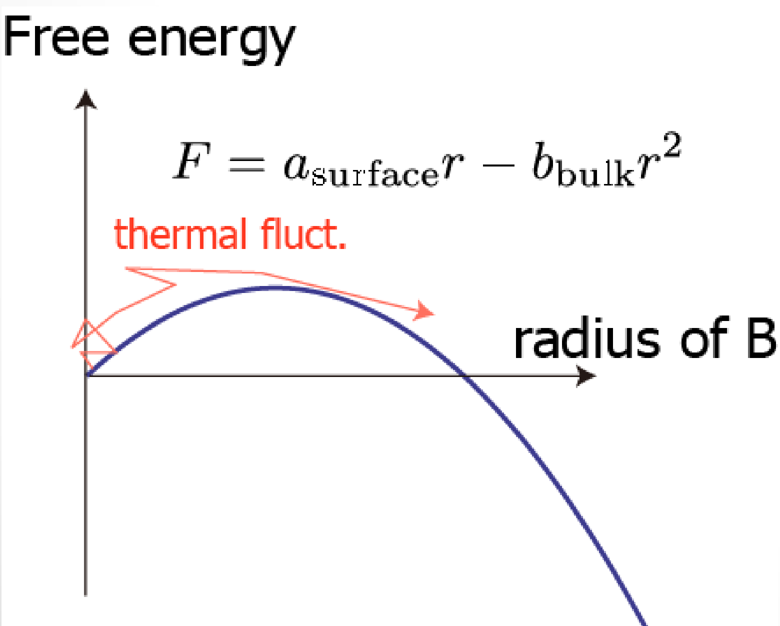
RG, fixed point,…

Classic case



Classic case

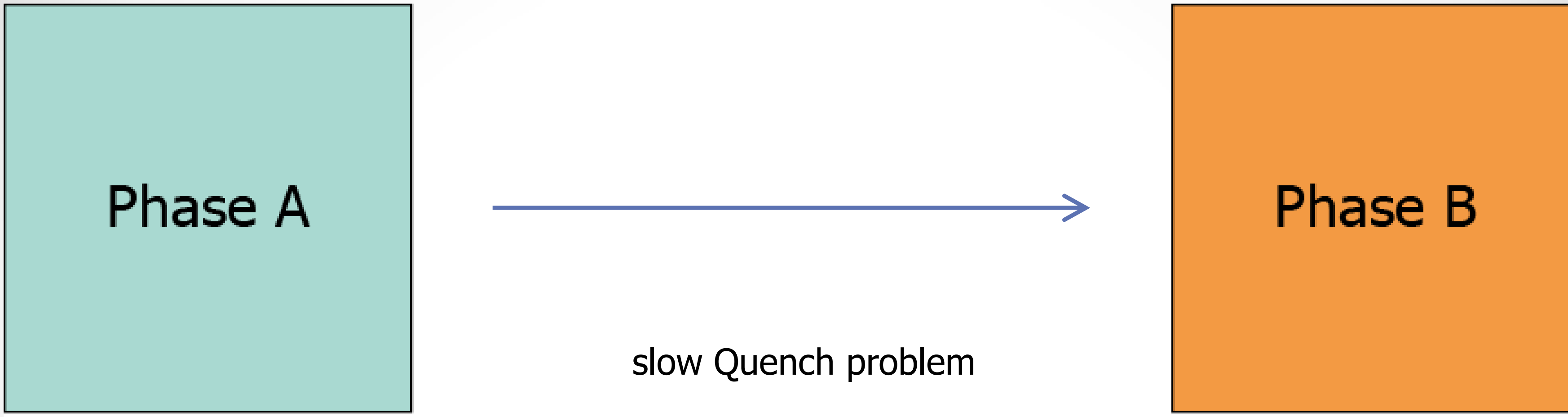




Production rate of B

$$P \propto e^{-\Delta_{\rm gap}/T}$$

Quantum case



slow Quench problem

condition A | condition B

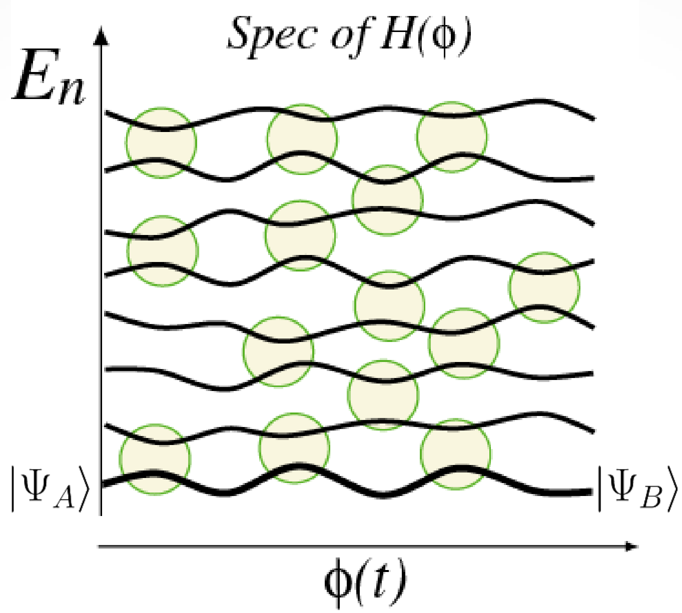
$|\Psi_A\rangle$ $$i\partial_t|\Psi(t)\rangle = H(\phi(t))|\Psi(t)\rangle$$ $|\Psi_B\rangle$

Quantum case

Phase A

condition A

$|\Psi_A\rangle$

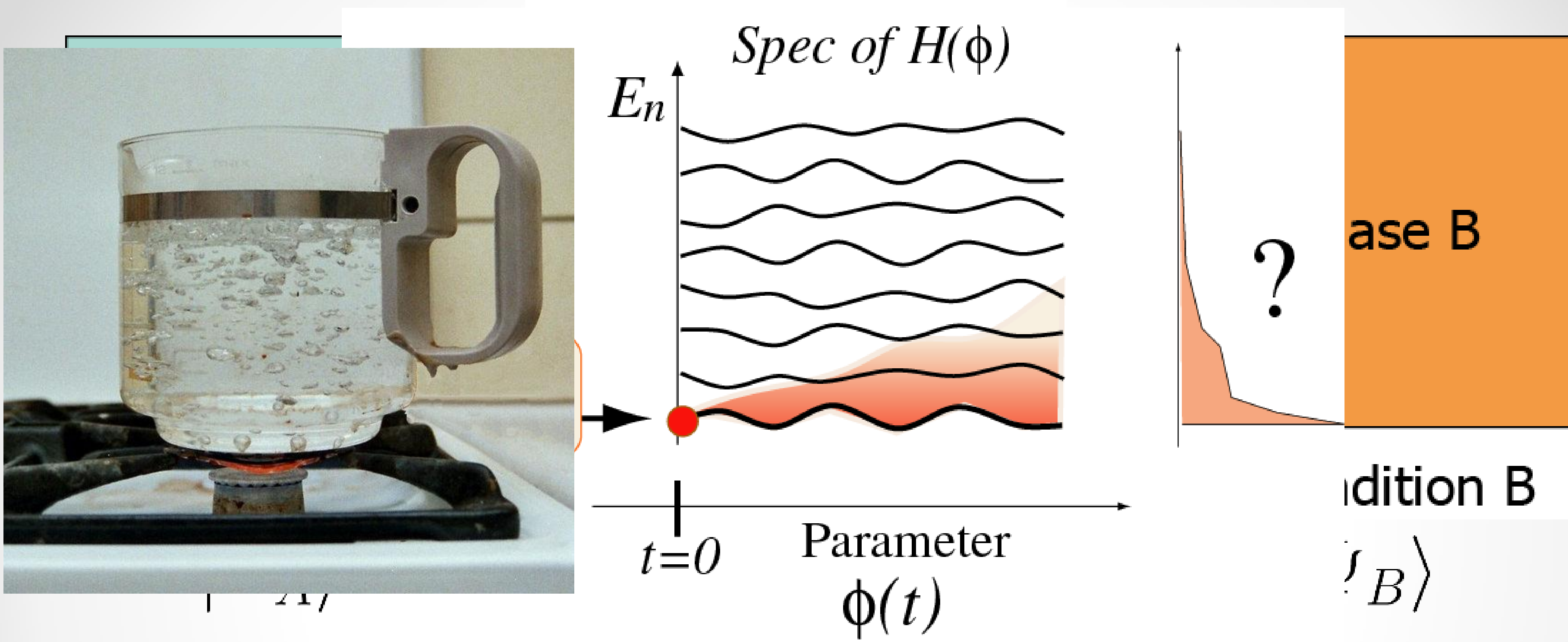


Phase B

condition B

$|\Psi_B\rangle$

Quantum case



$$|\Psi\rangle = c_B|\Psi_B\rangle + \sum_n c_n|\Psi_n\rangle$$

$$\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$$

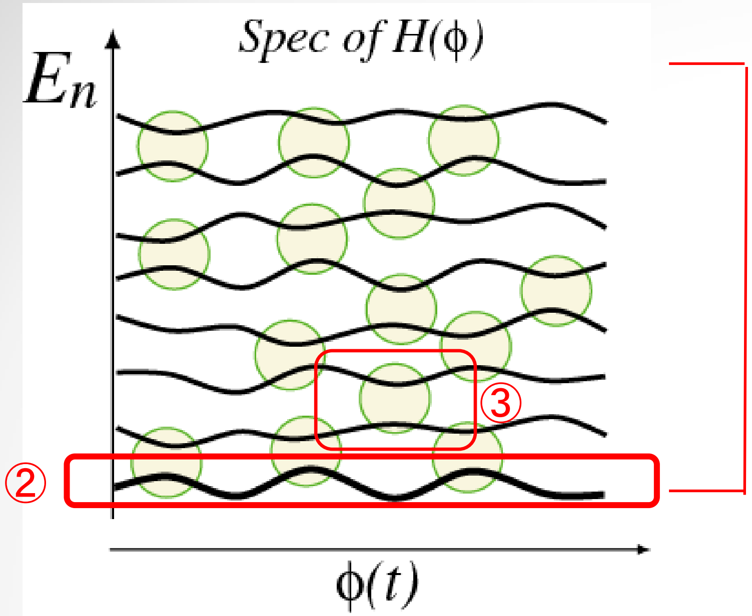
$$= \mathrm{diag}(|c_B|^2, |c_1|^2, |c_2|^2, \ldots) + \text{terms like } e^{i(E_i-E_j)t}c_i^* c_j$$

distribution ~ “thermal state”?

Examples

	A		B	excitation
Schwinger mechanism (Zener breakdown)	insulator		Noneq. steady state in *E*-field (if exists)	charge pair
Kibble-Zurek	normal		superfluid (broken U(1))	vortex
Takayoshi-Aoki-Oka '13 Takayoshi-Sato-Oka '14	Haldane-AKLT state (Symmetry protected topological state)	⟷	spin polarized state	?

⋮

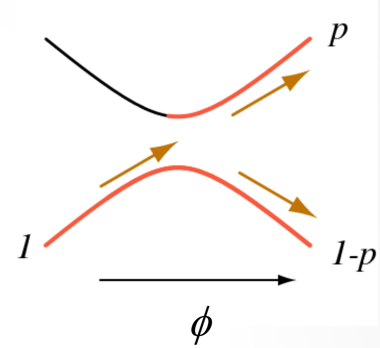


① Full dynamics

Quantum walk
in energy space

Oka, Konno, Aoki: PRL (2005)

③ production rate



two level approximation

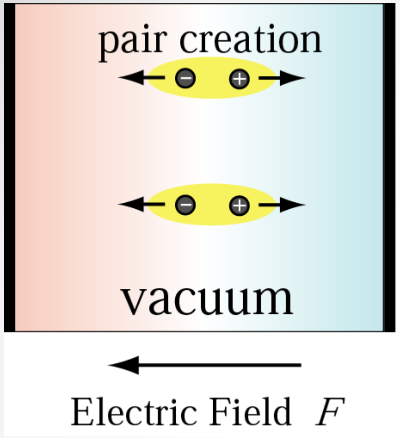
② vacuum decay rate/ Euler-Heisenberg Lagrangian

(fidelity, Loschmidt echo)

$$\Xi(t) = \langle 0; A(t)|\hat{T}e^{-i\int_0^t H(A(s))ds}|0;\phi(0)\rangle e^{i\int_0^t E_0(A(s))ds}$$

$$\to e^{i\mathcal{L}Vt}$$

$$\Gamma = 2\mathrm{Im}\mathcal{L} = \sum_{i=\text{decay channel}} P_i$$



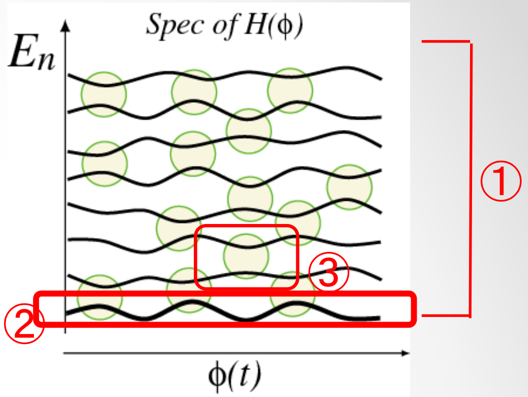
Schwinger mechanism (Zener breakdown)

QED semiconductor

② Euler Heisenberg 1936, Schwinger 1951 Zener 1932

③ Brezin Itzykson 1970, Popov 1970

$F(t) = F_0 \cos \Omega t$



interacting model in E-field

Hubbard model (closed ring)

① transient steady state

(1dim) Oka Aoki: PRL (2003), PRL(2005)
(DMFT) Eckstein, Oka, Werner: PRL (2010)
(Quantum Walk) Oka, Konno, Aoki: PRL (2005)

③ production rate of charge pairs

(Bethe) Oka Aoki, 2010, Oka 2012

$F(t) = F_0 \cos \Omega t$

② vacuum decay rate

(1dim dmrg) Oka Aoki: PRL(2005)

Large N SUSY QCD (large gluon bath)

steady state

Karch O'Bannon (2007)

Nakamura 2010, Karch Sondhi (2011)

③ production rate of charge pairs

Semenoff-Zarembo 2011

② vacuum decay rate/ Euler-Heisenberg Lagrangian

Hashimoto Oka 2013

① + dynamics $\tau_{\rm th} = \dfrac{\hbar}{k_B T_{\rm eff}}$

"Exact Nonequilibrium Steady State of an Open Hubbard Chain"

(MPS + Liouville) Prosen PRL 2014

Euler-Heisenberg effective Lagrangian

``Consequences of Dirac's Theory of Positrons" (English translation in arXiv)

W. Heisenberg and H. Euler, Z. Physik 98, 714 (1936) also Weisskopf (1936)

W. Heisenberg

H. Euler

(1909–1941)

Folgerungen aus der Diracschen Theorie des Positrons.

