平衡化した量子純粋状態が持つ
エンタングルメントエントロピーの普遍的な振る舞い

arXiv: 1703.02993 (Nat. Comm. in press)

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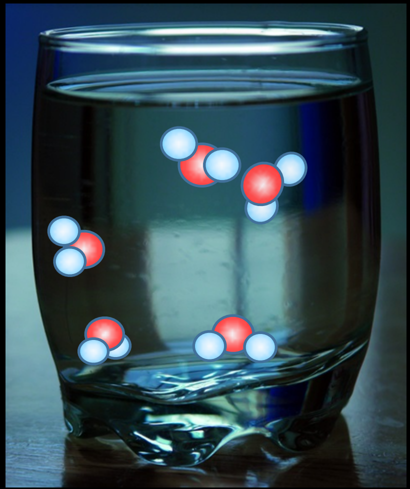
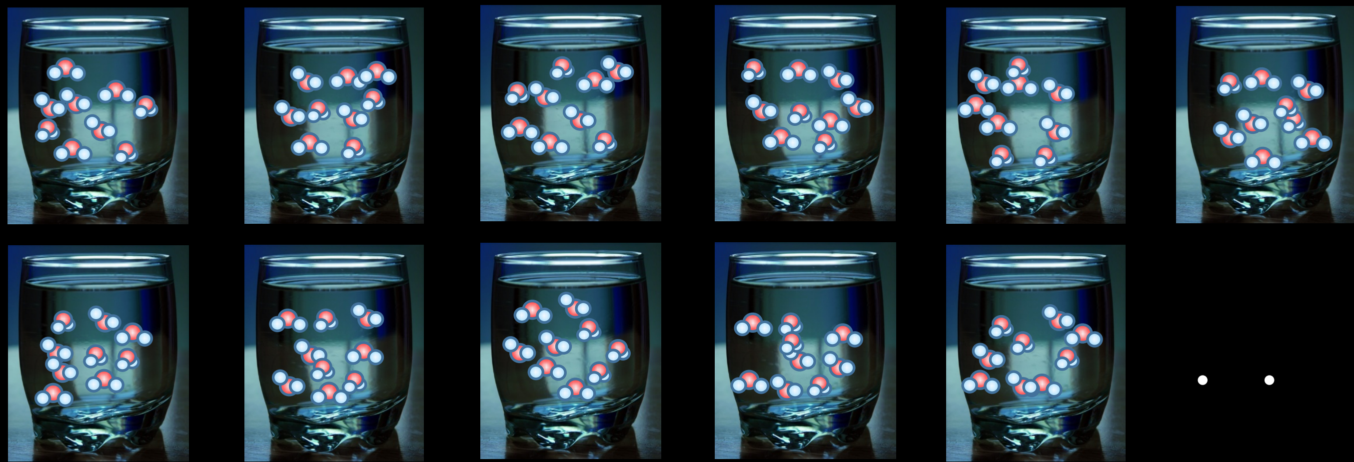
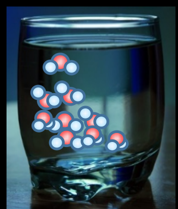
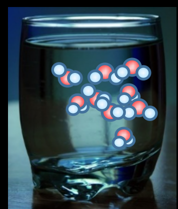
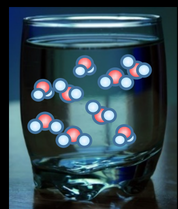
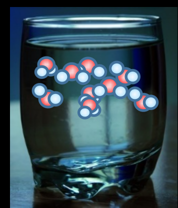
(ISSP) (ISSP) (IPMU, Tokyo Univ.)



✓ Ensemble average



✓ Superposition of all realizable states

$|\Psi\rangle = c_1$  $+ c_2$  $+ c_3$  $+ c_4$  $+ \cdots$

Thermal Pure Quantum States

SS and A. Shimizu, PRL (2012)
SS and A. Shimizu, PRL (2013)

The canonical thermal pure quantum (cTPQ) state at temperature $1/\beta$ is defined as

$$|\beta\rangle \equiv \frac{1}{\sqrt{Z}} \sum_i c_{ab} \exp\left[-\frac{1}{2}\beta\hat{H}\right] |i\rangle$$

Random number (c_{ab}) — High energy cut-off ($\exp\left[-\frac{1}{2}\beta\hat{H}\right]$) — Arbitrary basis ($|i\rangle$)

$Z \equiv \sum_i |c_i|^2 e^{-\beta E_i}$, $\{|a,b\rangle\}_{ab}$: arbitrary orthonormal basis

$\{c_i\}_i$: random complex numbers s.t. $c_i \equiv \frac{x_i + i y_i}{\sqrt{2}}$

(x_i and y_i obey normal distribution with mean = 0 and variance = 1)

✔ Equilibrium value

For $^\forall \epsilon > 0$,

$$\mathrm{P}\left(\left|\langle\beta|\hat{A}|\beta\rangle - \langle\hat{A}\rangle_\beta^{\mathrm{ens}}\right| \geq \epsilon\right) \leq \frac{1}{\epsilon^2} \frac{\langle(\Delta\hat{A})^2\rangle_{2\beta}^{\mathrm{ens}} + (\langle A\rangle_{2\beta}^{\mathrm{ens}} - \langle A\rangle_\beta^{\mathrm{ens}})^2}{\exp[2V\beta\{f(2\beta) - f(\beta)\}]}$$

$$\leq \frac{1}{\epsilon^2} \frac{V^{3m}}{\exp[O(V)]}$$

"Typicality"

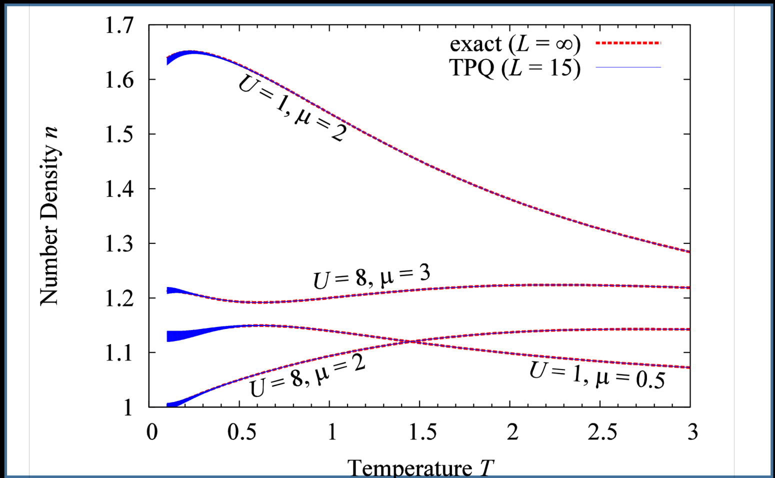
$f(\beta;V) \equiv \frac{F(\beta,V)}{V}$: Free energy density

$\langle\hat{A}\rangle_\beta^{\mathrm{ens}}$: Ensemble average, $\langle(\Delta\hat{A})^2\rangle_\beta^{\mathrm{ens}}$:Variance of $\hat{A}$

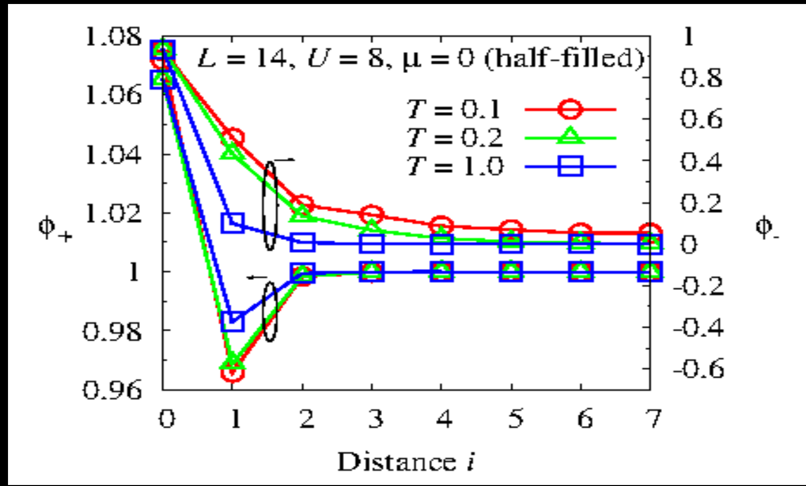
Numerical Applications of TPQ state

SS and A.Shimizu, arXiv (2013)
M. Hyuga, SS, K. Sakai, A.Shimizu ,PRB(2014)

