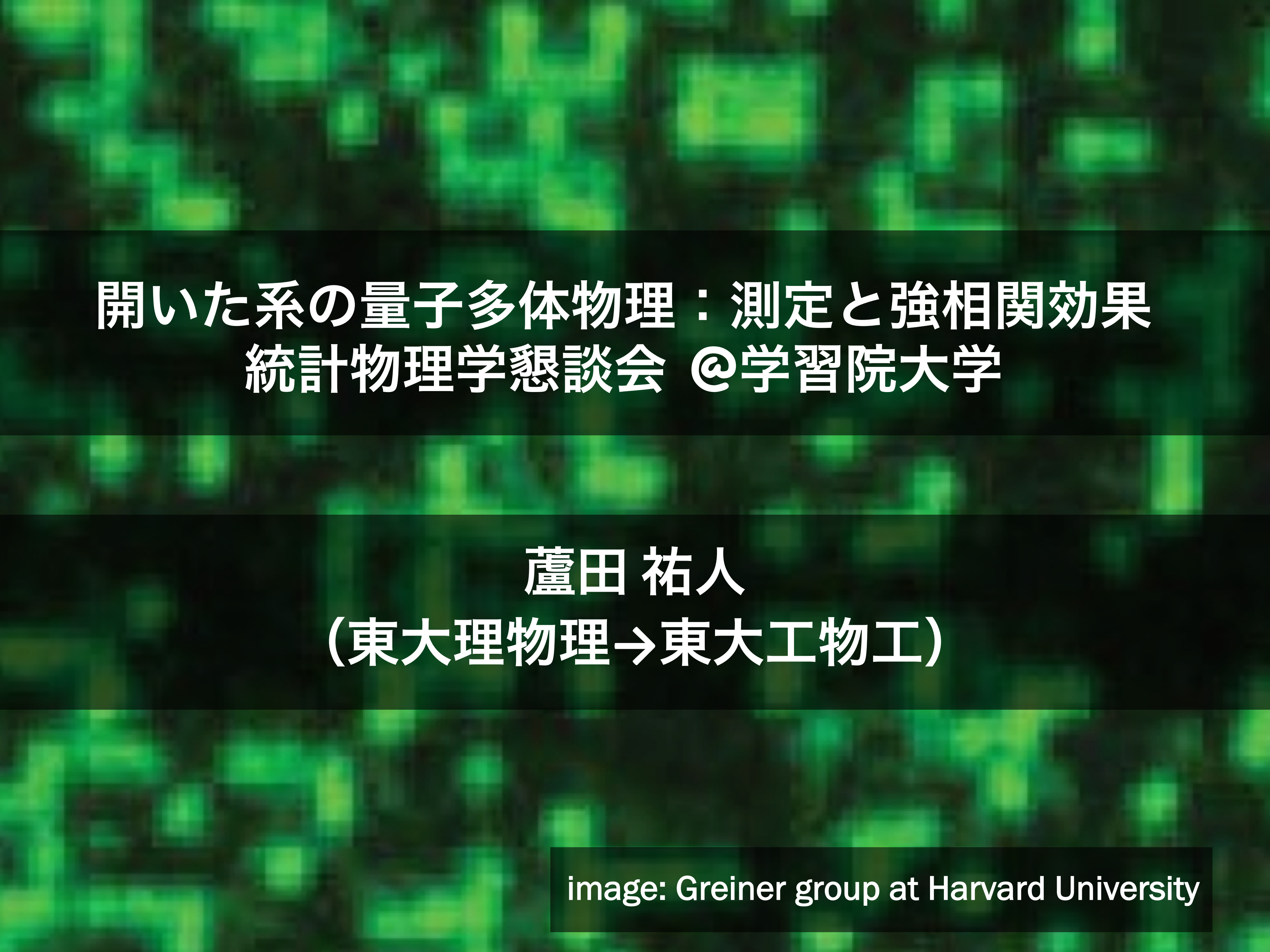


開いた系の量子多体物理：測定と強相関効果

統計物理学懇談会 ＠学習院大学

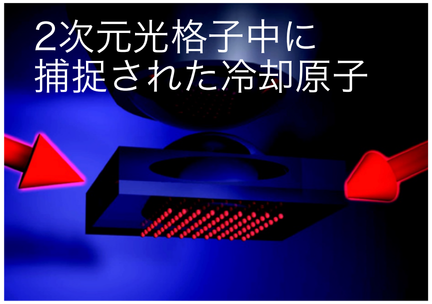
蘆田 祐人
（東大理物理→東大工物工）

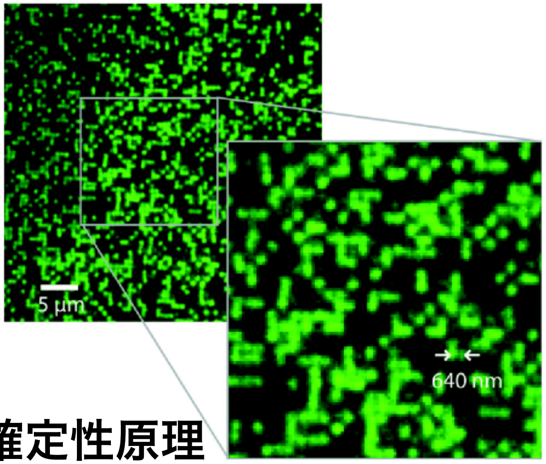
image: Greiner group at Harvard University

量子多体系の単一原子観測/制御技術の実現

- 従来の統計力学・物性物理学：大自由度系（多体系）のミクロな運動の詳細は観測/制御しないという仮定のもとに成立
- “マクロ”な量子多体系を１原子レベルで“ミクロ”に観測/制御