Von **W. Heisenberg** und **H. Euler** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur Paarerzeugung neigt, eine Abänderung der Maxwellschen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur wenig ändert. Es ergibt sich für das Feld eine Lagrange-Funktion:

effective Lagrangian

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2 - \mathfrak{B}^2) + \frac{e^2}{\hbar c}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}.$$

($\mathfrak{E}$, $\mathfrak{B}$ Kraft auf das Elektron.

$|\mathfrak{E}_k| = \frac{m^2c^3}{e\hbar} = \frac{1}{„137“}\frac{e}{(e^2/mc^2)^2} =$ „Kritische Feldstärke“.)

Ihre Entwicklungsglieder für (gegen $|\mathfrak{E}_k|$) kleine Felder beschreiben Prozesse der Streuung von Licht an Licht, deren einfachstes bereits aus einer Störungsrechnung bekannt ist. Für große Felder sind die hier abgeleiteten Feldgleichungen von den Maxwellschen sehr verschieden. Sie werden mit den von Born vorgeschlagenen verglichen.

W. Heisenberg and H. Euler, Z. Physik 98, 714 (1936)

Dirac fermion $L = \bar{\psi}(i\partial\!\!\!/ + eA\!\!\!/ - m)\psi$

$$\mathcal{L}(A_{ext}) = -i \ln \langle e^{-i\int A_\mu j^\mu} \rangle_0$$

$$= -i \ln \mathrm{Det}\left[i\partial\!\!\!/ + eA\!\!\!/ - m\right]/\mathrm{Vol.}$$

$$= \frac{1}{2}(\mathfrak{E}^2 - \mathfrak{B}^2) + \frac{e^2}{hc}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B}) \cdot \frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2 - \mathfrak{B}^2 + 2i(\mathfrak{E}\mathfrak{B})}\right) + \mathrm{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2 - \mathfrak{B}^2 + 2i(\mathfrak{E}\mathfrak{B})}\right) - \mathrm{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2 - \mathfrak{E}^2) \right\}$$

$$= \sum_{m,n=0}^{\infty} c_{m,n} F^m G^n + i\Gamma/2$$

imaginary part
= electron-positron pair production rate

$$\Gamma \sim \frac{e^2 E^2}{4\pi^3} \exp\left(-\pi \frac{m^2}{eE}\right)$$

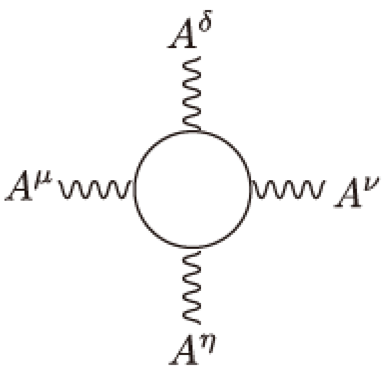
Cross correlation in nonlinear optics

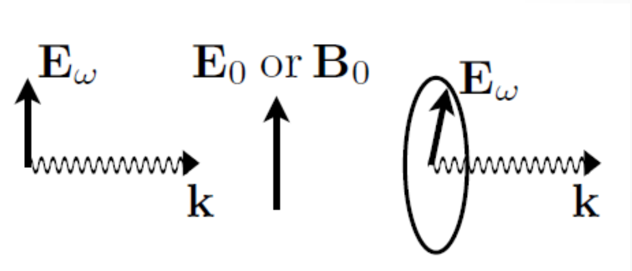
In Lorentz invariant material

$$\mathbf{D} = 2\varepsilon_0 c_{1,0}\mathbf{E} + \sqrt{\frac{\varepsilon_0}{\mu_0}}c_{0,1}\mathbf{B} + 2\varepsilon_0 c_{1,1}G\mathbf{E} + \sqrt{\frac{\varepsilon_0}{\mu_0}}c_{1,1}F\mathbf{B} + 4\varepsilon_0 c_{2,0}F\mathbf{E} + 2\sqrt{\frac{\varepsilon_0}{\mu_0}}c_{0,2}G\mathbf{B} \ldots$$

$$\mathbf{H} = 2c_{1,0}\frac{\mathbf{B}}{\mu_0} - \sqrt{\frac{\varepsilon_0}{\mu_0}}c_{0,1}\mathbf{E} + 2c_{1,1}G\frac{\mathbf{B}}{\mu_0} - \sqrt{\frac{\varepsilon_0}{\mu_0}}c_{1,1}F\mathbf{E} + 4\varepsilon_0 c_{2,0}F\frac{\mathbf{B}}{\mu_0} - 2\sqrt{\frac{\varepsilon_0}{\mu_0}}c_{0,2}G\mathbf{E}, \ldots$$

Lorentz invariants $F = \left(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}\right)$ $\quad G = \sqrt{\frac{\varepsilon_0}{\mu_0}}(\mathbf{E}\cdot\mathbf{B})$





Kerr effect $\quad \chi^{(3)} \propto c_{2,0}$

Dirac fermions show nontrivial nonlinear ME effect

Cotton Mouton effect
(birefringence induced by B)

$$\Delta n_{\mathrm{CM}} = n_{\parallel} - n_{\perp} = (c_{0,2} - 4c_{2,0})\frac{B_0^2}{\mu_0}$$

Relation to the Berry phase theory of polarization

TO Aoki PRL '05

EH effective Lagrangian

$$\mathcal{L}(A_{ext}) = -i \ln \langle e^{-i \int A_\mu j^\mu} \rangle_0$$

$$A^\mu = (Ex, \mathbf{0})$$

adiabatic limit (small E, long time)

Berry phase theory of polarization (Resta's twist operator version)

Resta (late 80s) '92, King-Smith Vanderbilt '93, .. Resta PRL 99

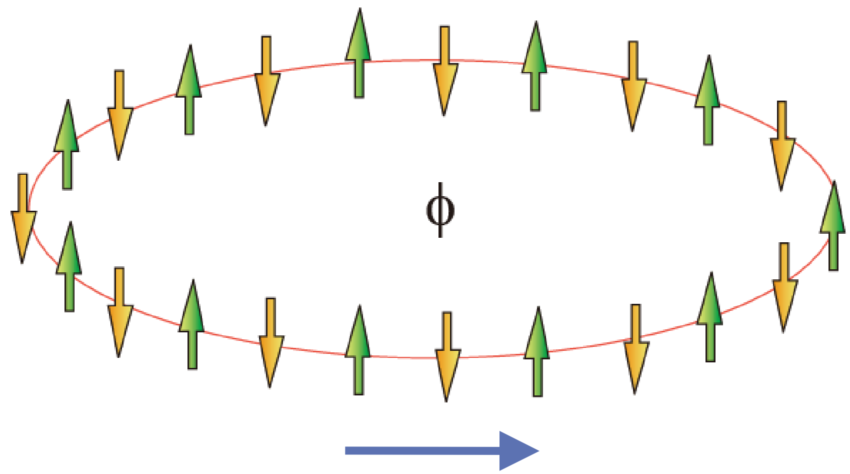
$$P = \frac{L}{2\pi} \mathrm{Im} \ln z \qquad z = \langle \Psi | e^{i(2\pi/L)\hat{X}} | \Psi \rangle$$

Dielectric breakdown in Mott insulators

(1dim) TO, Aoki: PRL (2003), PRL(2005)
(DMFT) Eckstein, TO, Werner: PRL (2010)

half-filled Hubbard model (1-dim)

$$H(t) = -\sum_i \left[e^{i\phi(t)} c_{i+1}^\dagger c_i + e^{-i\phi(t)} c_i^\dagger c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



electric field *F*

$$F(t) = -\frac{d\Phi(t)}{dt}$$

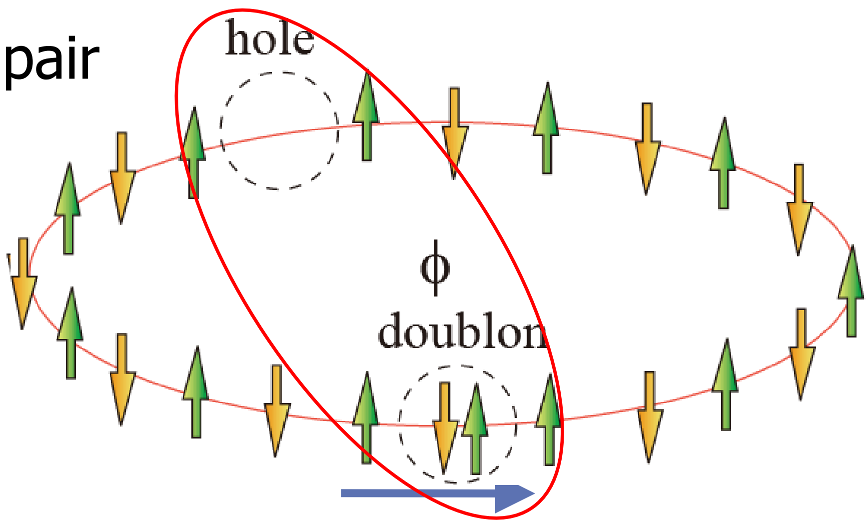
Dielectric breakdown in Mott insulators

(1dim) TO, Aoki: PRL (2003), PRL(2005),..
(DMFT) Eckstein, TO, Werner: PRL (2010)

half-filled Hubbard model (1-dim)

$$H(t) = -\sum_i \left[e^{i\phi(t)} c_{i+1}^\dagger c_i + e^{-i\phi(t)} c_i^\dagger c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Doublon-hole pair