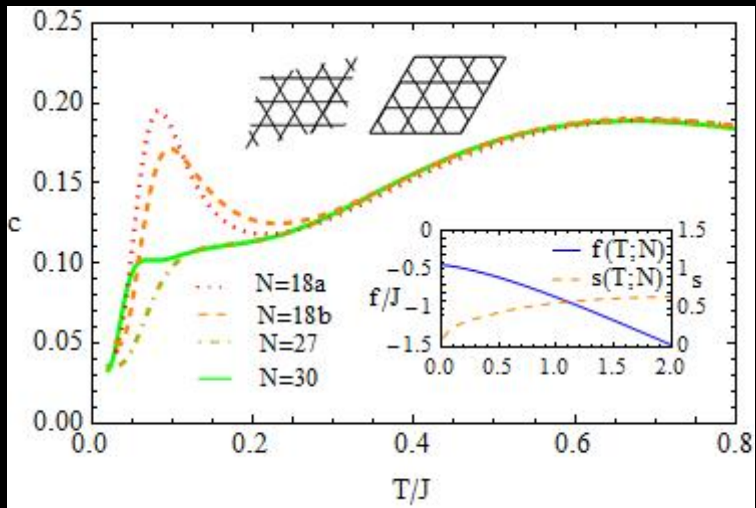
Number Density



Correlation Function



Specific Heat

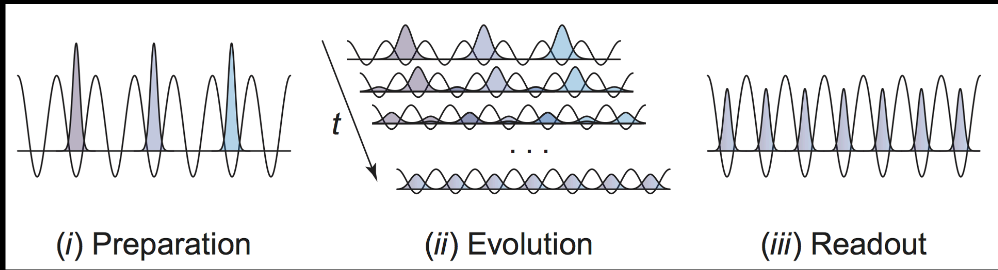


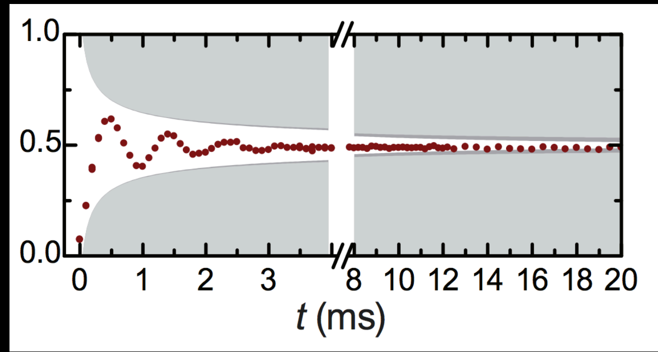
A single realization of the TPQ state gives equilibrium values of all macrocscopic quantities.

Is such a state metaphysical?

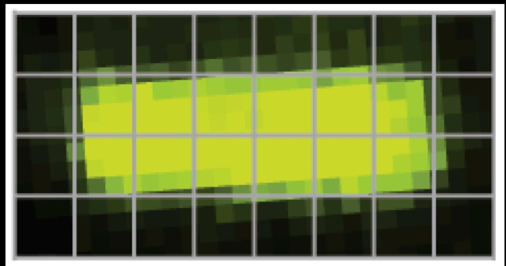
No, such states are realized in ultracold atoms experiments.

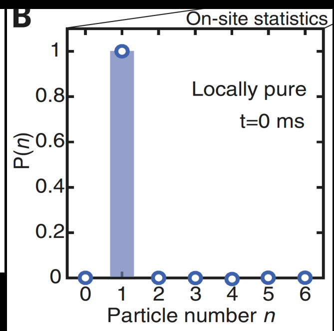
By I.Bloch's Group (S. Trotzky,et.al, Nat. Phys.(2012))





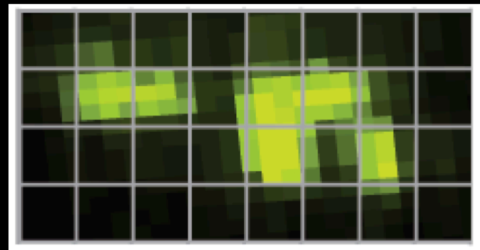
By M.Greiner's Group (A.Kaufman,et.al, Science(2016)
"Quantum thermalization through entanglement in an isolated many-body system")

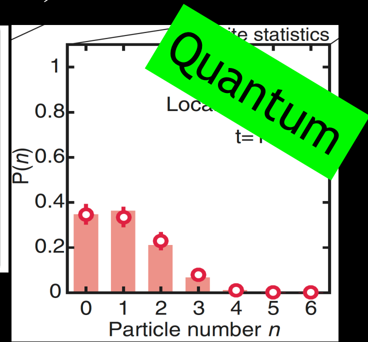




Initial state

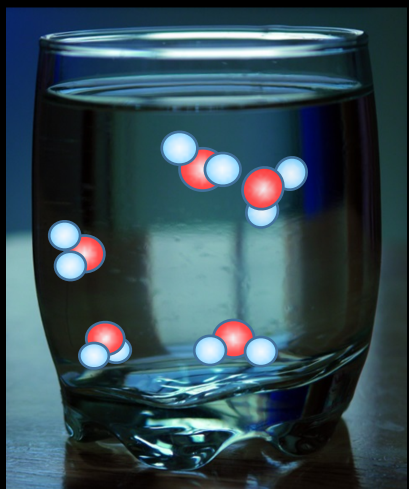
→



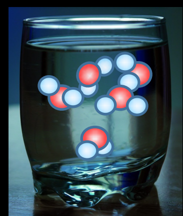
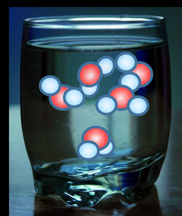
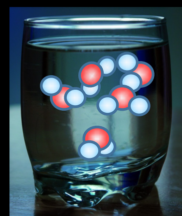
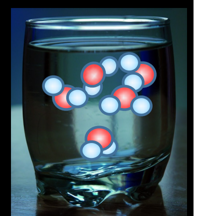


Quenched state

Not



But rather

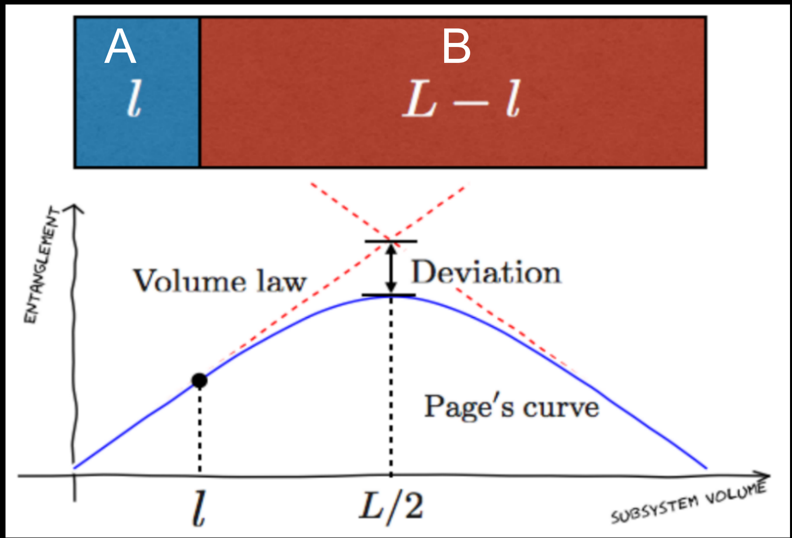
 $+c_2$  $+c_3$  $+c_4$ 

Bipartite Entanglement Entropy

We consider isolated quantum system

Hamiltonian: $\hat{H}$

Time evolution: $e^{-\frac{i}{\hbar}\hat{H}}|\psi\rangle$



When a system is devided into A and B;

Hilbert space: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

Quantum state: $\hat{\rho}_A \equiv \mathrm{tr}_B[|\psi\rangle\langle\psi|]$

Renyi entanglement entropy

$$S_n \equiv \frac{1}{1-n}\ln\left(\mathrm{tr}_A[\hat{\rho}_A^n]\right)$$

It quantifies a quantum correlation between A and B.

$$\xrightarrow{n\to 1} S_{\mathrm{vN}} \equiv \mathrm{tr}_A[\hat{\rho}_A \ln \hat{\rho}_A]$$

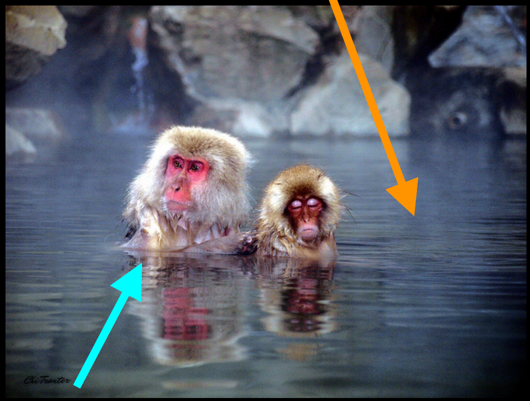
What is the role of the entanglement entropy in thermalization?

✓ When we look locally, the thermodynamic entropy is recovered as the entanglement entropy

✓ When we look grobally, quantum correlation breaks the correspondence between the thermal and entanglement entropy.