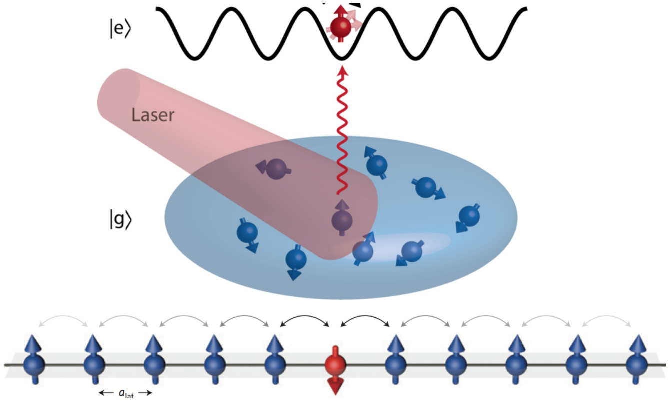
単一原子分解能の測定


2次元光格子中に
捕捉された冷却原子



ハイゼンベルグの不確定性原理
→ 測定の反作用が多体物性に本質的影響

単一原子・スピンの操作



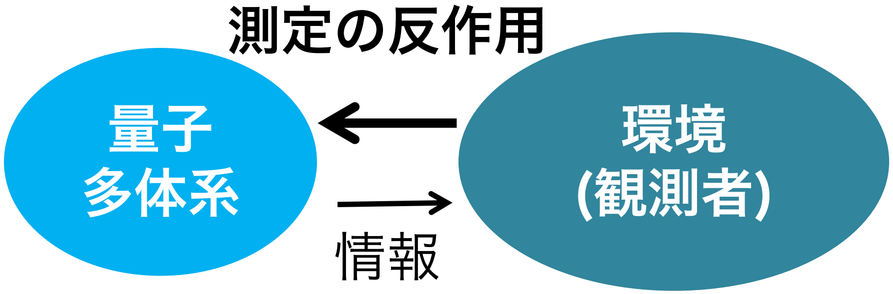
外部環境と量子スピンが強く結合
→ 固体物理で実現困難な非平衡開放系

量子物理学研究の新たな舞台
→ 既存の枠組みを超えた理論的基礎づけの必要性

量子多体系の単一原子観測/制御技術の実現

- 従来の統計力学・物性物理学：大自由度系（多体系）のミクロな運動の詳細は観測/制御しないという仮定のもとに成立
- “マクロ”な量子多体系を１原子レベルで“ミクロ”に観測/制御