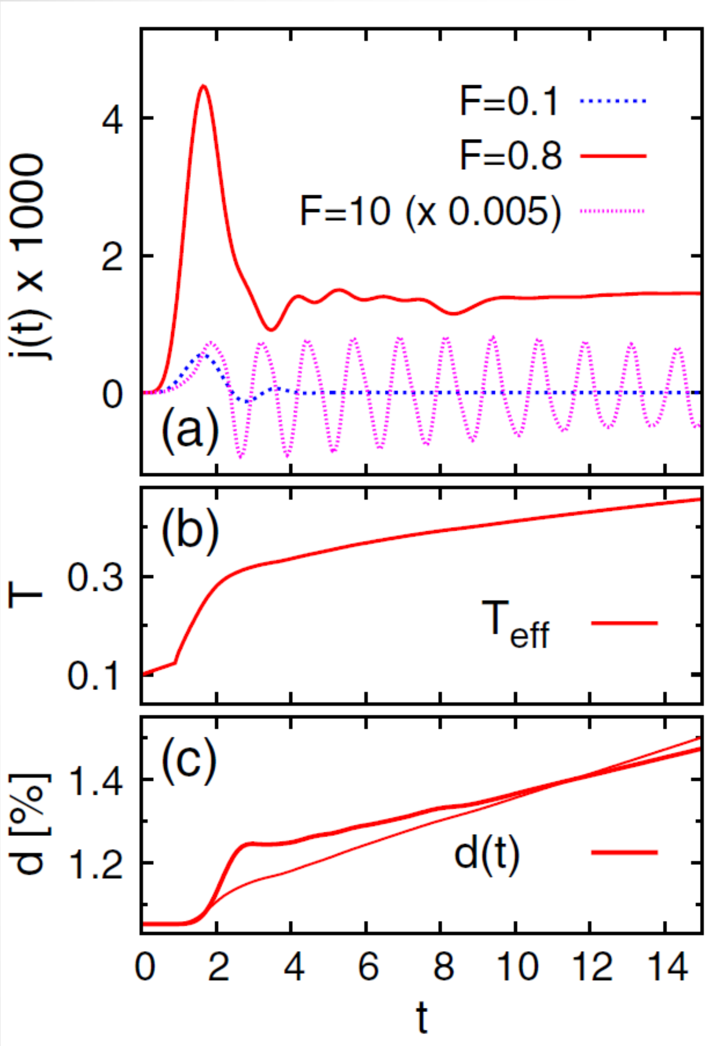


electric field *F*

$$F(t) = -\frac{d\Phi(t)}{dt}$$

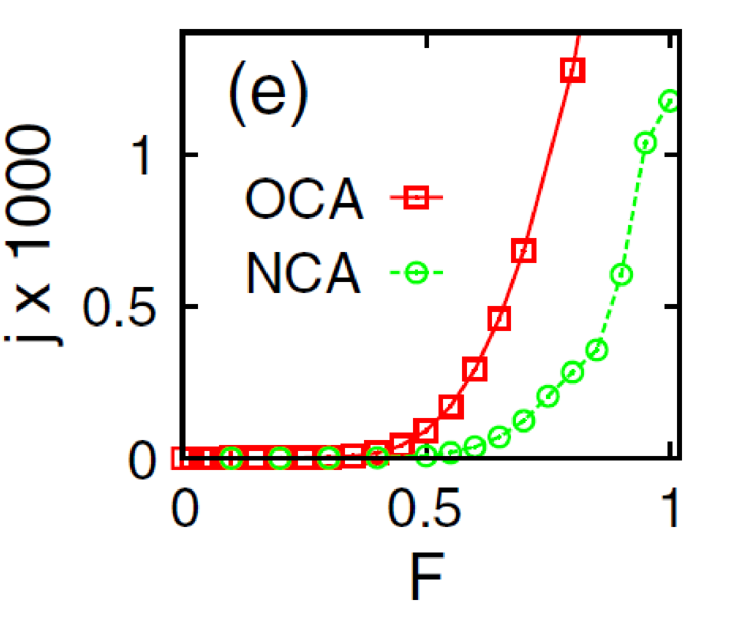
Nonequilibrium DMFT

Eckstein, TO, Werner PRL 2010
Eckstein, Werner PRB 2012



$$\dot{d}(t) = \Gamma_{\rm dh}$$

$$j/F \propto \Gamma_{\rm dh}$$



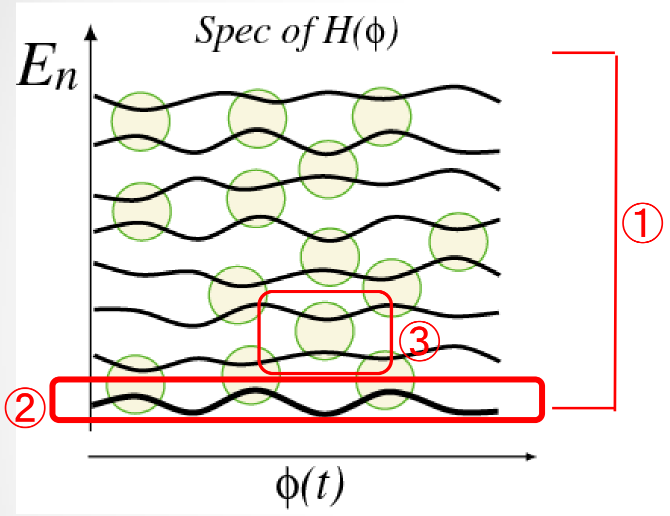
$$j_{\rm tun}(F) = F\sigma_{\rm tun}^{\infty}\exp(-F_{\rm th}/F)$$

threshold form

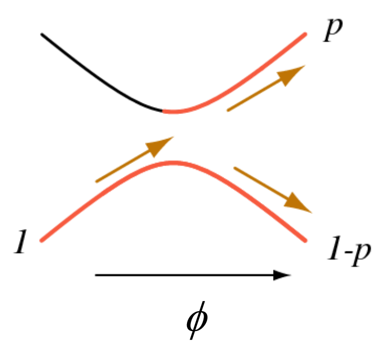
transient steady state

How do we solve this analytically?

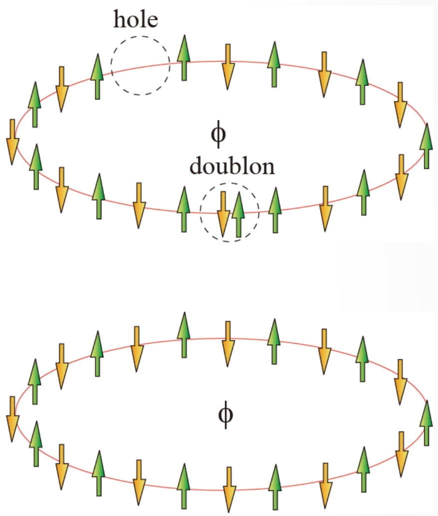
Hubbard model is exactly solvable
but one cannot obtain the matrix elements



production rate



two level approximation



1 pair

g.s.

ignore 2 pairs, 3 pairs, .., magnon

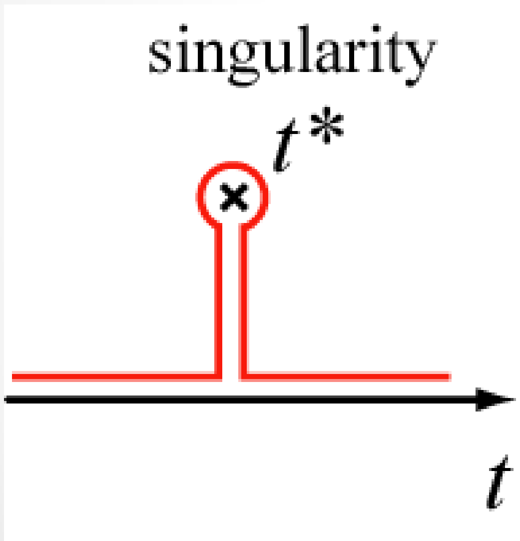
Imaginary time method (Landau-Dykhne theory)

Dykhne JETP (1962), Daviis, Pechukas, J.Chem.Phys. (1976)

Landau-Lifshitz *Quantum mechanics*

Matrix version of WKB approximation

$$H(t) = \begin{pmatrix} A(t) & B(t) \\ C(t) & D(t) \end{pmatrix}$$



1. Use complex time
2. Find the singular point

$$E_2(t^*) = E_1(t^*)$$

3. Tunneling probability

$$p = \exp\left(-2\mathrm{Im}S_{1,2}/\hbar\right)$$

$$S_{1,2} = \int_{t_0}^{t^*} dt'[E_2(\Phi(t')) - E_1(\Phi(t'))]$$

imaginary part of the dynamical phase

Generalized Landau-Zener formula

Imaginary time method + Bethe ansatz

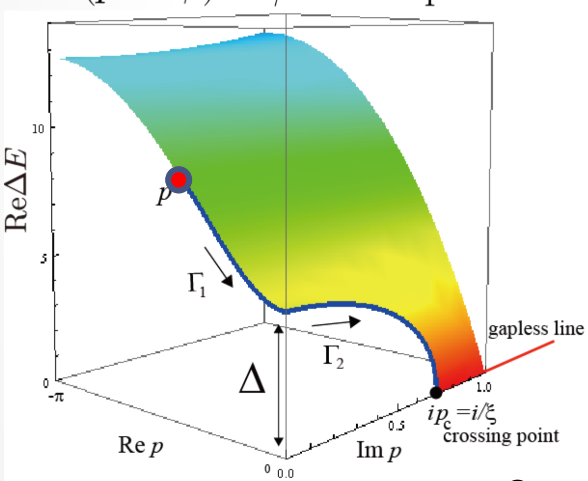
Momentum resolved dh-pair creation rate

$$\mathcal{P}_p = \exp\left(-2\mathrm{Im}\int_\Gamma \Delta E(p-\Phi)\frac{-1}{F(\Phi)}d\Phi\right)$$

ΔE: d-h energy, Γ: complex path, F: Jacobian   Oka PRB 2012

complex momentum (~WKB)

$\Delta E(p-\phi) \quad \phi \in \mathrm{complex}$

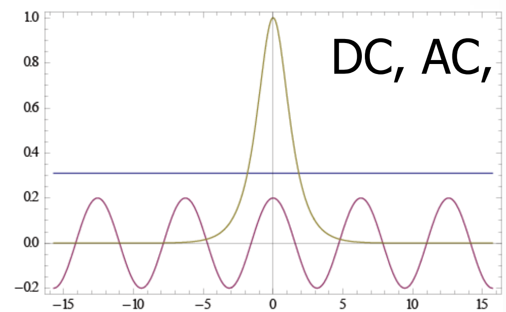


U=8

$$\varepsilon_h(k) = U/2 + 2\cos k + 2\int_0^\infty \frac{d\omega}{\omega}\frac{J_1(\omega)\cos(\omega\sin k)e^{-U\omega/4}}{\cosh(\omega U/4)}$$

$$p_h(k) = \frac{\pi}{2} - k + 2\int_0^\infty \frac{d\omega}{\omega}\frac{J_0(\omega)\sin(\omega\sin(k))}{1+\exp(U\omega/2)}$$

electric field $F(t)$



DC, AC, pulse

type	$F(t)$	$F(\Phi)$	attempt frequency f
DC-field	F_0	F_0	$F_0/2\pi$
AC-field	$F_0 \sin\Omega t$	$\pm\sqrt{F_0^2 - \Omega^2\Phi^2}$	Ω/π
single pulse	$F_0\cosh^{-2}(t/\sigma)$	$F_0\left(1-\frac{\Phi^2}{\sigma^2 F_0^t}\right)$	1(single process)

1D Mott insulator

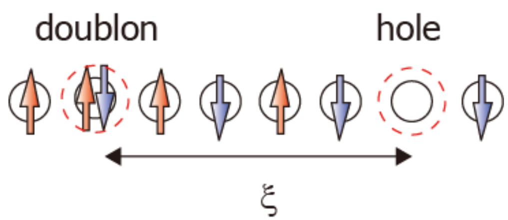
DC-limit

Oka-Aoki 2010

$$\mathcal{P}_p = \exp\left(-\pi \frac{F_{\rm th}}{F_0}\right)$$

No p-dependence

$$F_{\rm th} \sim \Delta_{\rm Mott}/\xi$$



ξ: dh-pair size (correlation length) ∼1, 2sites

Estimate for 1d Mott insulators

	τ(eV)	U(eV)	a(Å)	$\Delta_{\rm Mott}$(eV)	$\xi(a)$	$E_{\rm th}$(MV/cm)
ET-F_2TCNQ	0.1	1	10	0.7	1.1	3
[Ni(cnxn)$_2$Br]Br_2	0.22	2.4	5	1.6	1.0	16
Sr_2CuO_3	0.52	3.1	4	1.5	2.1	9

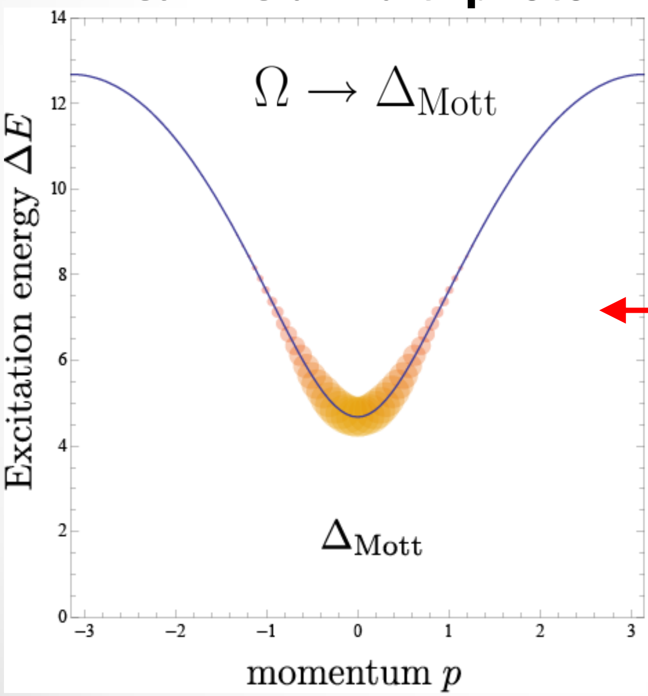
AC-field: Keldysh crossover

$$F(t) = F_0 \cos(\Omega t)$$

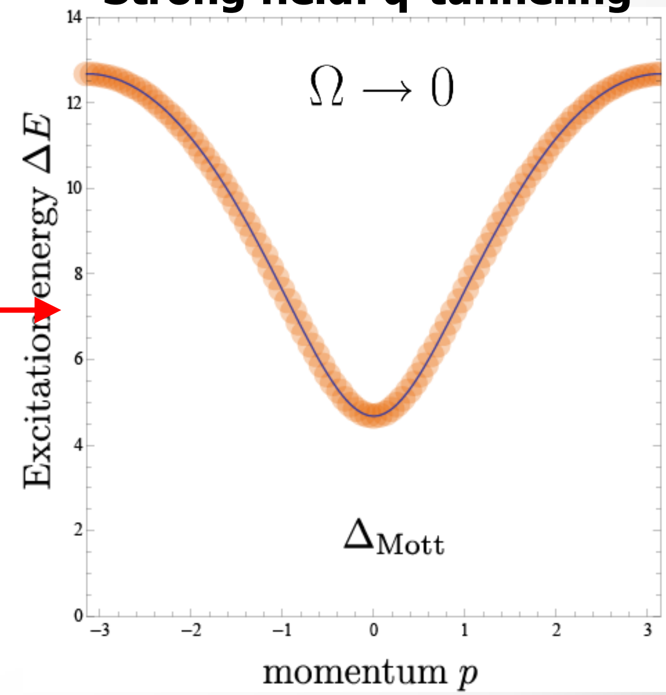
$$\mathcal{P}_{p=0} \rightarrow \begin{cases} \left(\frac{F_0 \xi}{b\Omega}\right)^{2\frac{\Delta_{\mathrm{Mott}}}{\Omega}} & \gamma \gg 1, \quad \text{mulit-photon} \\ \exp\left(-\frac{\pi}{2}\frac{\Delta_{\mathrm{Mott}}}{\xi F_0}\left(1 - \frac{\pi}{16}\gamma^2 + \ldots\right)\right) & \gamma \ll 1, \quad \text{tunneling} \end{cases}$$