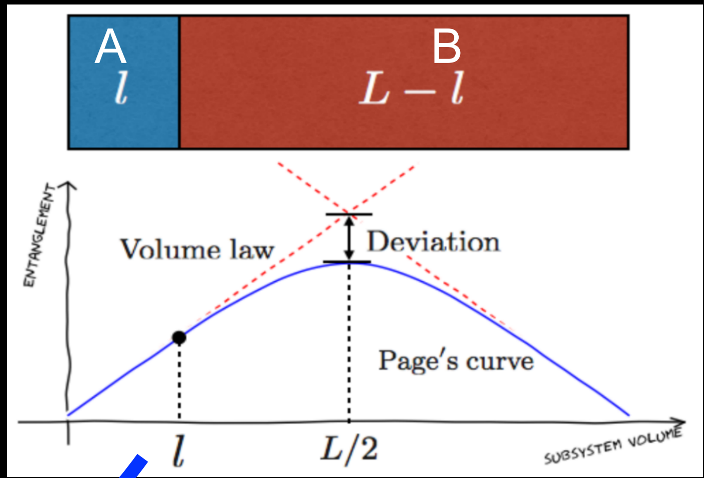
What is the role of the entanglement entropy in thermalization?

Bob



Alice

Thermalization



$l \ll L$

$l \sim L/2$

Alice

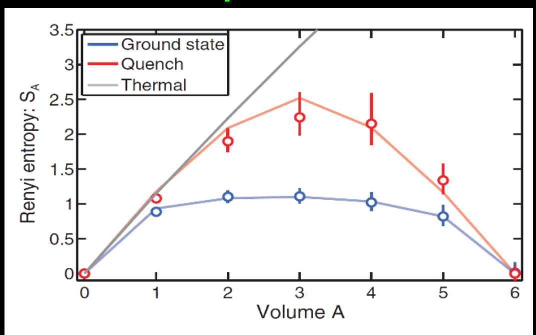
Bob

Quantum Correlation

Chi Tranter "snow monkeys in hot spring Japan"
https://www.flickr.com/photos/chitranter/14212780326/

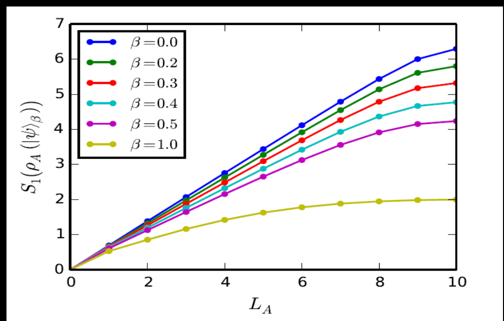
Steve Harris "dsc_7150-monkeys.jpg"
https://www.flickr.com/photos/steveharris/47049240/

Experiment



(Kaufman et.al., Science, 2016)

Energy eigenstates



(Garrison et.al, arXiv, 2015)

And more… (P. Calabrese and J. Cardy
J. Stat Mech 2007
T.Takayanagi and T. Ugajin
JHEP 2010 …..)

In this talk, I focus on…
✓Volume-law entanglement, i.e., the state has finite energy density
✓The second ($n=2$) Renyi entropy

Renyi Entropy of TPQ state

Let's calculate the Renyi entropy for the TPQ state;

$$|\beta\rangle \equiv \frac{1}{\sqrt{Z}} \sum_{a,b} c_{ab} \ \exp\left[-\frac{1}{2}\beta\hat{H}\right] \ |a,b\rangle \qquad Z \equiv \sum_i |c_i|^2 e^{-\beta\hat{H}}$$

Reduced density matrix

$$\hat{\rho}_A \equiv \mathrm{tr}_B\left[|\beta\rangle\langle\beta|\right]$$
$$= \frac{1}{Z}\sum_{a_1,a_2,a_3,a_4,b_1,b_2,b_3} c_{a_2b_2} c^*_{a_3b_3}$$
$$\times\langle a_1,b_1|e^{-\frac{1}{2}\beta\hat{H}}|a_2,b_2\rangle\langle a_3,b_3|e^{-\frac{1}{2}\beta\hat{H}}|a_4,b_1\rangle$$

Therefore, we can calculate $\mathrm{tr}_A[\hat{\rho}^n_A]$:

$$\mathrm{tr}_A[\hat{\rho}^n_A] = \frac{1}{Z^n}\sum_{\{a^j_i\}_{i,j},\{b^j_i\}_{i,j}} c_{a^1_2b^1_2} c^*_{a^1_3b^1_3} c_{a^2_2b^2_2} c^*_{a^2_3b^2_3}\cdots c_{a^1_2b^n_2} c^*_{a^1_3b^n_3}$$
$$\times\langle a^1_1,b^1_1|e^{-\frac{1}{2}\beta\hat{H}}|a^1_2,b^1_2\rangle\langle a^1_3,b^1_3|e^{-\frac{1}{2}\beta\hat{H}}|a^2_1,b^1_1\rangle$$
$$\times\langle a^2_1,b^2_1|e^{-\frac{1}{2}\beta\hat{H}}|a^2_2,b^2_2\rangle\langle a^2_3,b^2_3|e^{-\frac{1}{2}\beta\hat{H}}|a^3_1,b^2_1\rangle$$
$$\cdots\times\langle a^n_1,b^n_1|e^{-\frac{1}{2}\beta\hat{H}}|a^n_2,b^n_2\rangle\langle a^n_3,b^n_3|e^{-\frac{1}{2}\beta\hat{H}}|a^1_1,b^n_1\rangle$$

⟶ Just consider n=2 (&3) case in this talk.

Renyi Entropy

2nd Renyi

$$\overline{\mathrm{tr}_A[\hat{\rho}_A^2]} = \frac{1}{Z^2} \sum_{\{a_i\}_i,\{b_i\}_i} \overline{c_{a_2 b_2} c^*_{a_3 b_3} c_{a_5 b_5} c^*_{a_6 b_6}}$$
$$\times \langle a_1, b_1 | e^{-\frac{1}{2}\beta \hat{H}} | a_2, b_2 \rangle \langle a_3, b_3 | e^{-\frac{1}{2}\beta \hat{H}} | a_4, b_1 \rangle$$
$$\times \langle a_4, b_4 | e^{-\frac{1}{2}\beta \hat{H}} | a_5, b_5 \rangle \langle a_6, b_6 | e^{-\frac{1}{2}\beta \hat{H}} | a_1, b_4 \rangle$$

Using

Randomness in cTPQ state

$$\overline{c_{a_2 b_2} c^*_{a_3 b_3} c_{a_5 b_5} c^*_{a_6 b_6}} = \delta_{(a_2 b_2),(a_3 b_3)} \delta_{(a_5 b_5),(a_6 b_6)} + \delta_{(a_2 b_2),(a_6 b_6)} \delta_{(a_5 b_5),(a_3 b_3)},$$

we get

$$\overline{\mathrm{tr}_A[\hat{\rho}_A^2]} = \frac{1}{Z^2} \sum_{\{a_i\}_i,\{b_i\}_i} \langle a_1, b_1 | e^{-\beta \hat{H}} | a_4, b_1 \rangle \langle a_4, b_4 | e^{-\beta \hat{H}} | a_1, b_4 \rangle$$
$$+ \langle a_1, b_1 | e^{-\beta \hat{H}} | a_1, b_4 \rangle \langle a_4, b_4 | e^{-\beta \hat{H}} | a_4, b_1 \rangle$$
$$= \frac{1}{Z^2} \left(\mathrm{tr}_A \left[\mathrm{tr}_B [e^{-\beta \hat{H}}]^2 \right] + \mathrm{tr}_B \left[\mathrm{tr}_A [e^{-\beta \hat{H}}]^2 \right] \right)$$

2^{nd} Renyi Entropy

We evaluate 2^{nd} Renyi entropy in statistical-mechanical way:

$$\overline{S_2} = -\ln\left[\frac{\mathrm{tr}_A\left[\mathrm{tr}_B[e^{-\beta\hat{H}}]^2\right] + \mathrm{tr}_B\left[\mathrm{tr}_A[e^{-\beta\hat{H}}]^2\right]}{Z(\beta)^2}\right]$$

Step1) For large $Z(\beta)$, by using transfer matrices, we can prove

$\mathrm{tr}_A\left[\mathrm{tr}_B[e^{-\beta\hat{H}}]^2\right] = P \times Z_A(2\beta)Z_B(\beta)^2$, and $Z(\beta) = Q \times Z_A(\beta)Z_B(\beta)$

P, Q: const.

Therefore, $\overline{S_2} = -\ln\left(\frac{Z_A(2\beta)}{Z_A(\beta)^2} + \frac{Z_B(2\beta)}{Z_B(\beta)^2}\right) + \ln R$ $\quad R \equiv Q/P$

Step2) For large d, the free energy is extensive, i.e., $\frac{Z_A(2\beta)}{Z_A(\beta)^2} = Sa^{-\ell}$

Extensivity of density matrix

S: const.