観測下の量子多体系


測定の反作用

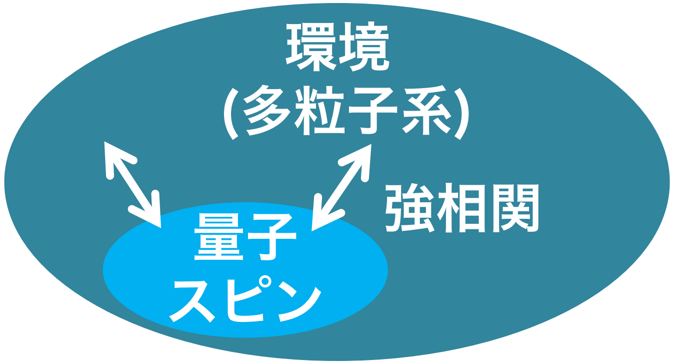
量子
多体系

環境
(観測者)

情報

ハイゼンベルグの不確定性原理
→ 測定の反作用が多体物性に本質的影響

環境と強く結合した開放量子系


環境
(多粒子系)

強相関

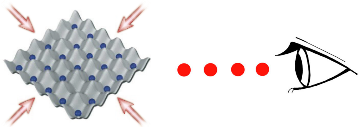
量子
スピン

外部環境と量子スピンが強く結合
→ 固体物理で実現困難な非平衡開放系

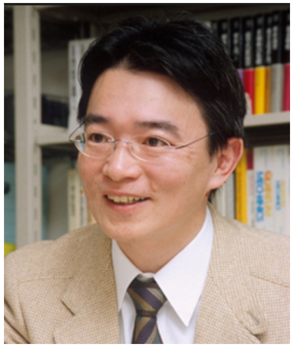
量子物理学研究の新たな舞台
→ 既存の枠組みを超えた理論的基礎づけの必要性

Collaborators

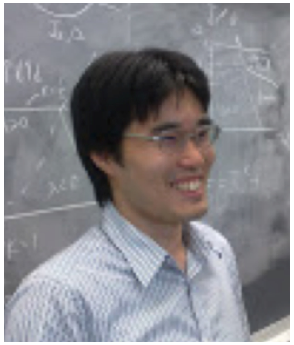
I: Quantum many-body systems under continuous observation



Advisor:
M. Ueda
(Tokyo)

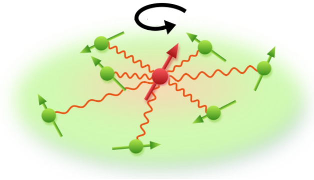


S. Furukawa
(Tokyo)



K. Saito
(Keio)

II: Quantum systems strongly correlated with environment



E. Demler
(Harvard)

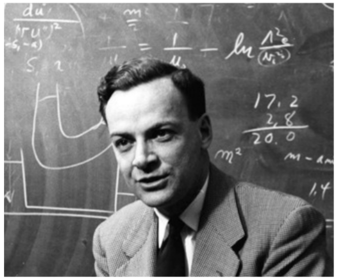
J. I. Cirac
(MPQ)

T. Shi
(MPQ→CAS)

M. C. Bañuls
(MPQ)

R. Schmidt
(Harvard→MPQ)

L. Tarruell
(ICFO)



*“Ultimately, -in the great future- we can arrange the atoms the way we want; the very atoms, all the way down! **What would happen if we could arrange the atoms one by one the way we want them?**”*

Richard Feynman (1959, Caltech)

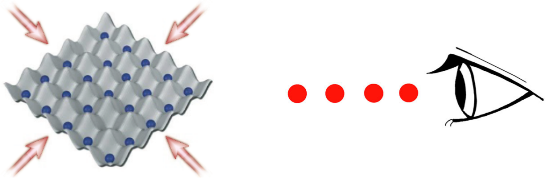
Part 1. Quantum many-body systems under continuous observation