Keldysh parameter $\gamma = \Omega / \xi F_0$

Weak field: multi-photon

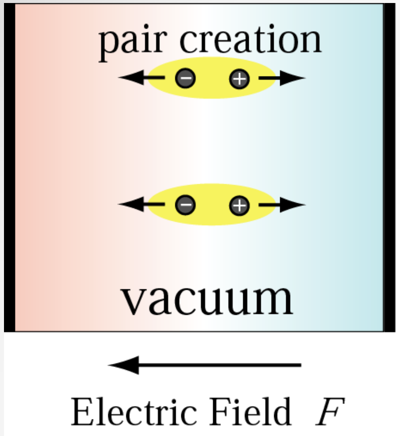


Strong field: q-tunneling



~ femto-second pump-probe

~ nonlinear transport



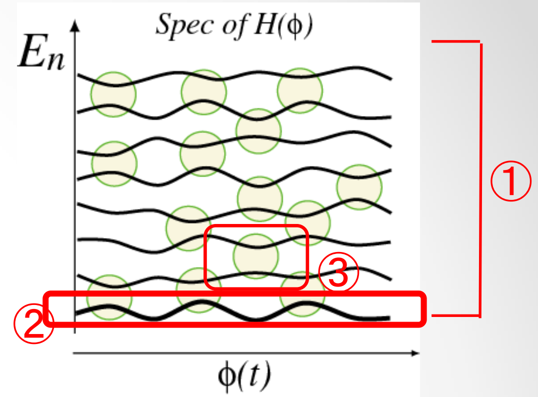
Schwinger mechanism (Zener breakdown)

QED　semiconductor

② Euler Heisenberg 1936, Schwinger 1951　Zener 1932

③ Brezin Itzykson 1970, Popov 1970

$$F(t) = F_0 \cos \Omega t$$



interacting model in *E*-field

Hubbard model　(closed ring)

① transient steady state

(1dim) Oka Aoki: PRL (2003), PRL(2005)
(DMFT) Eckstein, Oka, Werner: PRL (2010)

③ production rate of charge pairs

(Bethe) Oka Aoki, 2010, Oka 2012

$$F(t) = F_0 \cos \Omega t$$

② vacuum decay rate

(1dim dmrg) Oka Aoki: PRL(2005)

Large *N* SUSY QCD　(large gluon bath)

steady state　Karch O'Bannon (2007)

Nakamura 2010, Karch Sondhi (2011)

③ production rate of charge pairs

Semenoff-Zarembo 2011

② vacuum decay rate/ Euler-Heisenberg Lagrangian

→ Hashimoto Oka 2013

① + dynamics　$\tau_{\rm th} = \dfrac{\hbar}{k_B T_{\rm eff}}$

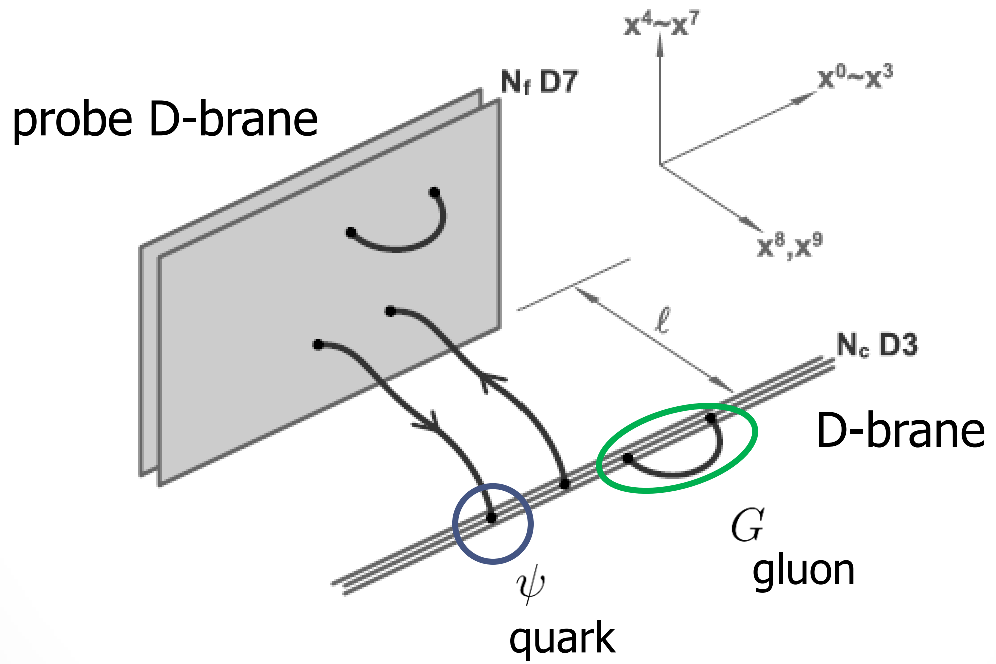
"Exact Nonequilibrium Steady State of an Open Hubbard Chain"

(MPS + Liouville) Prosen PRL 2014

Gauge/gravity duality using the D3/D7 configuration

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i i(\gamma^\mu D_\mu)_{ij}\psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \mathrm{SUSY\ partners}$$

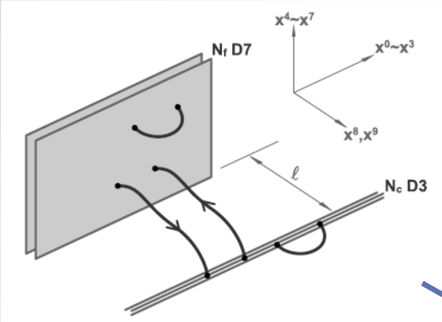
= low energy theory of the D3/D7 configuration

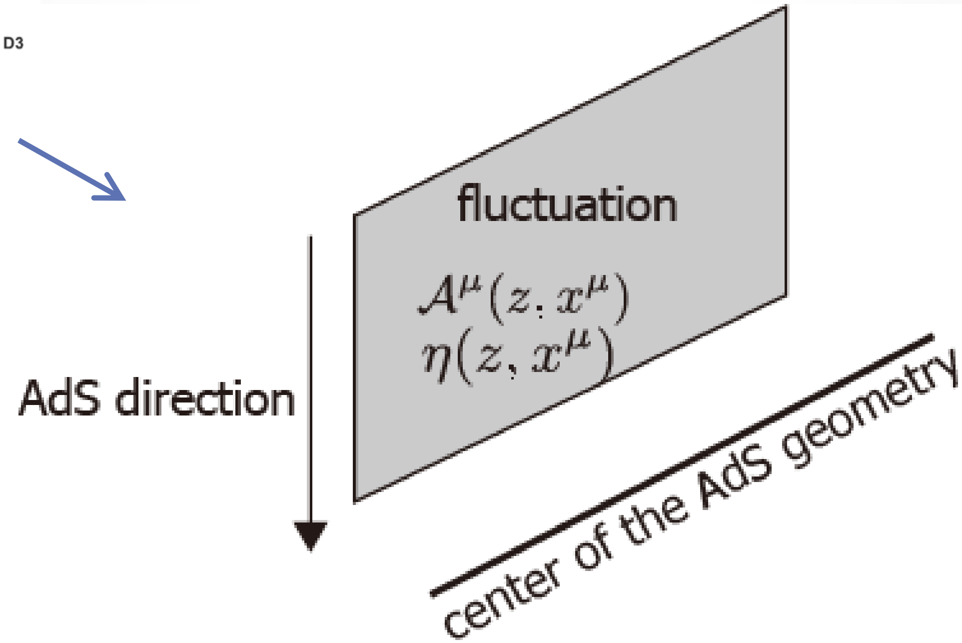


review: Erdmenger *et al.* 0711.4467
Kim *et al.* 1205.4852

$N_c \to \infty$ limit

SUSY Yang Mills: Maldecena '99
SYM+quark: Karch Katz '02
Sakai Sugimoto '04





Classical field theory described by the Dirac-Born-Infeld (DBI) action

$$S_{\rm DBI} = -T_p \int d\sigma e^{-\Phi} \sqrt{-\det(g_{mn} + 2\pi\alpha' \mathcal{F}_{mn})}$$

$$\mathcal{F}_{mn} = \partial_m \mathcal{A}_n - \partial_n \mathcal{A}_m$$

"nonlinear Maxwell equation + AdS metric"

review: Erdmenger *et al*. 0711.4467
Kim *et al*. 1205.4852

Gauge/gravity correspondence

Gubser, Klebanov, Polyakov '98, Witten '98

$$\left\langle e^{i\int d^dx A_\mu^{\text{ext}}(x) J^\mu(x)} \right\rangle = e^{iS_{\text{DBI}}(\mathcal{A}^*;A^{\text{ext}})_{\text{on shell}}}$$

on shell $\longleftrightarrow$ $\mathcal{A}^*$ is the solution of the equation of motion

Euler-Heisenberg Lagrangian from Gauge / Gravity correspondence

$$\langle e^{i\int d^dx A_\mu^{\rm ext}(x) J^\mu(x)} \rangle = e^{iS_{\rm DBI}(\mathcal{A}^*; A^{\rm ext})_{\rm on\ shell}}$$

GKP '98, Witten '98

We want the following

$$\langle e^{i\int d^dx A_\mu^{\rm ext}(x) J^\mu(x)} \rangle_0 = ?$$

Euler-Heisenberg Lagrangian from Gauge / Gravity correspondence

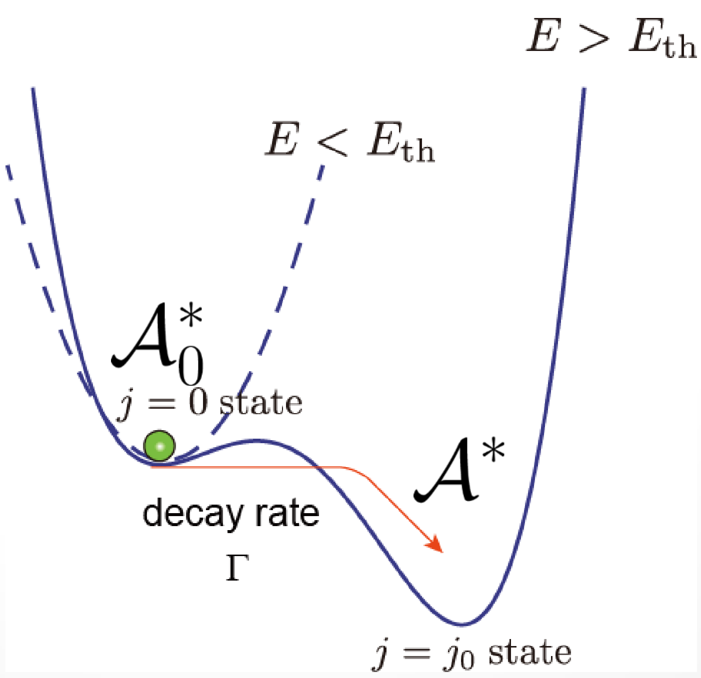
$$\langle e^{i\int d^dx A_\mu^{\rm ext}(x) J^\mu(x)} \rangle = e^{iS_{\rm DBI}(\mathcal{A}^*; A^{\rm ext})_{\rm on\ shell}}$$

GKP ’98, Witten ‘98

We want the following

$$\langle e^{i\int d^dx A_\mu^{\rm ext}(x) J^\mu(x)} \rangle_0 = e^{iS_{\rm DBI}(\mathcal{A}_0^*; A^{\rm ext})_{\rm off\ shell}}$$