Finally, we get $\overline{S_2} = -\ln\left(a^{-\ell} + a^{-L+\ell}\right) + \ln K$ $\quad K \equiv R/S$

$$= \ell \ln a - \ln\left(1 + a^{-L+2\ell}\right) + \ln K$$

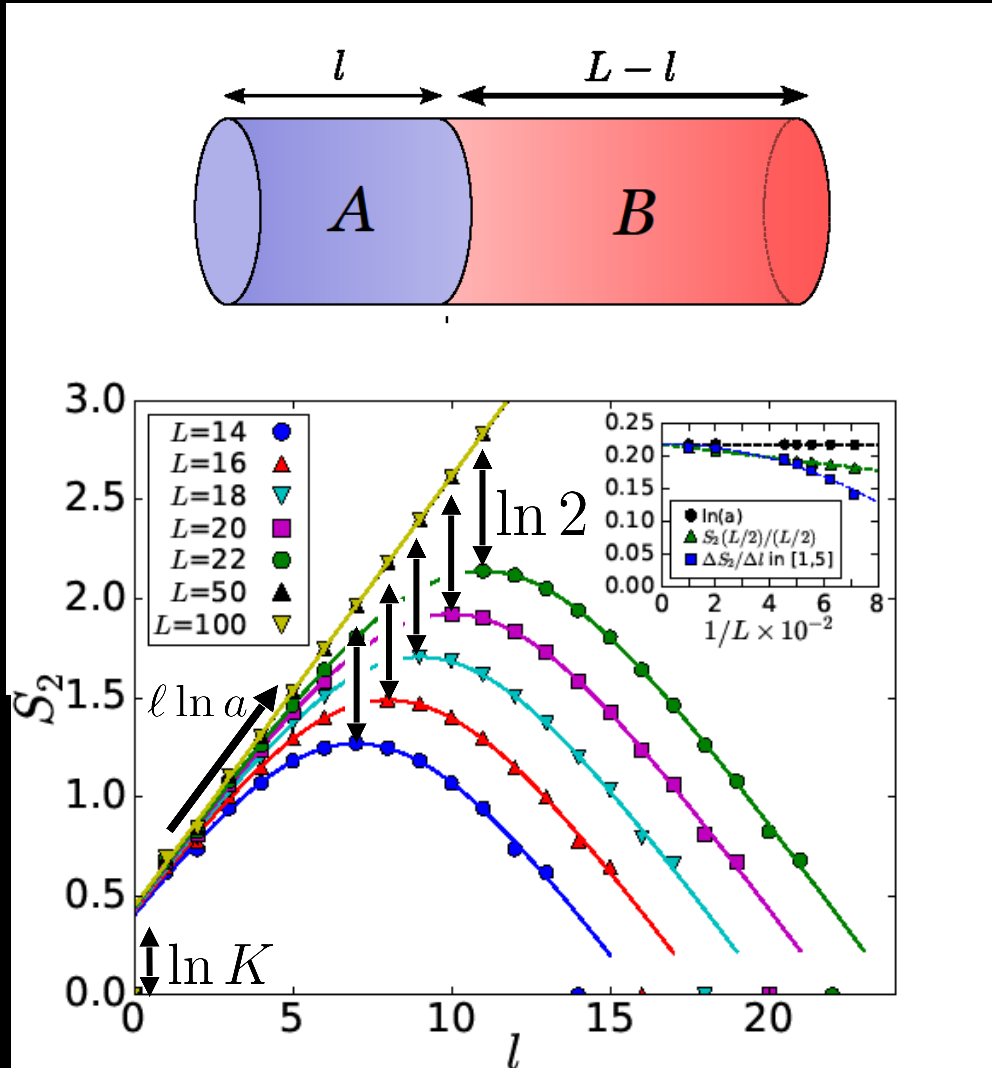
Analytical Result:
2nd Renyi Entropy of cTPQ states

$$\overline{S_2} = \boxed{\ell \ln a} - \boxed{\ln\left(1 + a^{-L+2\ell}\right)} + \boxed{\ln K}$$

Volume law

Deviation around $\ell \sim \frac{L}{2}$

Constant offset



Key of derivation

- ✓ Randomness in cTPQ state
- ✓ Extensivity of density matrix

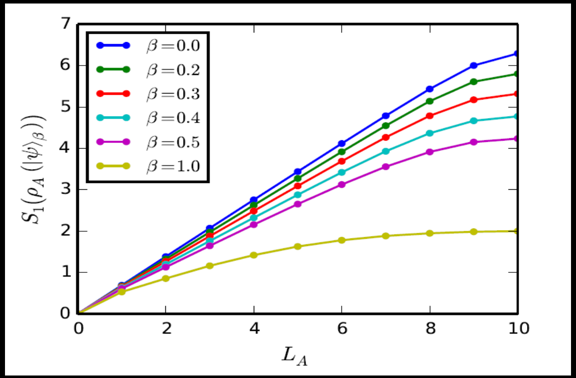
- ✓ 2nd Renyi entropy is characterized just by two parameters, a and K.
- ✓ Right at the middle, $\ell \sim \frac{L}{2}$, the deviation is always $\ln 2$, independent of β.

Numerical calculation

Entanglement entropy of the cTPQ states

$$\overline{S_2} = \ell \ln a - \ln\left(1 + a^{-L+2\ell}\right) + \ln K$$

Entanglement entropy of excited pure quantum states



(Garrison and Grover, arXiv, 2015)

We will use our analytical result for the cTPQ states as a fitting function for other excited pure states.

→ It works when the state is scrambled, and doesn't when it isn't.

Numerical calculation 1: Energy eigenstates

Energyeigenstate Thermalization Hypothesis (ETH) (Deutsch, Srednicki, Rigol) :

In nonintegrable systems, a single energy eigenstate is expected to represent a thermal equilibrium state.

Popular expectation in ETH: At $\ell << L$, $\mathrm{tr}_B[|n\rangle\langle n|] \simeq e^{-\beta \hat{H}}$.

$|n\rangle$: Energy eigenstate

β : Inverse temperature estimated from Energy

However, $\mathrm{tr}_B[|n\rangle\langle n|]$ and $e^{-\beta \hat{H}}$ will be very different at $\ell \sim L$

How about entanglement?

→ We will test our fitting function for energy eigenstates.

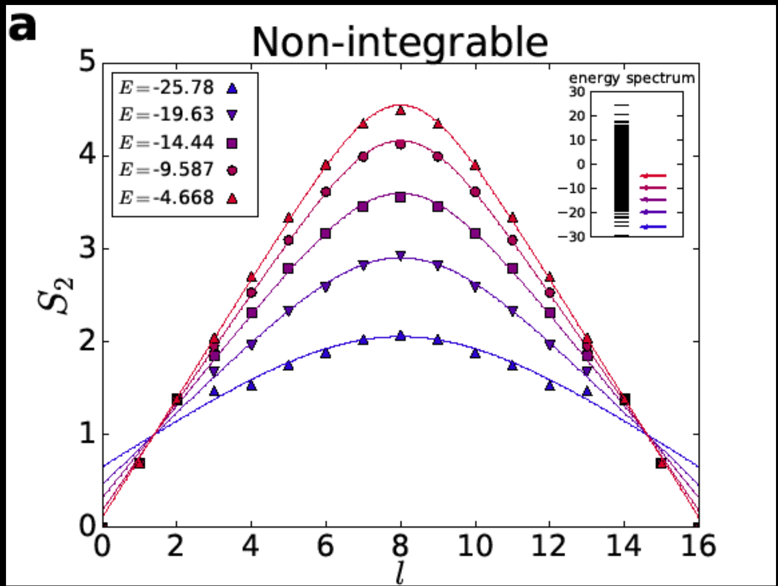
$$\overline{S_2} = \ell \ln a - \ln\left(1 + a^{-L+2\ell}\right) + \ln K$$

Numerical calculation 1: Energy eigenstates

$$\hat{H} = \sum_i S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z + J_2(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + S_i^z S_{i+2}^z)$$

Non-integrable $\Delta = 0, J_2 = 0.45$

Integrable $\Delta = 2, J_2 = 0$



Dots: numerical data
Line: Fittings

Energy eigenstates agree with the function: $\overline{S_2} = \ell \ln a - \ln\left(1 + a^{-L+2\ell}\right) + \ln K$

- ✓ At $\ell \ll L$, volume law. Consistent to Energy eigenstate thermalization hypothesis (ETH)
- ✓ At $\ell \sim L$, S_2 still exhibits generic behavior predictable by our formula

Numerical calculation 1: Energy eigenstates

$$\hat{H} = \sum_i S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z + J_2(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + S_i^z S_{i+2}^z)$$