Review of theory of continuous measurement of quantum systems

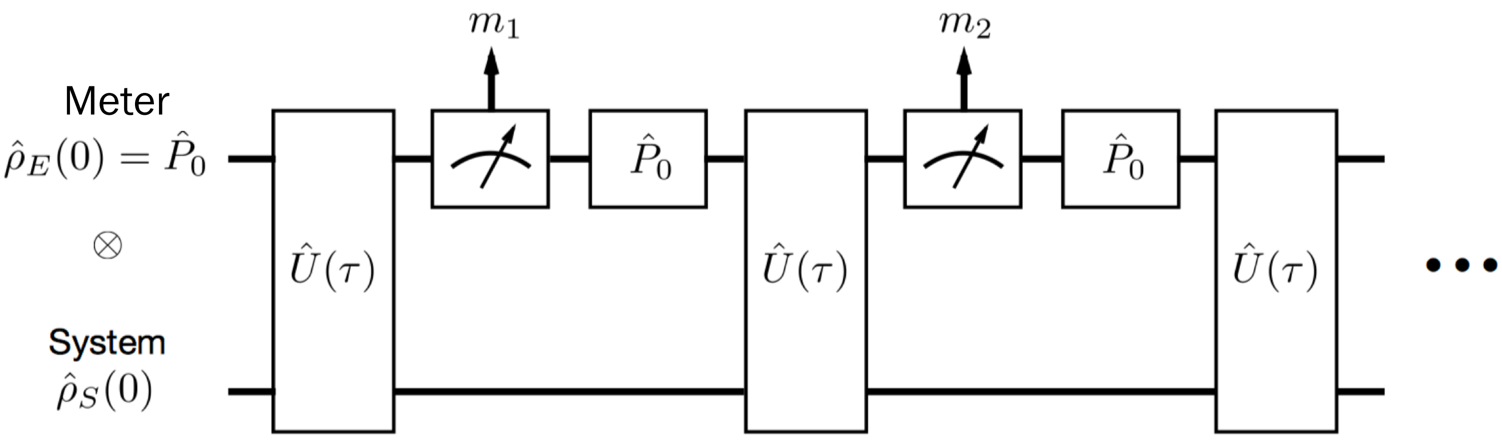
Quantum critical phenomena

Out-of-equilibrium dynamics

Thermalization



Continuous Monitoring of Quantum Systems: Quantum Trajectory



$$\hat{U}(\tau) = e^{-i\hat{H}_{\mathrm{tot}}\tau}, \quad \hat{H}_{\mathrm{tot}} = \hat{H} + \hat{V}, \quad \hat{V} = v\sum_{m=1}^{M} \hat{L}_m \otimes \hat{A}_m + \mathrm{H.c.}$$

We take continuous measurement limit: $\tau \to 0$ with keeping $\gamma \propto v^2\tau$ finite.

There are two possibilities as a measurement outcome m

(i) Quantum jump process $m \neq 0$

$$\mathcal{E}_m(\hat{\rho}) = \mathrm{Tr}_{\mathrm{M}}[\hat{P}_m \hat{U}(\tau)\hat{\rho}_{\mathrm{tot}}\hat{U}^\dagger(\tau)\hat{P}_m] \simeq \gamma\tau\hat{L}_m\hat{\rho}\hat{L}_m^\dagger$$

(ii) No-jump process $m = 0$ **: non-Hermitian evolution**

$$\mathcal{E}_0(\hat{\rho}) \simeq (1 - i\hat{H}_{\mathrm{eff}}\tau)\hat{\rho}(1 + i\hat{H}_{\mathrm{eff}}^\dagger\tau), \quad \hat{H}_{\mathrm{eff}} = \hat{H} - (i\gamma/2)\sum_m \hat{L}_m^\dagger\hat{L}_m$$