Hashimoto Oka JHEP `13



Euler-Heisenberg Lagrangian of the N=2 SUSY QCD in the large N_c limit

Hashimoto Oka JHEP `13, Hashimoto Sonoda Oka *in prep.*

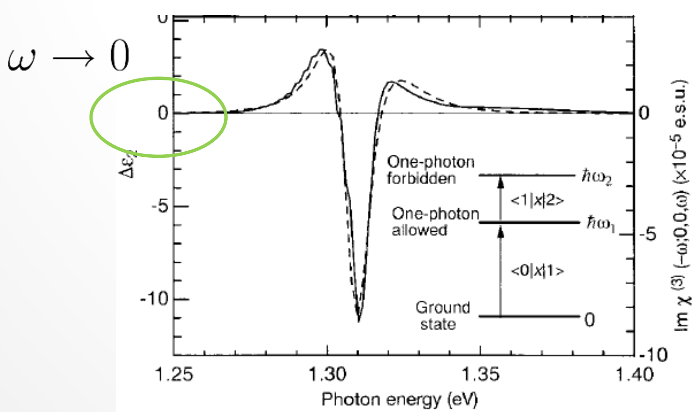
$$\mathcal{L}_{\rm EH}^{\rm SQCD} = -\mu_7 \int dz \frac{R^5}{z^5}\sqrt{(1-\beta F_{0z}^2)(1+\beta f \mathbf{B}^2) - \beta f \mathbf{E}^2 - (\beta f)^2(\mathbf{E}\cdot\mathbf{B})^2}$$

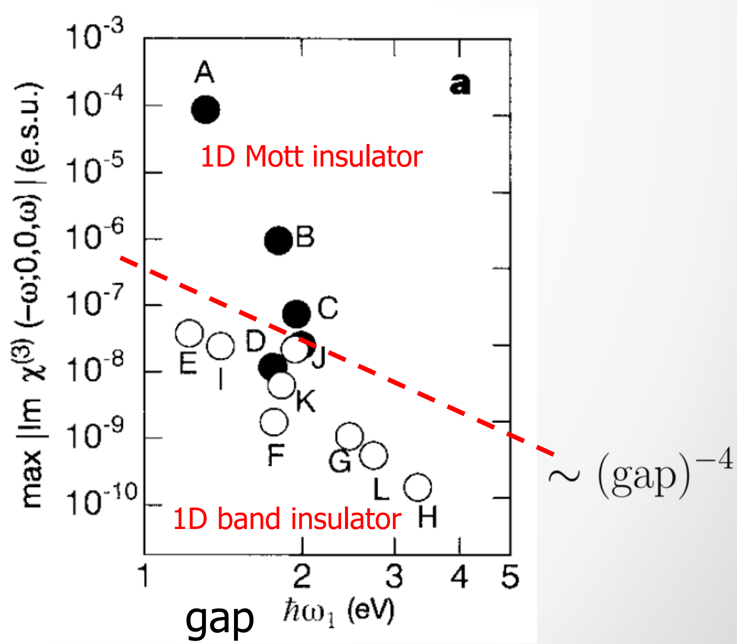
$$\beta = \frac{(2\pi\alpha')^2}{R^4}z^4, \;\; f = \left(\frac{R^4}{\eta z^2 + R^4}\right)^2$$

Example: Kerr/Cotton Mouton effects

$$c_{2,0} = \frac{1}{3\cdot 2^7 \pi^4}\frac{N_c \lambda}{m_q^4}$$

$$c_{2,0} \sim \chi^{(3)}(0;0,0,0)$$





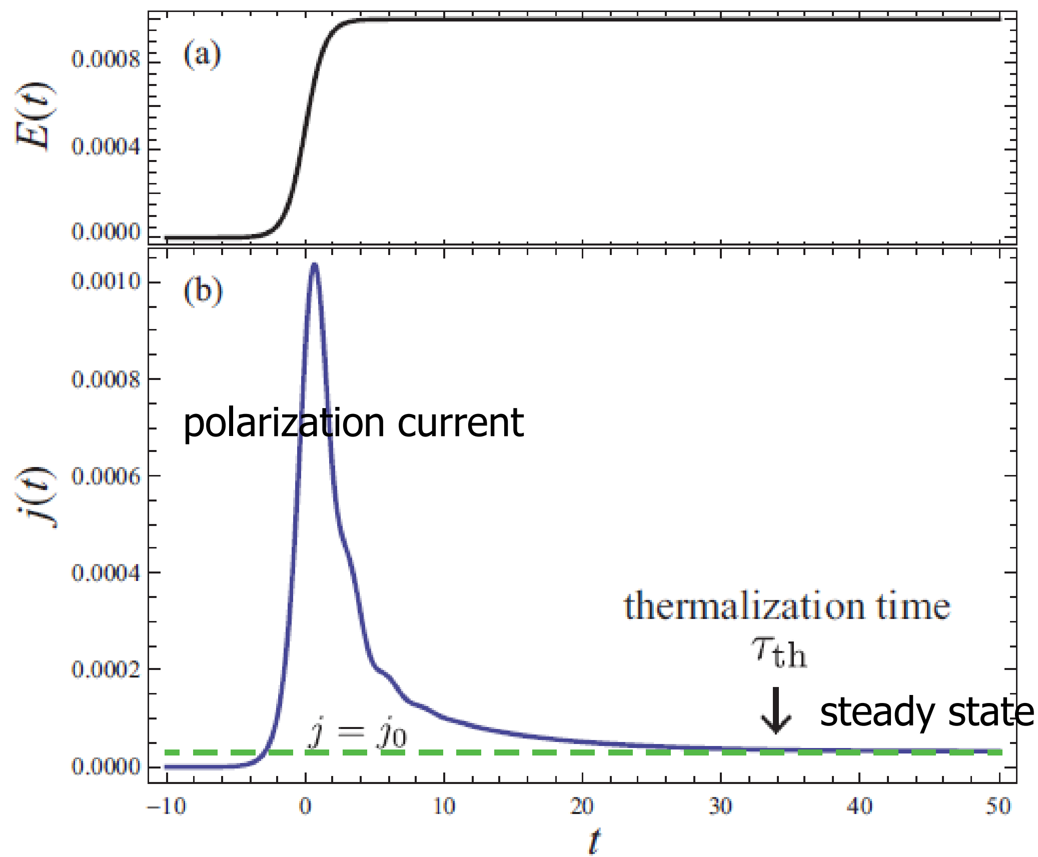
Kishida *et al.* Nature 2000

Time dependent calculation in the gapless case

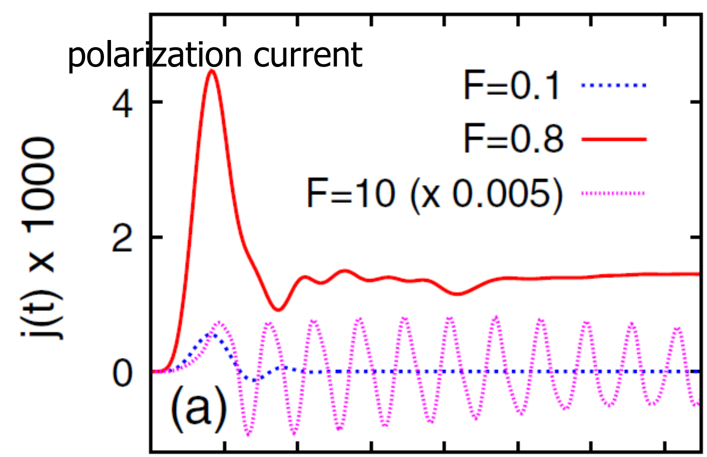
solve EOM

$$-\partial_z\left(\frac{\sqrt{1+\frac{z^6}{R^4}d^2}\partial_z A_1}{z\sqrt{1-\frac{z^4}{R^4}\{(\partial_0 A_1)^2-(\partial_z A_1)^2\}}}\right)+\partial_0\left(\frac{\sqrt{1+\frac{z^6}{R^4}d^2}\partial_0 A_1}{z\sqrt{1-\frac{z^4}{R^4}\{(\partial_0 A_1)^2-(\partial_z A_1)^2\}}}\right)=0$$

$$A_x = -\int^t E(s)ds + h(t,z)$$



current in the Hubbard model



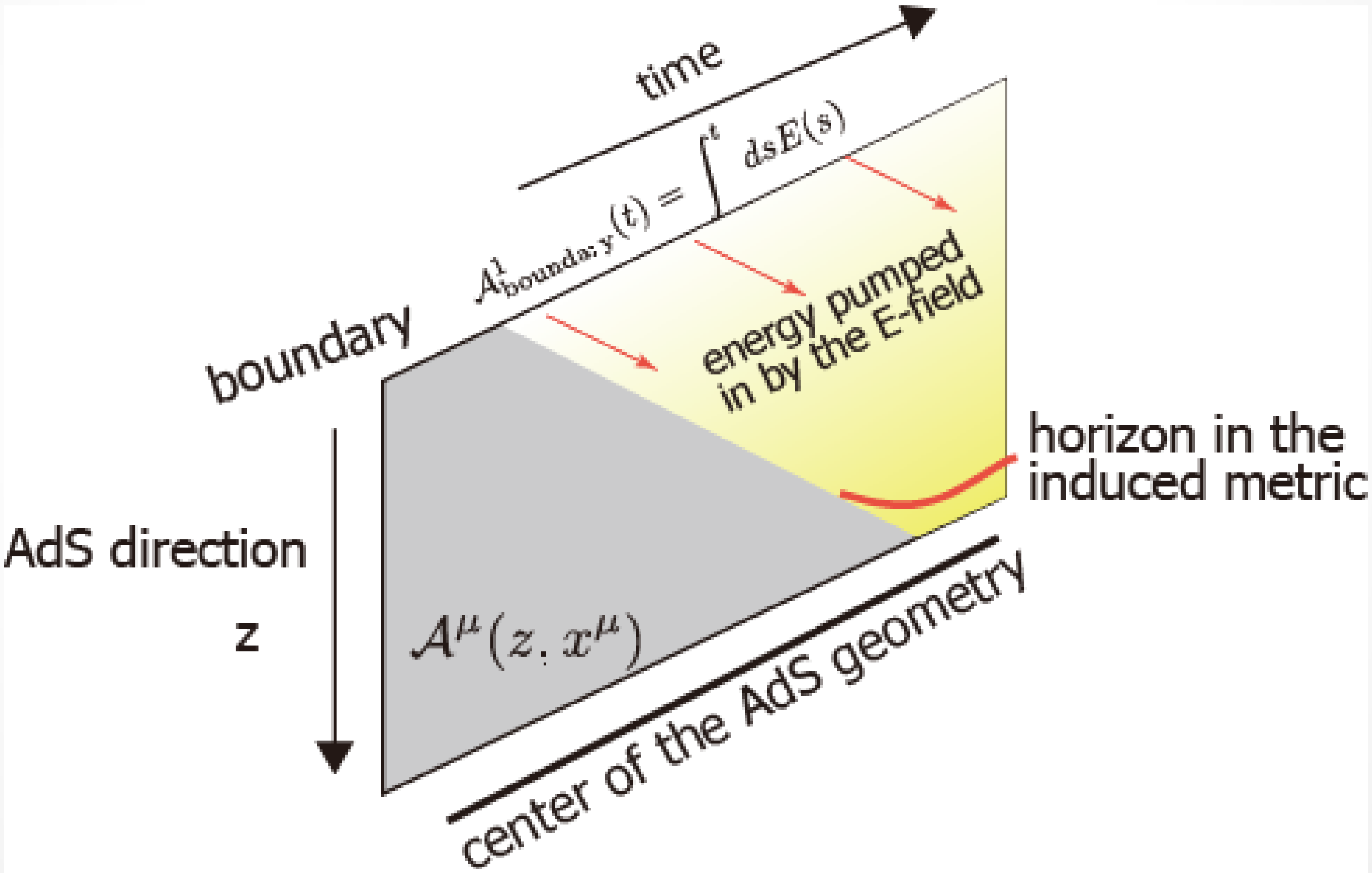
Eckstein TO Werner PRL '10

Hashimoto TO JHEP '13

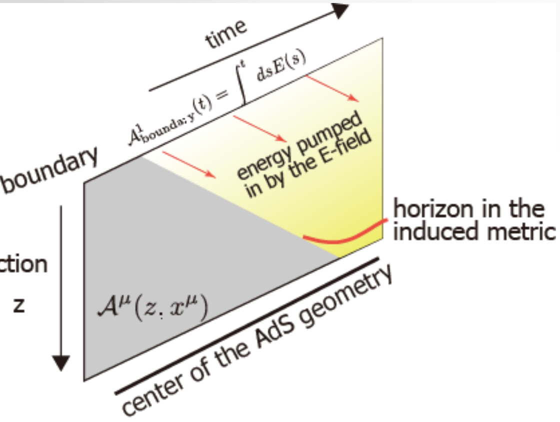
Time dependent calculation in the gapless case