Non-integrable $\Delta = 0, J_2 = 0.45$

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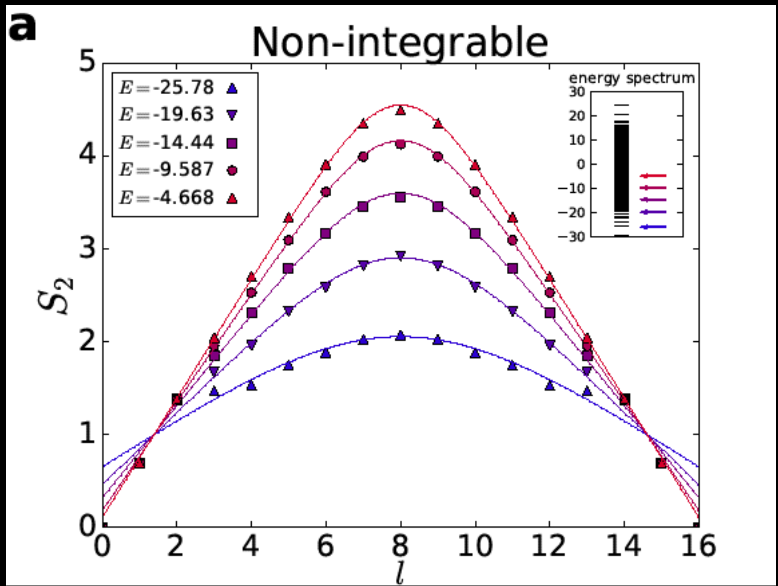
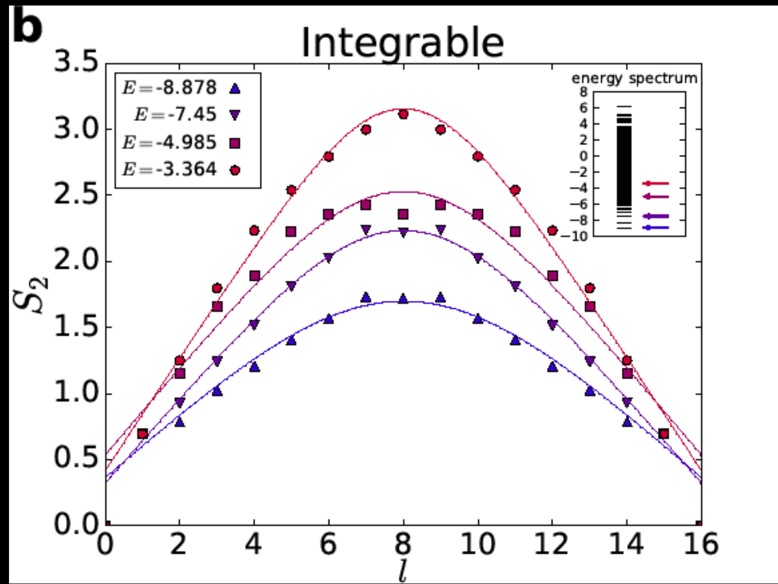




Dots: numerical data
Line: Fittings

In the integrable model, there are a lot of energy eigenstates for which the fittings don't work.

→ Let's see this difference quantitatively.

Numerical calculation 1: Energy eigenstates

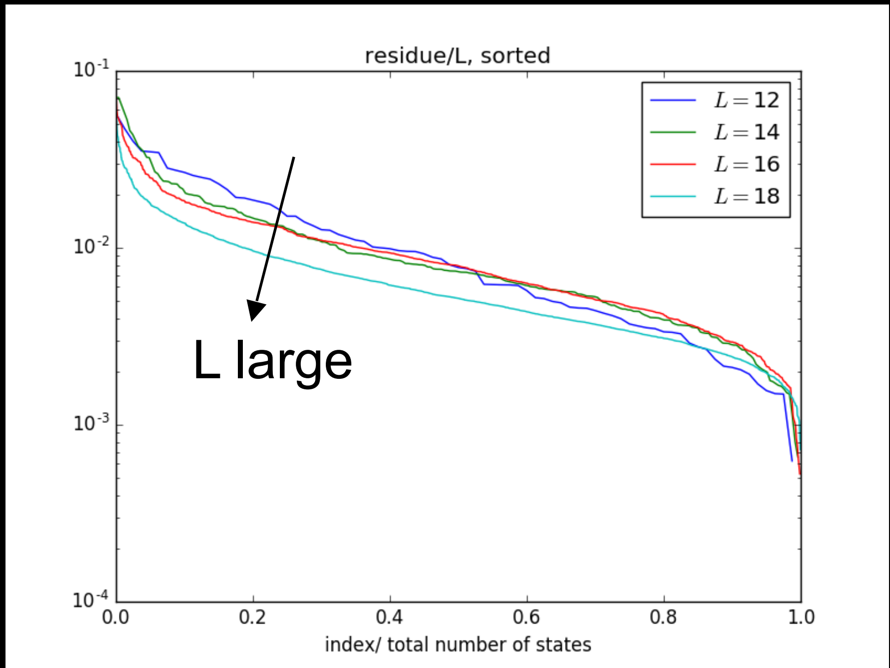
$$\hat{H} = \sum_i S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z + J_2(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + S_i^z S_{i+2}^z)$$

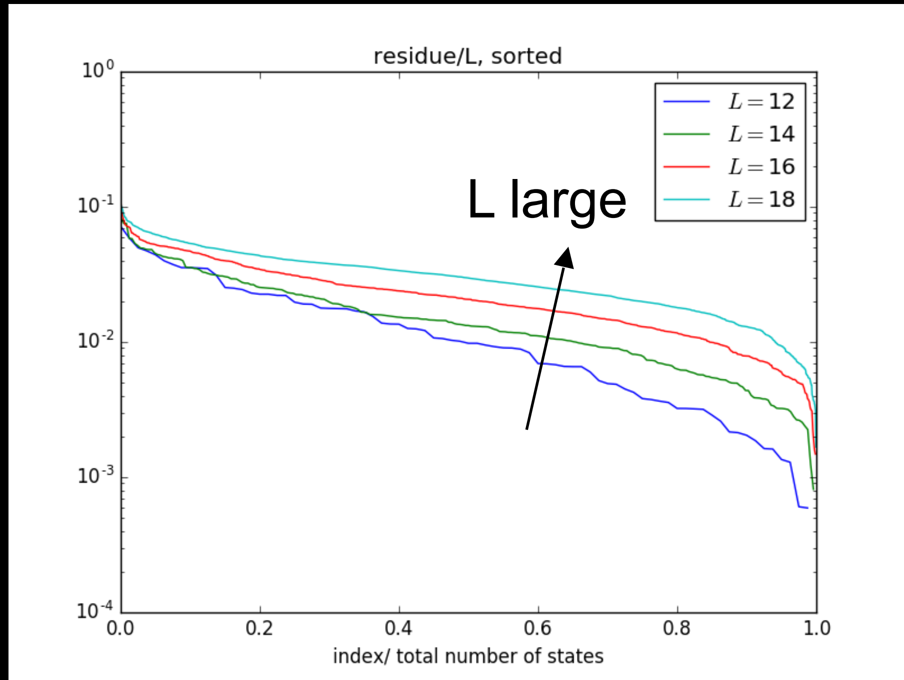
Non-integrable $\Delta = 0, J_2 = 0.45$

Integrable $\Delta = 2, J_2 = 0$

Bad

Good





Vertical axis: Residues of fittings of energy eigenstates

Horizontal axis: Index of eigenstates. Eigenstates are sorted in order of their residues, i.e., percentile.

Non-integrable: Fittings improves as L increases. Leads to typical behavior.
Integrable: Fittings gets worse as L increases, maybe due to many conserved quantities. Cf) L.Vidmar,L.Hackl,E.Bianchi, M.Rigol PRL.119,020601(2017)

Effect of energy fluctuation in microcanonical energy shell

Cf) T.Grover et.al, arXiv 1503.00729 &1709.08784, A.Dymarsky, N.Lashkari,& H.Liu, arXiv 1611.08764

Correction from energy fluctuation

$$\rho_{\rm mc/2} \equiv {\rm Tr}_B |n\rangle\langle n| \simeq \frac{e^{-\beta \hat{H}}}{Z}?$$

von Neumann Entropy

$$S_{\rm vN}(\hat{\rho}_{\rm mc}) = S_{\rm vN}(\frac{e^{-\beta \hat{H}}}{Z})$$

$$S_{\rm vN}(\hat{\rho}) \equiv -{\rm Tr}_A[\rho {\rm ln} \rho]$$

However, energy fluctuation is different

$$\langle n|(\Delta \hat{H})^2|n\rangle = 0$$

$$\langle \beta|(\Delta \hat{H})^2|\beta\rangle = O(L)$$

Renyi entropy has the correction which comes from the energy fluctuation…

$$S_2(\hat{\rho}_{\rm mc}) \neq S_2(\frac{e^{-\beta \hat{H}}}{Z})$$

(T.Grover et.al, arXiv 1709.08784, A.Dymarsky, N.Lashkari,& H.Liu, arXiv 1611.08764)