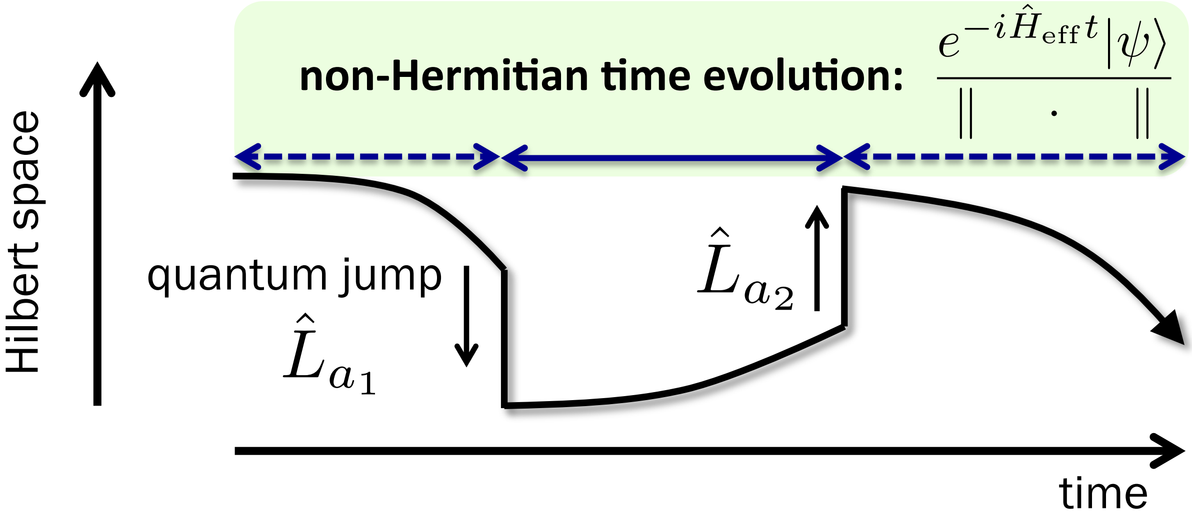
Continuous Monitoring of Quantum Systems: Quantum Trajectory

Stochastic time-evolution under Continuous observation

$$d|\psi\rangle = \underbrace{-i\hat{H}|\psi\rangle dt}_{\text{unitary evolution}} \underbrace{\overset{\text{(norm preservation)}}{- \sum_a \frac{1}{2}\left(\hat{L}_a^\dagger \hat{L}_a - \langle \hat{L}_a^\dagger \hat{L}_a\rangle\right)|\psi\rangle dt}}_{\text{non-unitary evolution without quantum jumps}} \underbrace{+ \sum_a \left(\frac{\hat{L}_a|\psi\rangle}{\sqrt{\langle \hat{L}_a^\dagger \hat{L}_a\rangle}} - |\psi\rangle\right) dN_a}_{\text{stochastic quantum jumps}}$$

*Taking ensemble average *E* over dN_a, $\hat{\rho} = E[|\psi\rangle\langle\psi|]$ obeys Lindblad master eq.

Dalibard, Castin, Mølmer; Carmichael (1993)

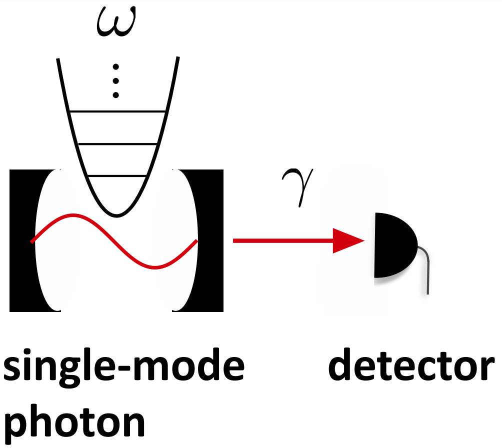


non-Hermitian time evolution: $\frac{e^{-i\hat{H}_{\rm eff}t}|\psi\rangle}{\|\quad\cdot\quad\|}$