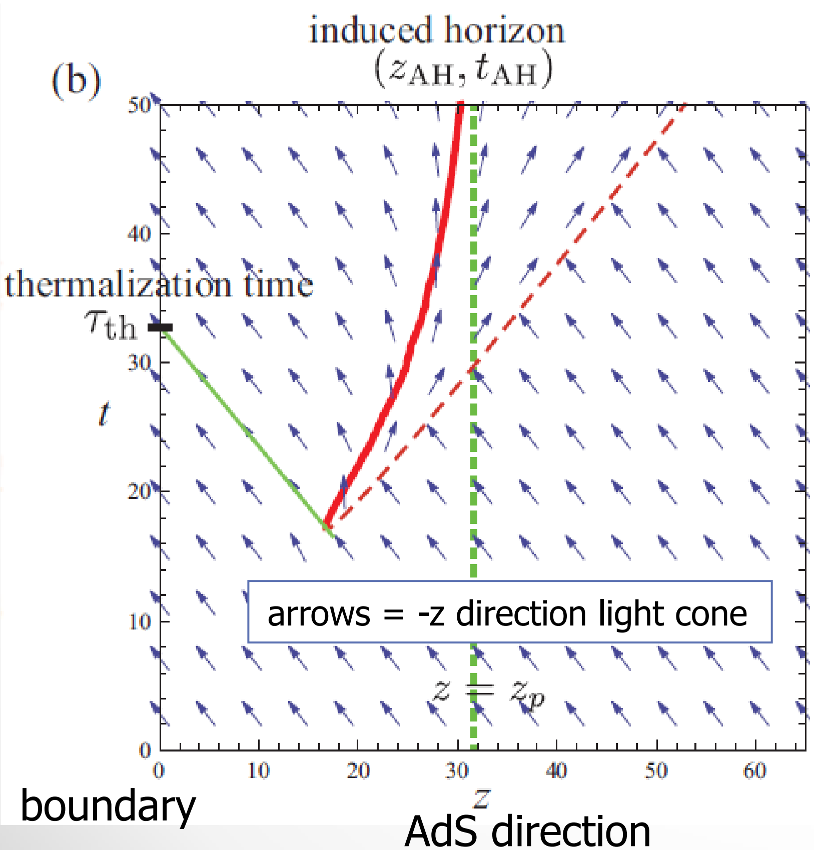
solve EOM

$$-\partial_z\left(\frac{\sqrt{1+\frac{z^6}{R^4}d^2}\partial_z A_1}{z\sqrt{1-\frac{z^4}{R^4}\{(\partial_0 A_1)^2-(\partial_z A_1)^2\}}}\right)+\partial_0\left(\frac{\sqrt{1+\frac{z^6}{R^4}d^2}\partial_0 A_1}{z\sqrt{1-\frac{z^4}{R^4}\{(\partial_0 A_1)^2-(\partial_z A_1)^2\}}}\right)=0 \qquad A_x=-\int^t E(s)ds+h(t,z)$$

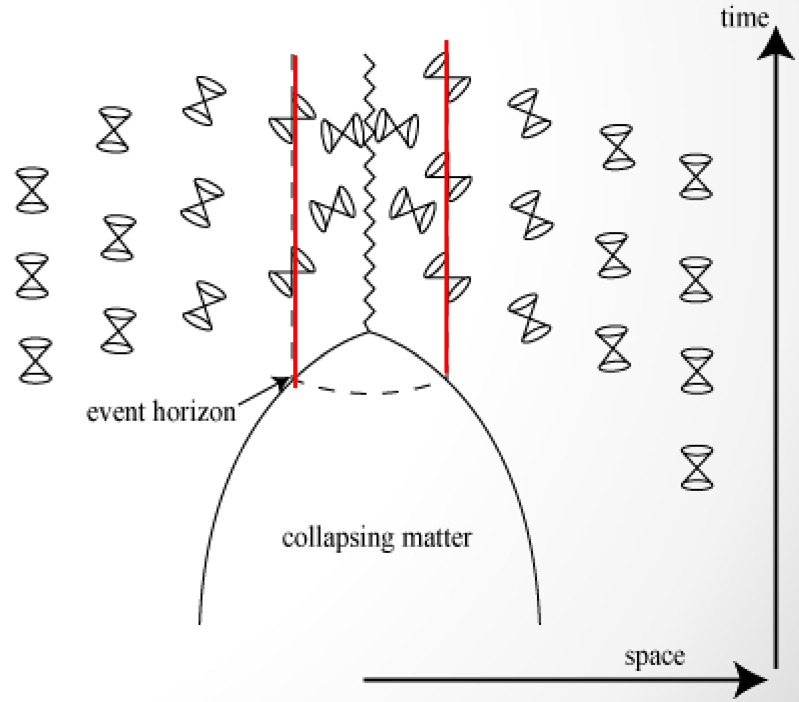


Horizon formation and the effective Hawking temperature





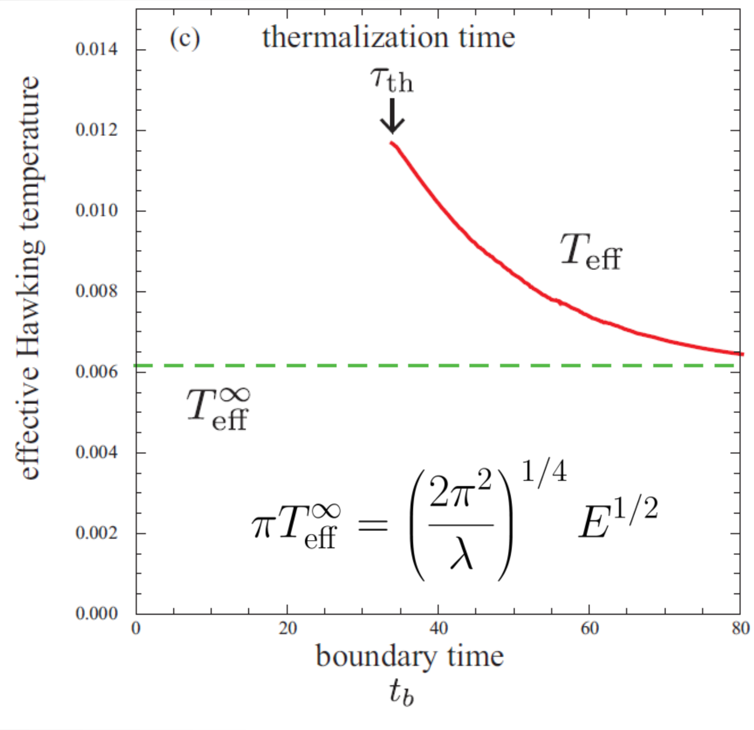
cf) Formation of black hole and horizon



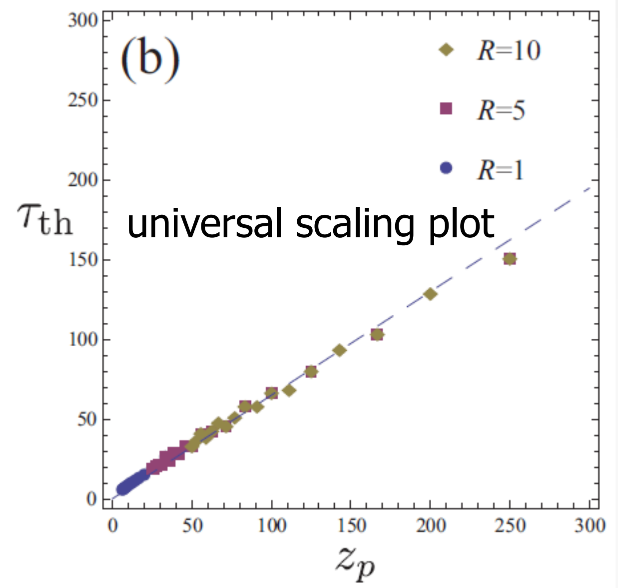
Hashimoto TO JHEP '13

Time evolution of the Hawking temperature

Effective temperature



dissipation source = gluons



$$\tau_{\rm th} = a\frac{\hbar}{k_B T_{\rm eff}}$$

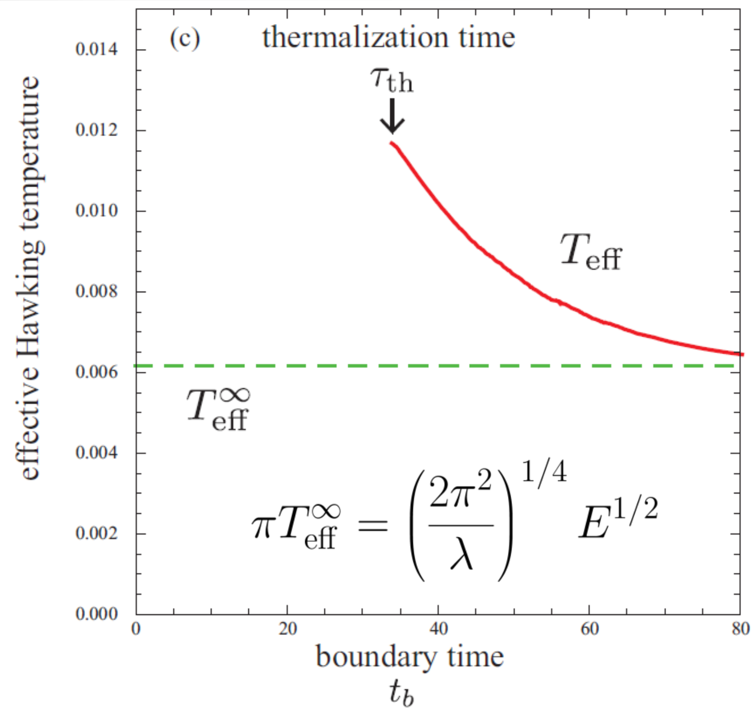
Planckian thermalization

(This can be obtained simply by dimensional analysis)

Hashimoto Oka JHEP '13

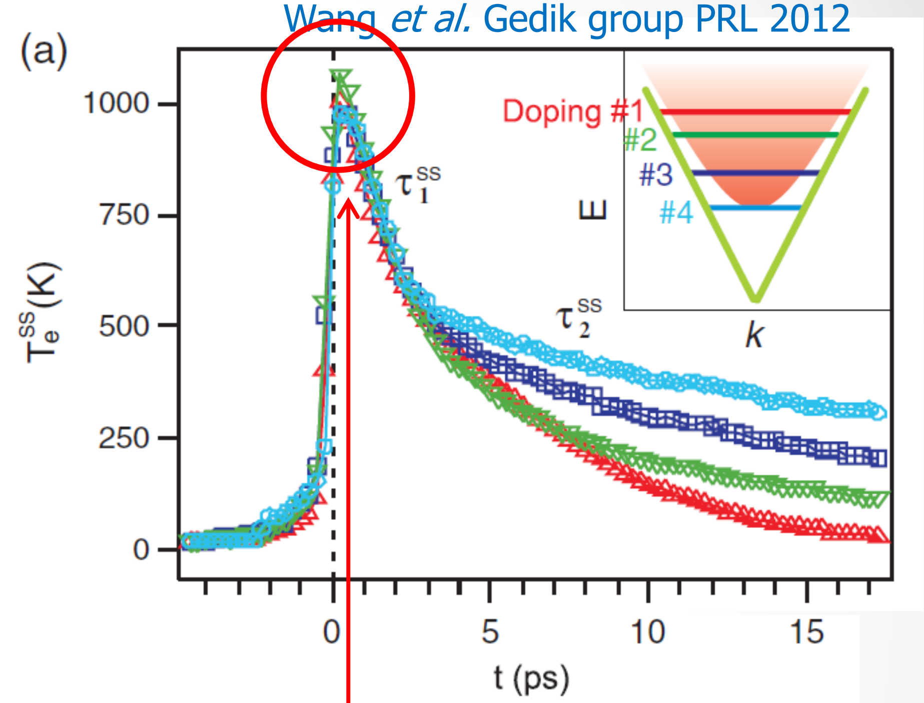
Comparison with experiment??

Effective temperature



dissipation source = gluons

Wang *et al.* Gedik group PRL 2012



dissipation source = acoustic phonons?

laser induced temperature ~ 100-1000K

$$\tau_{\rm th} = a\frac{\hbar}{k_B T_{\rm eff}} \sim 0.01 - 0.1{\rm ps}$$

Might explain the ultrafast ``thermalization" (good fit to the Fermi-function)

Hashimoto Oka JHEP '13

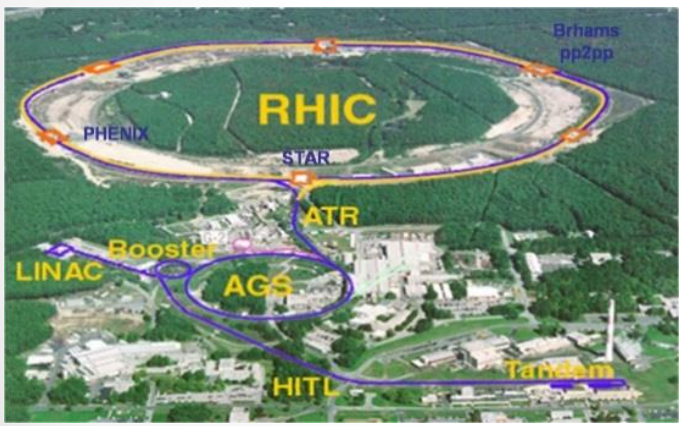
Summary and perspectives

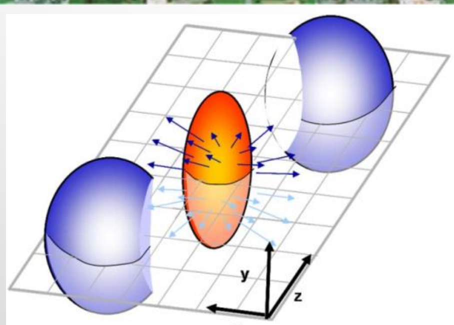
Universal relations

1. Nonlinear optical response in QCD (confinement phase)

2. Planckian thermalization

$$\tau_{\rm th} = a\frac{\hbar}{k_B T_{\rm eff}}$$





"Fast thermalization puzzle" in QGP formation

= QGP is described by a low viscosity fluid from a very fast time scale
= very very fast thermalization