Numerical calculation 1: Energy eigenstates

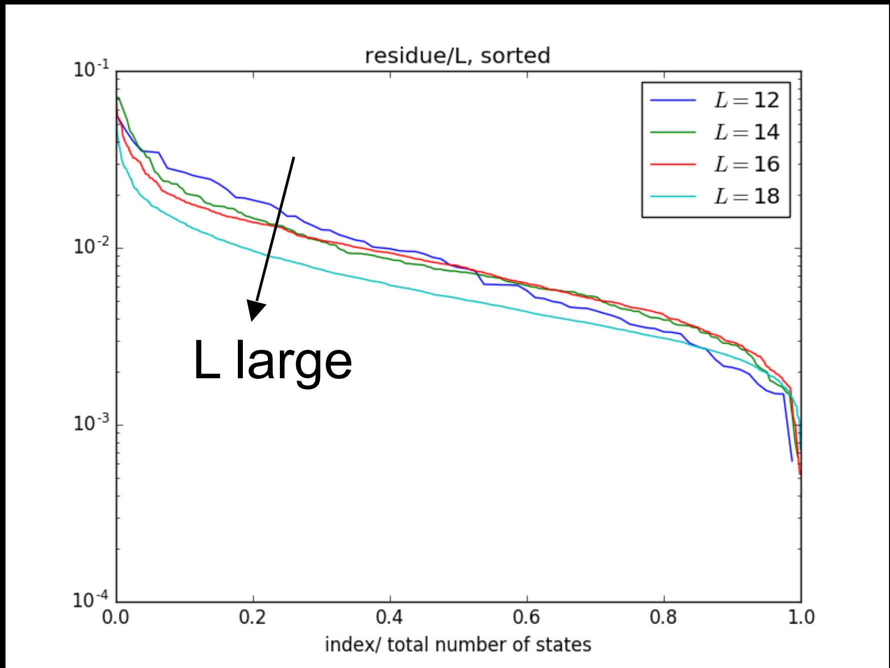
$$\hat{H} = \sum_i S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z + J_2(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + S_i^z S_{i+2}^z)$$

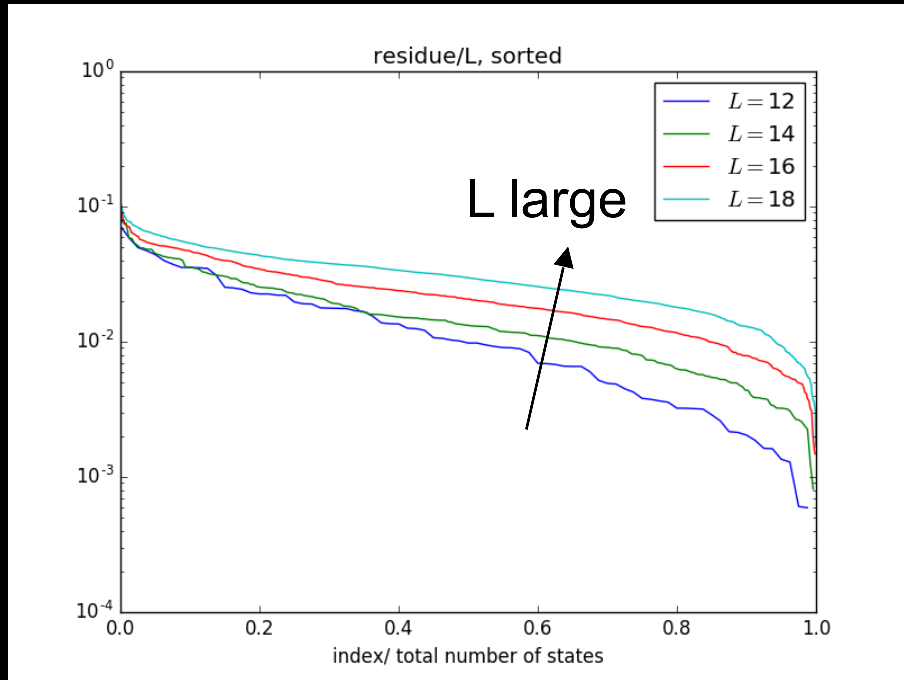
Non-integrable $\Delta = 0, J_2 = 0.45$

Integrable $\Delta = 2, J_2 = 0$

Bad

Good





Consider microcanonical-type TPQ state $|E\rangle \equiv \frac{1}{\sqrt{D(E)}} \sum_{n \in \mathrm{energyshell}} c_n |n\rangle$

2nd Renyi entropy is $\overline{S}_2 = -\ln\left[\mathrm{tr}_A\left(\mathrm{tr}_B\left(\hat{\rho}_{\mathrm{mc}}\right)^2\right) + \mathrm{tr}_B\left(\mathrm{tr}_A\left(\hat{\rho}_{\mathrm{mc}}\right)^2\right)\right]$ (T.Grover et.al, arXiv 1709.08784)

This correction in non-integrable model is subtle effect only appear in eigenstates and in large L (and is easily fixable).
By contrast, the behavior is completely different in integrable model

Numerical calculation 2: States after quantum quench

Quench protocol Initial state: $|\uparrow\downarrow\uparrow\downarrow\cdots\uparrow\downarrow\rangle$

We suddenly change Hamiltonian from $\hat{H} = \hat{1}$

to $\hat{H} = \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$ ← Integrable Hamiltonian

In integrable systems, a state after quench may relaxes to some stationary state, but it never thermalize.

Thermalize?

Yes! — It locally relaxes to…

Thermal state (Gibbs ensemble)

$e^{-\beta \hat{H}}$, characterized only by temperature $1/\beta$.

No ☹ — It locally relaxes to…

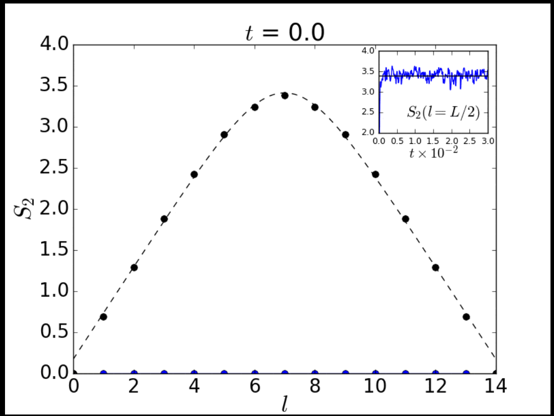
Generalized Gibbs Ensemble (GGE)

$e^{-\beta \hat{H} + \sum_n \lambda_n I_n}$, characterized by temperature $1/\beta$ and many integrals of motions I_n λ_n: Lagrangian multiplier

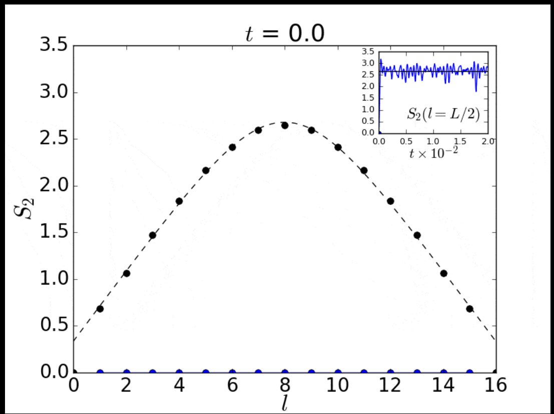
How about entanglement?

Numerical calculation 2: States after quantum quench

Non-integrable $\hat{H} = \sum_i \overrightarrow{S}_i \cdot \overrightarrow{S}_{i+1} + 0.5(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + S_i^z S_{i+2}^z)$



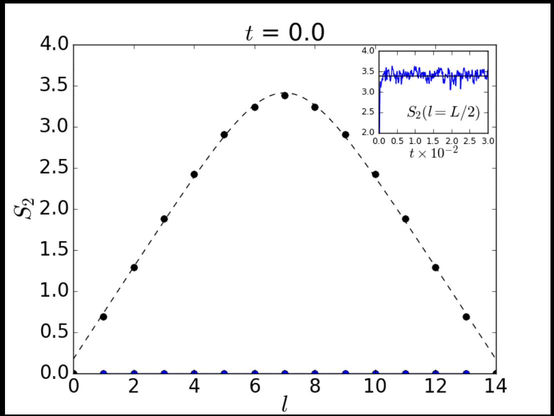
Integrable $\hat{H} = \sum_i \overrightarrow{S}_i \cdot \overrightarrow{S}_{i+1}$

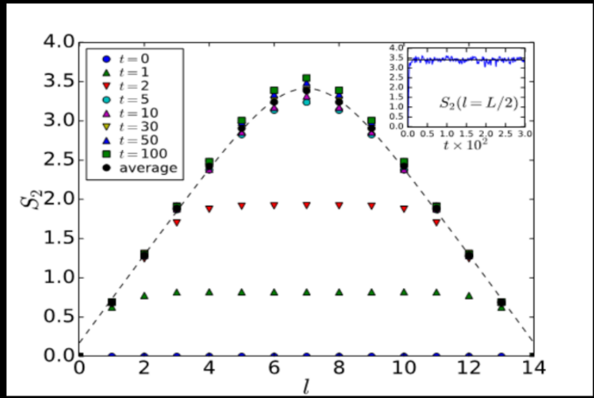


After the entanglement entropy saturates, it oscillates around our predicted curve.