(effective non-Hermitian Hamiltonian)

$$\hat{H}_{\rm eff} = \hat{H} - (i\gamma/2)\sum_m \hat{L}_m^\dagger \hat{L}_m$$

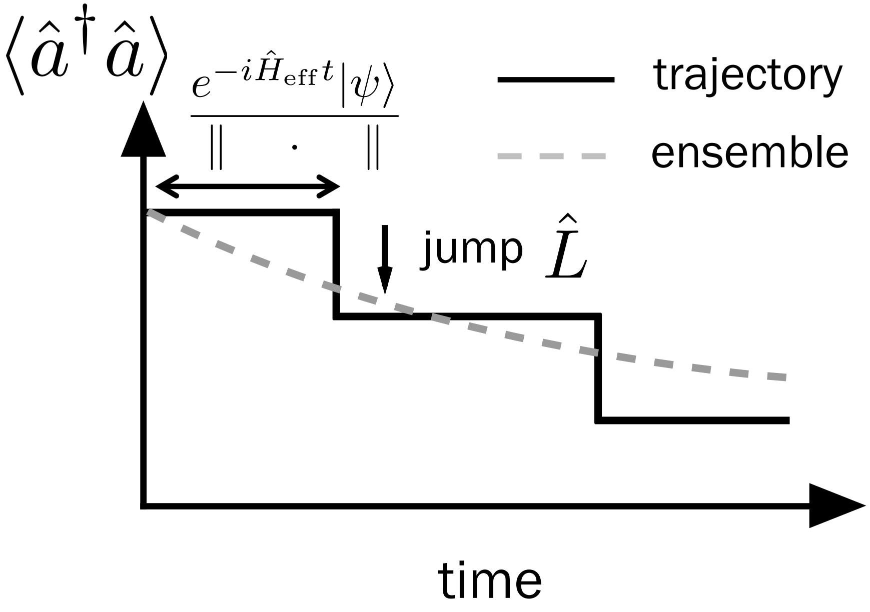
Minimal example: continuous observation of cavity photons



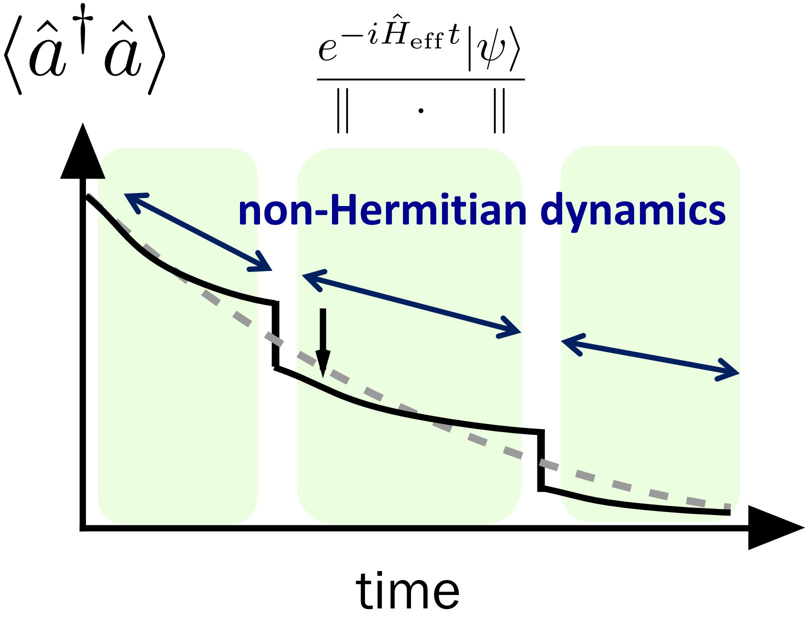
Effective Hamiltonian : $\hat{H}_{\mathrm{eff}} = \left(\omega - \frac{i\gamma}{2}\right)\hat{a}^{\dagger}\hat{a}$