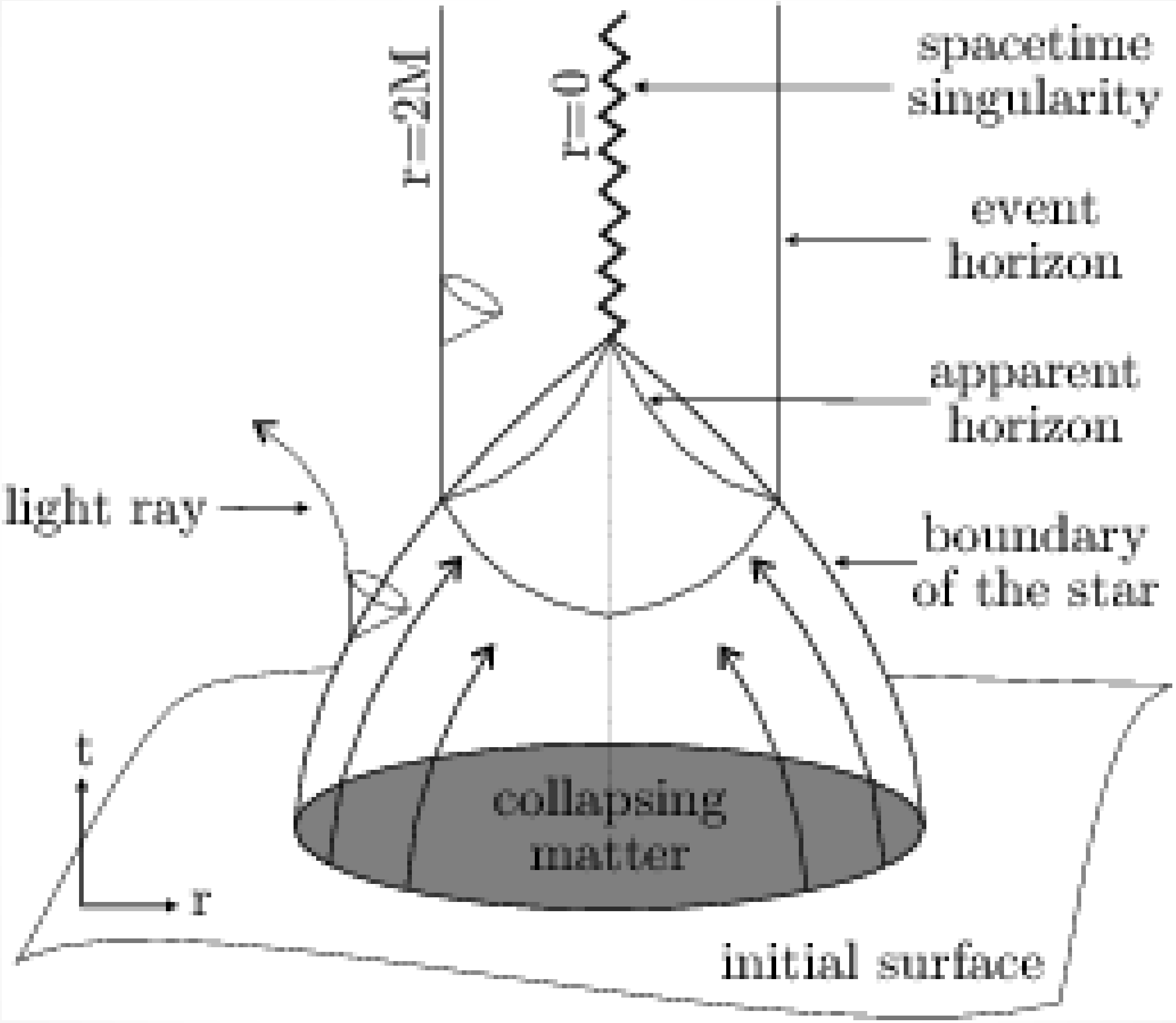
Strong E and B fields at ion collision

$$\pi T_{\rm eff}^{\infty} = \left(\frac{2\pi^2}{\lambda}\right)^{1/4} E^{1/2}$$

$$\tau_{\rm th} \leq 1 \ [{\rm fm/c}]$$

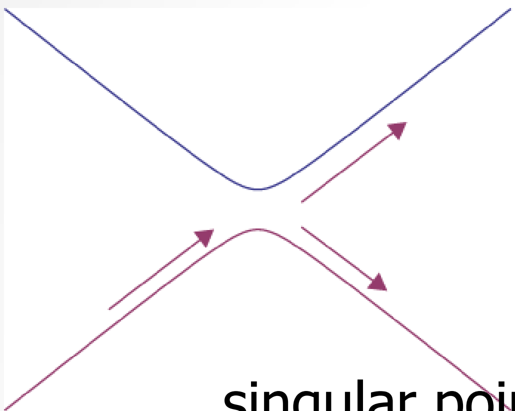
..consistent with experiment

(Note: Our theory do not describe the Yang-Mills thermalization)


r=2M
r=0
spacetime singularity
event horizon
apparent horizon
light ray
boundary of the star
t
r
collapsing matter
initial surface

Dirac model = Landau-Zener model

$$\mathcal{L} = \bar{\psi}\left[(i\partial_\mu - A_\mu)\gamma^\mu - m\right]\psi$$



singular point

$$t^* = \pm i\Delta_k/F$$

DC field

$$H(t) = \begin{pmatrix} Ft & \Delta_k \\ \Delta_k & -Ft \end{pmatrix} \qquad \Delta_k = \sqrt{m^2 + k_\perp^2}$$

$$S_{1,2} = \int_{t_0}^{t^*} dt'[E_2(\Phi(t')) - E_1(\Phi(t'))]$$
$$= i\int_{t_0}^{\Delta_k/F} dt'[\sqrt{-F^2t^2 + \Delta_k^2}]$$
$$= i\pi\frac{\Delta_k^2}{2F}$$

$$p_k = e^{-\pi\Delta_k^2/F}$$

k integral

$$\Gamma/V = 2\mathrm{Im}\mathcal{L} \sim \frac{e^2E^2}{4\pi^3}\exp\left(-\pi\frac{m^2}{eE}\right)$$

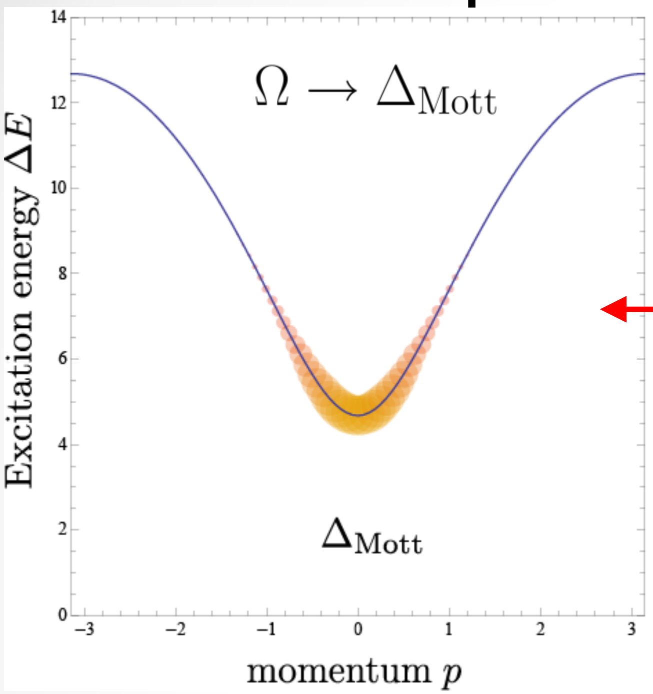
Heisenberg-Euler (1936)

Schwinger Phys. Rev.82, (1951)

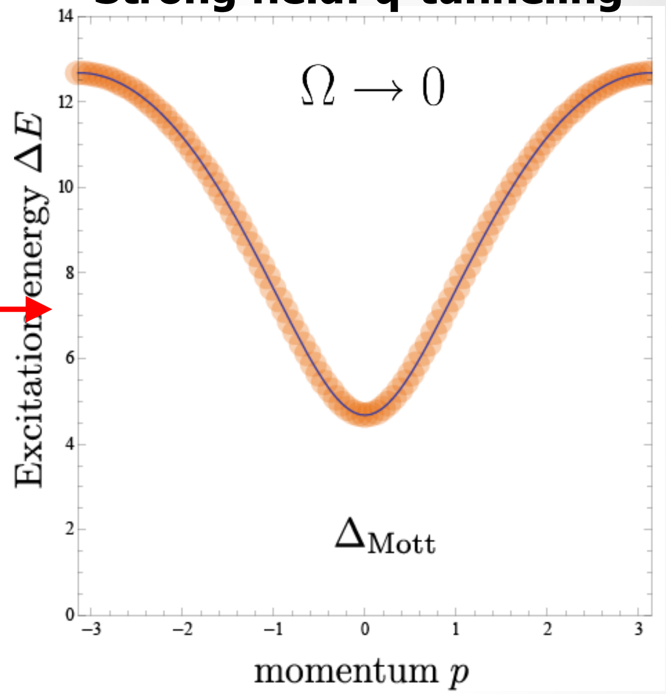
3+1 Dirac system

Keldysh crossover and the creation rate

Weak field : multi-photon



Strong field: q-tunneling



~ femto-second pump-probe

~ nonlinear transport

T=infinite state

produced carriers do not contribute to transport

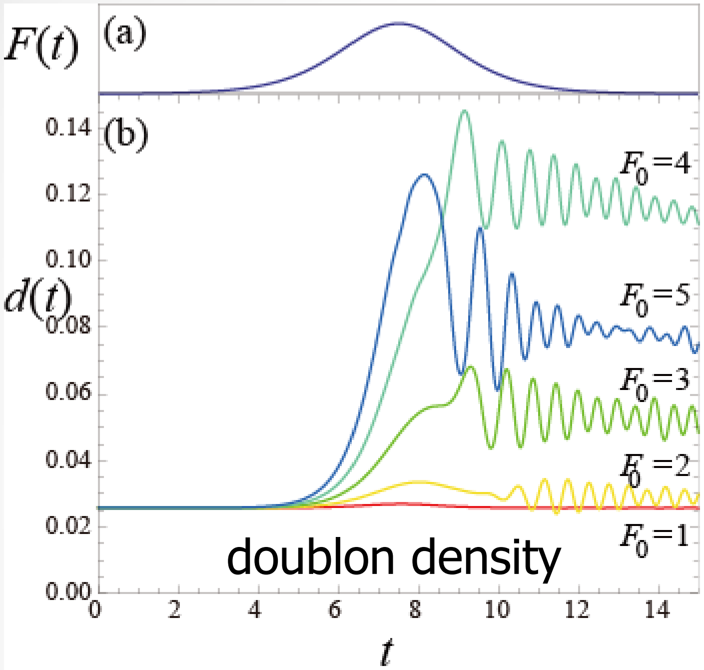
$$\sigma(T_{\mathrm{eff}} \to \infty) = 0$$

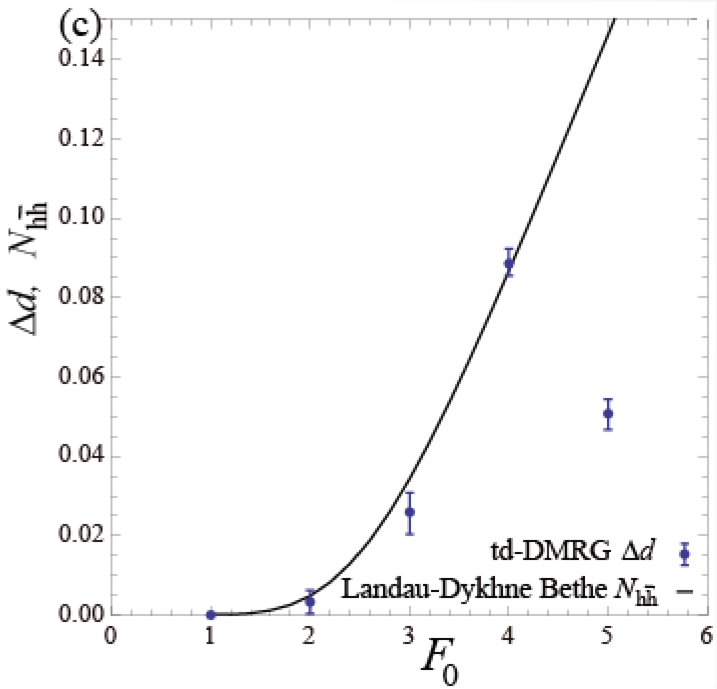
$$J \sim \mathcal{P}_p F + \sigma(T_{\mathrm{eff}}) d(t) F$$

In realistic systems, these excitations should cooled down

Comparison with numerical result (1d Hubbard)

pulse fields





doublon density (td-DMRG) $d(t) = \frac{1}{L}\sum_i \langle n_{i\uparrow} n_{i\downarrow}\rangle$

production rate (Bethe ansatz) $\Gamma = \int_{-\pi}^{\pi} \frac{dp}{2\pi} \mathcal{P}_p$