Numerical calculation: States after quantum quench

Non-integrable $\hat{H} = \sum_i \overrightarrow{S}_i \cdot \overrightarrow{S}_{i+1}$ $+ 0.5(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + S_i^z S_{i+2}^z)$

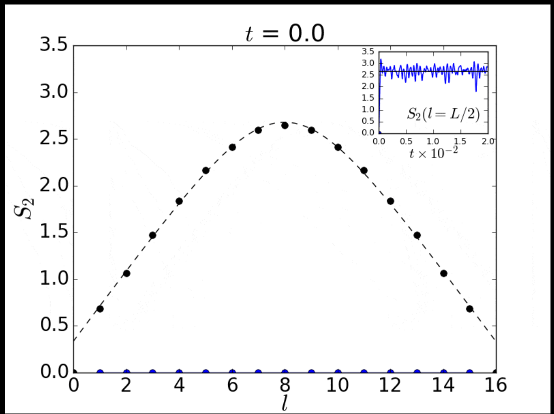


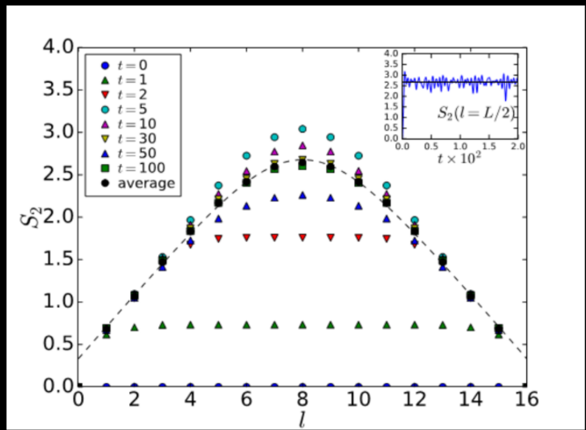


After the entanglement entropy saturates, it oscillates around our predicted curve.

Integrable $\hat{H} = \sum_i \overrightarrow{S}_i \cdot \overrightarrow{S}_{i+1}$

The time averages agree with our fitting.





Even the state has O(L) local integrables of motions, $e^{-\beta\hat{H}+\sum_n \lambda_n I_n}$,
2nd Renyi entropy is still characterized only by two parameters!

Why it works?

Key of derivation

✓ Randomness in cTPQ state

Extensivity of density matrix

In the case of a stationary state after a quench, the quantum state is

$$|\psi\rangle = \sum_n e^{-\frac{i}{\hbar}E_n} a_n |n\rangle$$

And the 2nd Renyi entropy is

$$S_2 = -\ln\left(\mathrm{tr}_A\left(\sum_{i,j,k,l} e^{-\frac{i}{\hbar}(E_i - E_j + E_k - E_l)} a_i a_j^* a_k a_l^* \mathrm{tr}_B\left(|i\rangle\langle j|\right)\mathrm{tr}_B\left(|k\rangle\langle l|\right)\right)\right).$$

$$\downarrow$$

$$\delta_{i,j}\delta_{k,l} + \delta_{i,l}\delta_{j,l}$$

If $E_i \neq E_j$ for $i \neq j$ and $E_i - E_j \neq E_k - E_j$ for $i \neq k$ and $j \neq l$,

$$S_2 = -\ln\left[\mathrm{tr}_A\left(\mathrm{tr}_B\left(\hat{\rho}_{\mathrm{dia}}\right)^2\right) + \mathrm{tr}_B\left(\mathrm{tr}_A\left(\hat{\rho}_{\mathrm{dia}}\right)^2\right)\right]$$

Why it works?

Key of derivation

- ✓ Randomness in cTPQ state
- ✓ Extensivity of density matrix

$$S_2 = -\ln\left[\mathrm{tr}_A\left(\mathrm{tr}_B\left(\hat{\rho}_{\mathrm{dia}}\right)^2\right) + \mathrm{tr}_B\left(\mathrm{tr}_{\mathrm{A}}\left(\hat{\rho}_{\mathrm{dia}}\right)^2\right)\right]$$

In the case of non-integrable model,

Thermal state (Gibbs ensemble)

$e^{-\beta\hat{H}}$, characterized only by temperature $1/\beta$.

Since $\langle\psi|(\Delta\hat{H})^2|\psi\rangle = O(L)$,

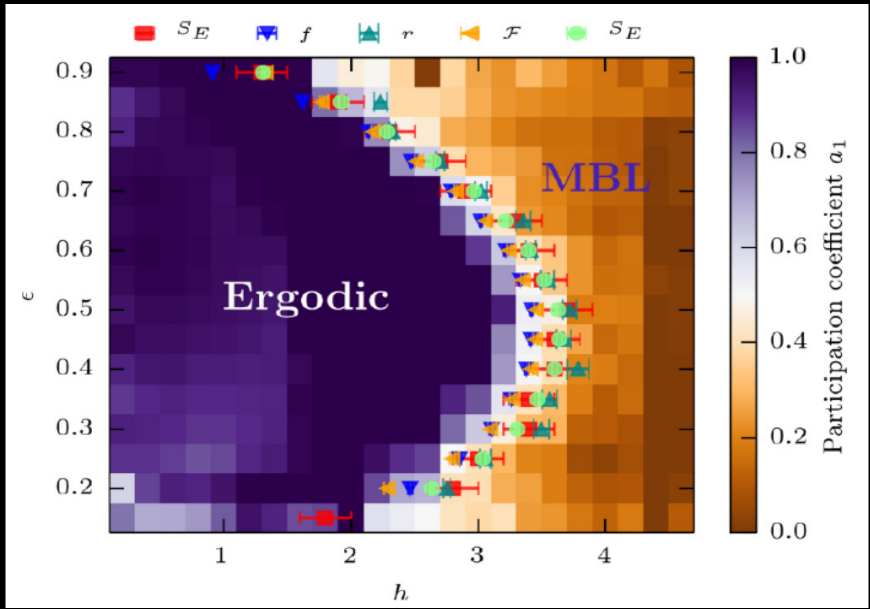
$$\mathrm{tr}_A\left(\mathrm{tr}_B\left(\hat{\rho}_{\mathrm{dia}}\right)^2\right) = Ka^{-a}$$

In the case of integrable model,

Generalized Gibbs Ensemble (GGE)

$e^{-\beta\hat{H}+\sum_n \lambda_n I_n}$, characterized by temperature $1/\beta$, and many integrals of motions I_n λ_n: Lagrangian multiplier

Numerical calculation 3: Many-body Localization



$$\hat{H} = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + h_i S_z$$

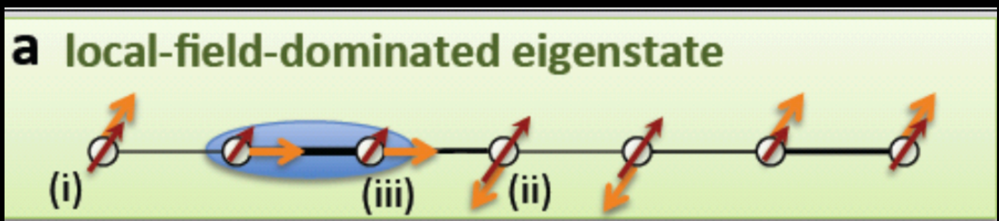
$\{h_i\}_i$: drawn from a uniform distribution$[-h, h]$

It shows ETH-MBL transition.

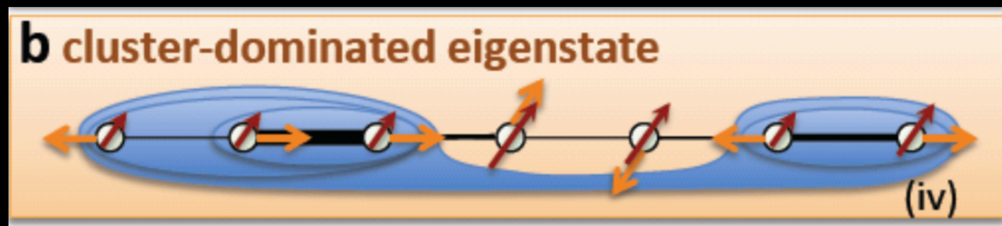
$h_c = 3.62, \ \nu = 0.80$ (values at ε=0.5)

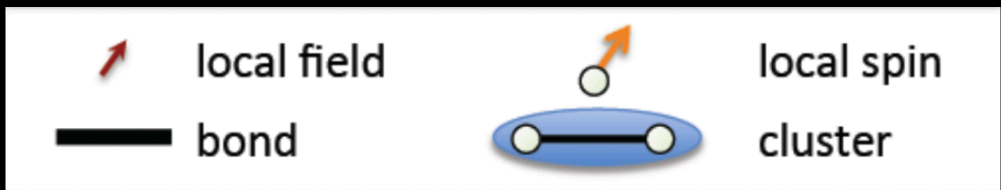
(D. J. Luis, N. Laflorencie, and F. Alet, PRB(R) 2015)

MBL phase



ETH phase





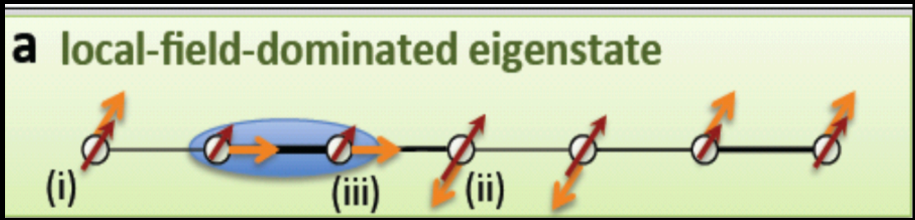
(D. Pekker, G. Refael, E. Altman, E.A.Demler, And V. Oganesyan,, PRX 2013)

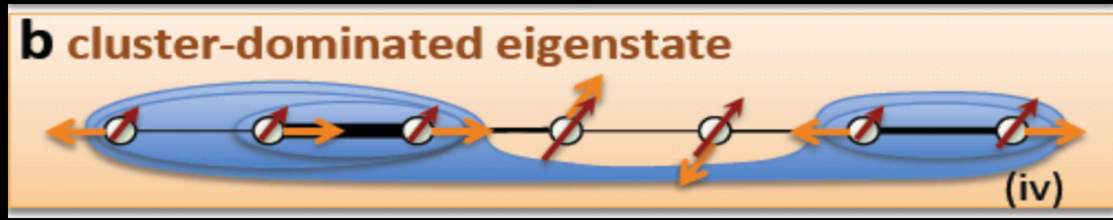
It is an eigenstate transition (dynamical transition), which cannot be captured by the equilibrium values of ensembles.