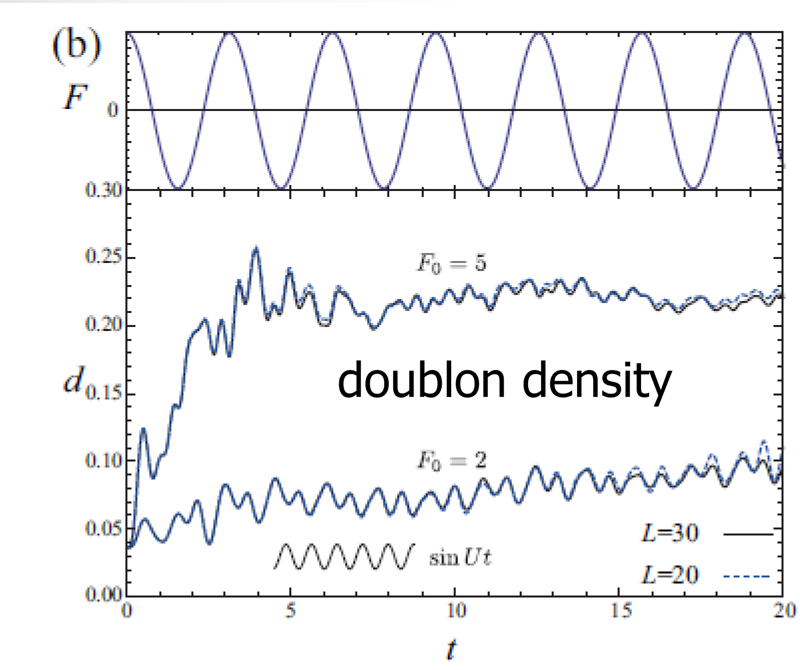
deviation at large F

Landau-Dyhkne formula ignores quantum interefrence in multi-tunneling events (Stokes phenomena)

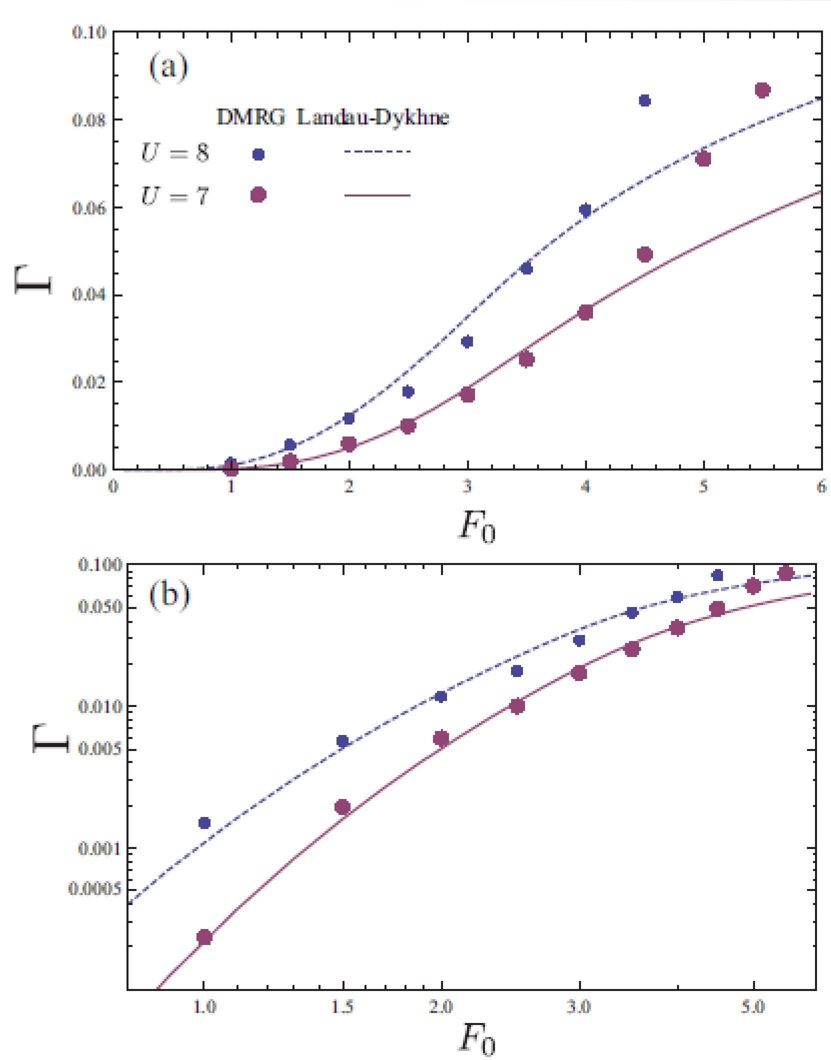
Comparison with numerical results (AC-field)

$$F(t) = F_0 \cos \Omega t$$

$$\Omega = 1$$

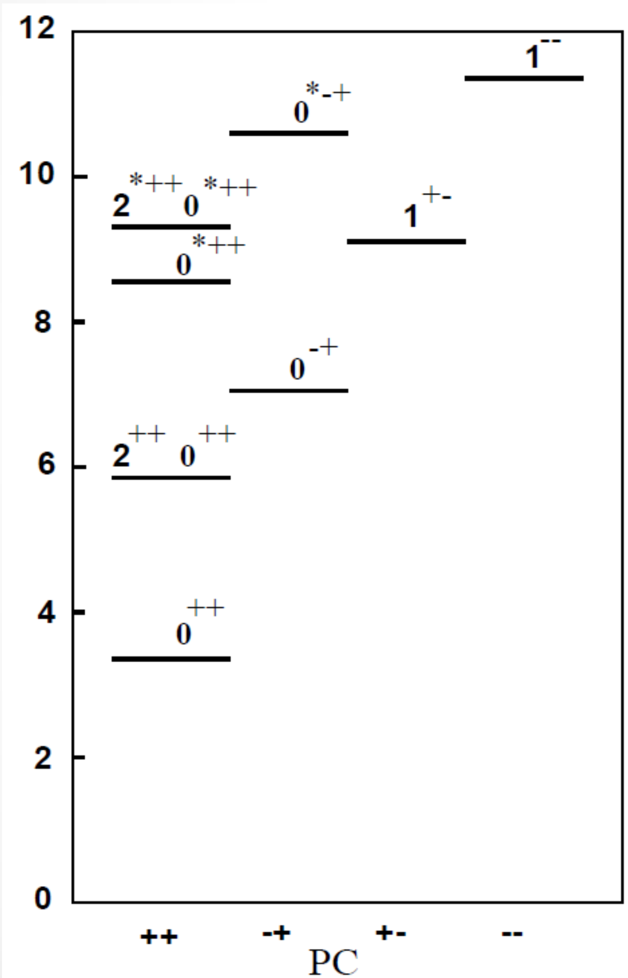


$$d \sim \Gamma t$$



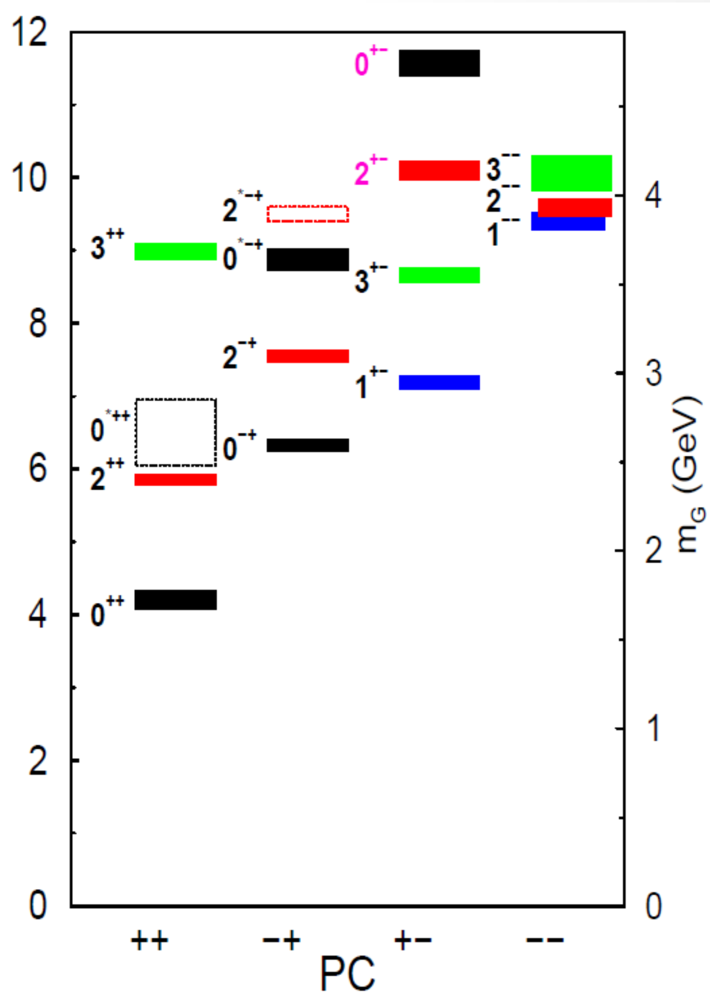
3-2 Superstring: better than simulations?

Superstring



[Brower,Mathur,Tan (03)]

Lattice



[Morningstar,Peardon (99)]

3-3 Superstring: better than simulations?

Radii of proton/neutron

[Sakai,Sugimoto,KH (0806.3122)]

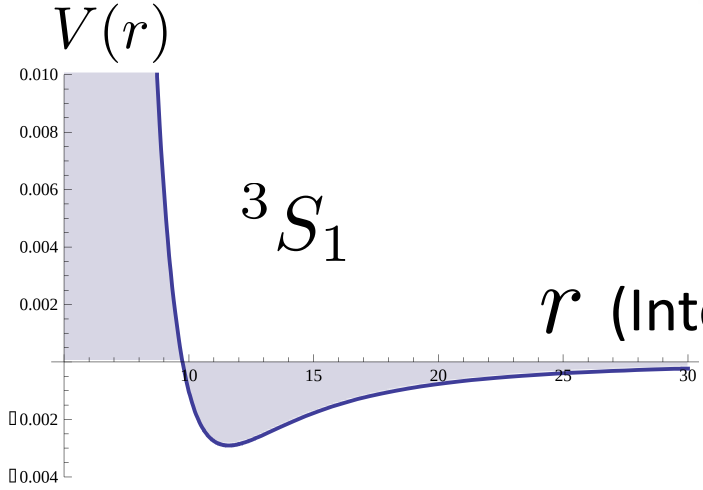
	Superstring	Experiment	
$\langle r^2 \rangle_{E,\mathrm{p}}$	$(0.74\ \mathrm{fm})^2$	$(0.875\ \mathrm{fm})^2$	
$\langle r^2 \rangle_{E,\mathrm{n}}$	0	$-0.116\ \mathrm{fm}^2$	
$\langle r^2 \rangle_A^{1/2}$	0.54 fm	0.674 fm	
μ_p	2.2	2.79	
μ_n	– 1.3	–1.91	
g_A	0.73	1.27	
$g_{\pi NN}$	7.5	13.2	
$g_{\rho NN}$	5.8	4.2 – 6.5	Lattice
$\mu_{\Delta^{++}}$	4.4	3.7 – 7.5	4.99
$\mu_{\Delta^{+}}$	2.3	–	2.49
$\mu_{\Delta^{0}}$	0.20	–	0.06
$\mu_{\Delta^{-}}$	–1.9	–	–2.45

3-4 Superstring: better than simulations?

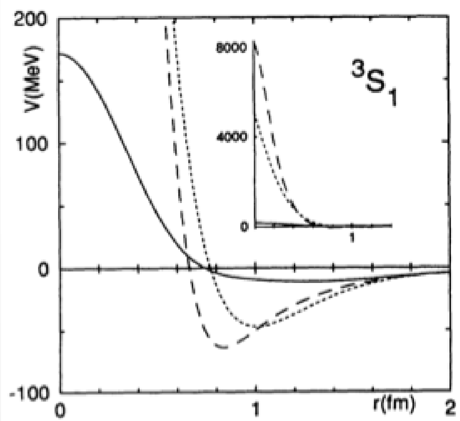
[Sakai,Sugimoto,KH (0901.4449)]

Nuclear forces

Superstring :

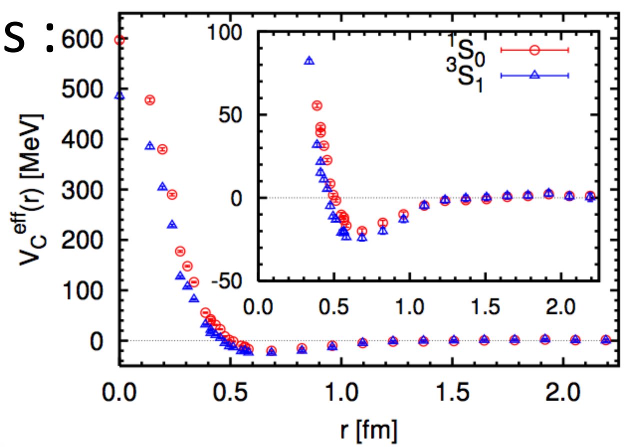


Experiments:



[Stoks,Klomp,Terheggen,deSwart ('94)]

Lattice QCD simulations :



[Aoki,Ishii,Hatsuda ('07)]