Numerical calculation 3: Many-body Localization

What kind of eigenstates?

ETH phase





MBL phase

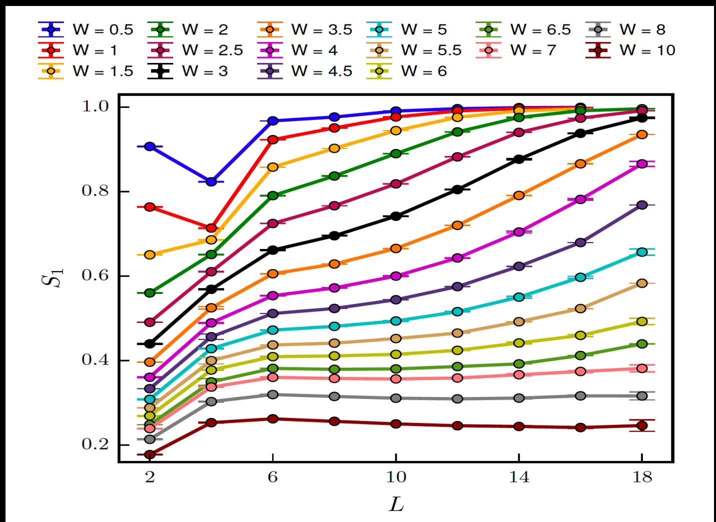
$$|\uparrow\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \otimes |\downarrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\uparrow\rangle$$

Area law

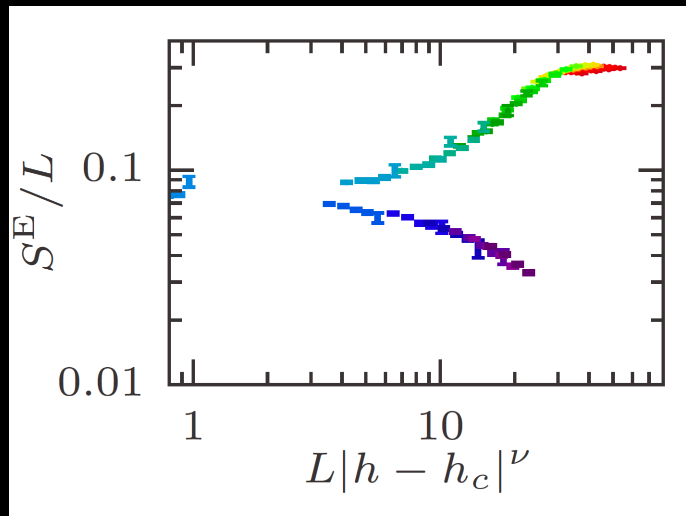
$$\frac{1}{\sqrt{D}}\Big(\frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle) \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$
$$+\frac{1}{\sqrt{2}}(|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle)) \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\downarrow\rangle$$
$$+\frac{1}{\sqrt{3}}(|\downarrow\uparrow\uparrow\rangle + |\downarrow\downarrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle) \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) + \cdots\Big)$$

Volume law

Numerical caululations

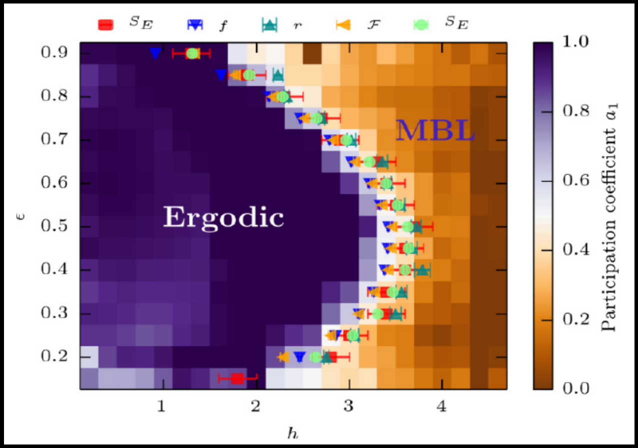


(V.Khemani,S.P.Lim,D.N.Sheng. and D.A.Huse, PRB 2017)



(D. J. Luis, N. Laflorencie, and F. Alet, PRB(R) 2015)

Numerical calculation 3: Many-body Localization



$$\hat{H} = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + h_i S_z$$

$\{h_i\}_i$: drawn from a uniform distribution $[-h, h]$

It shows ETH-MBL transition.
At the middle of the spectrum,

$$h_c = 3.62, \ \nu = 0.80$$

(D. J. Luis, N. Laflorencie, and F. Alet, PRB(R) 2015)

Problem: Harris bound requires $\nu \geq 2$, and thus this result violates the bound…

(A. Chandran, C. R. Laumann, and V. Oganesyan, arXiv, 2015)

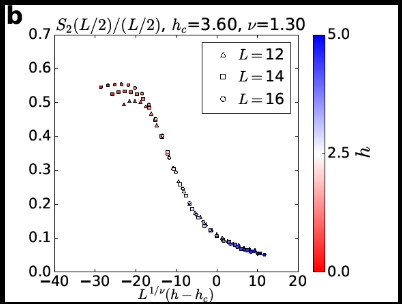
Our formula helps to estimate the slope of the volume-law precisely.

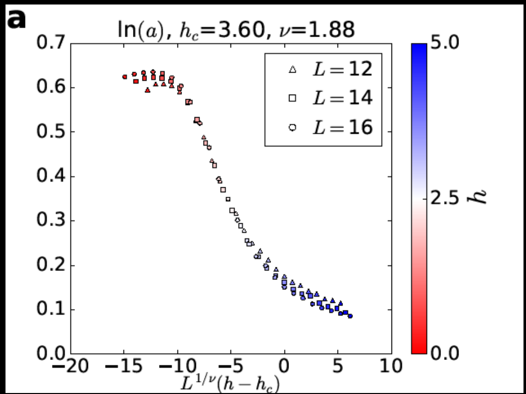
$$\overline{S_2} = \ell \ln a - \ln\left(1 + a^{-L+2\ell}\right) + \ln K$$

→

Our formula becomes a remedy: $\nu = 1.88$

(Cf Conventional way: $\nu = 1.30$)





Estimation of the critical exponent ν is so improved that the value is very close to satisfy Harris bound $\nu \geq 2$.

Summary

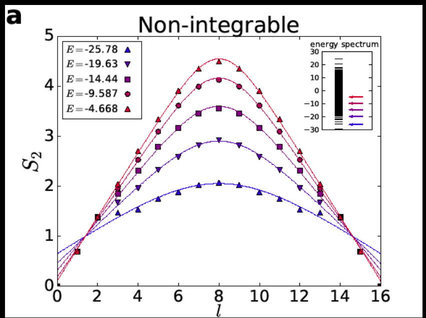
arXiv: 1703.02993

TPQ state: $|\beta\rangle \equiv \frac{1}{\sqrt{Z}} \sum_i z_i \; \exp\left[-\frac{1}{2}\beta \hat{H}\right] \; |i\rangle$

Renyi Entropy: $S_n \equiv \frac{1}{1-n} \ln\left(\mathrm{tr}_A[\hat{\rho}_A^n]\right)$

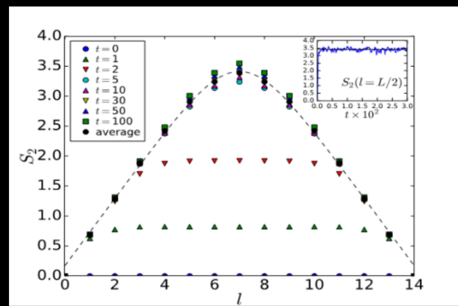
✓ Universal structure of Renyi entropy is obtained at finite temperature

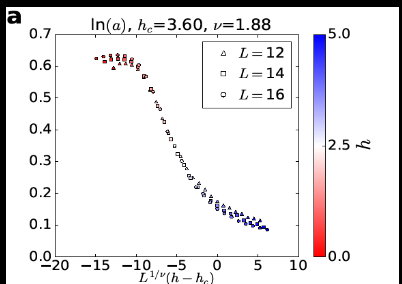
$$\overline{S_2} = \ell \ln a - \ln\left(1 + a^{-L+2\ell}\right) + \ln K$$



✓ 2nd Renyi of Energy eigenstates obeys our prediction

✓ 2nd Renyi of State after Quench to integrable obeys our prediction





✓ ETH-MBL transition is detected accurately