jump operator : $\hat{L} = \sqrt{\gamma}\hat{a}$

example 1: Fock state

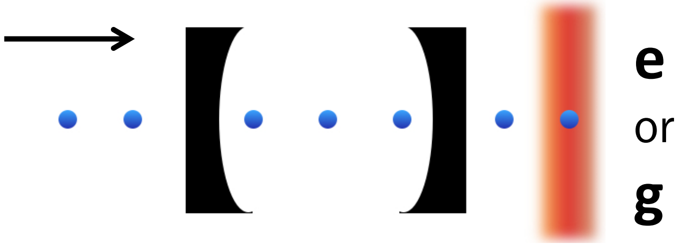


example 2: Squeezed state



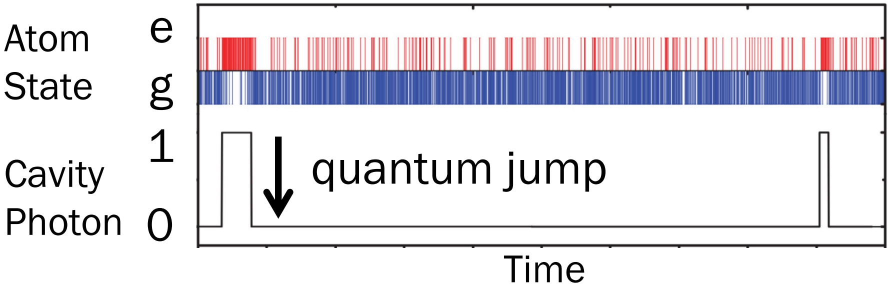
Experiments: continuous observation of small quantum systems

Microwave cavity photon



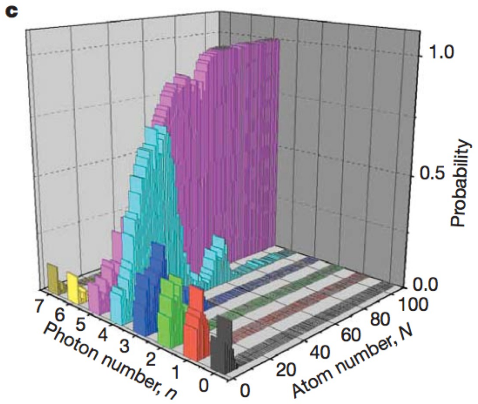
Serge Haroche
2012 Nobel Prize

observation of quantum jumps



Gleyzes et al., Nature 446, 297 (2007)

observation of wavefunction collapse



Guerlin et al., Nature 448, 889 (2007)

Superconducting qubit

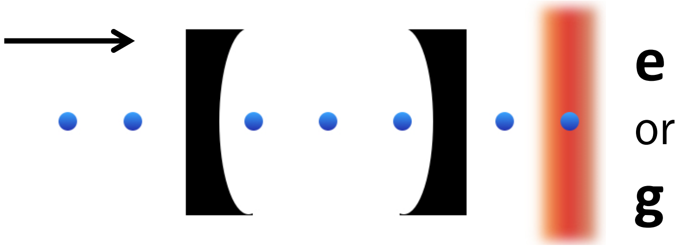
Vijay et al., PRL 106, 110502 (2011)

Quantum dots

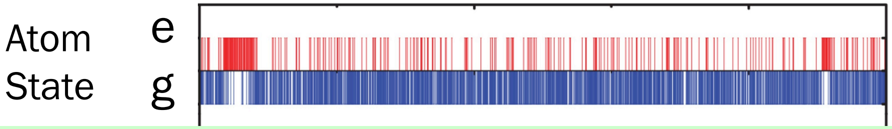
Vamivakas et al., Nature 467, 297 (2010)

Experiments: continuous observation of small quantum systems

Microwave cavity photon

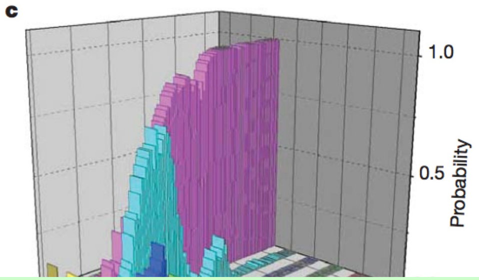


observation of quantum jumps



Serge Haroche
2012 Nobel Prize

observation of wavefunction collapse



Restricted to quantum systems with small degrees of freedom

Superconducting qubit

Vijay et al., PRL 106, 110502 (2011)

Quantum dots

Vamivakas et al., Nature 467, 297 (2010)

Quantum gas microscopy

Offers a new approach to quantum many-body systems via *in-situ* imaging of ultracold atoms

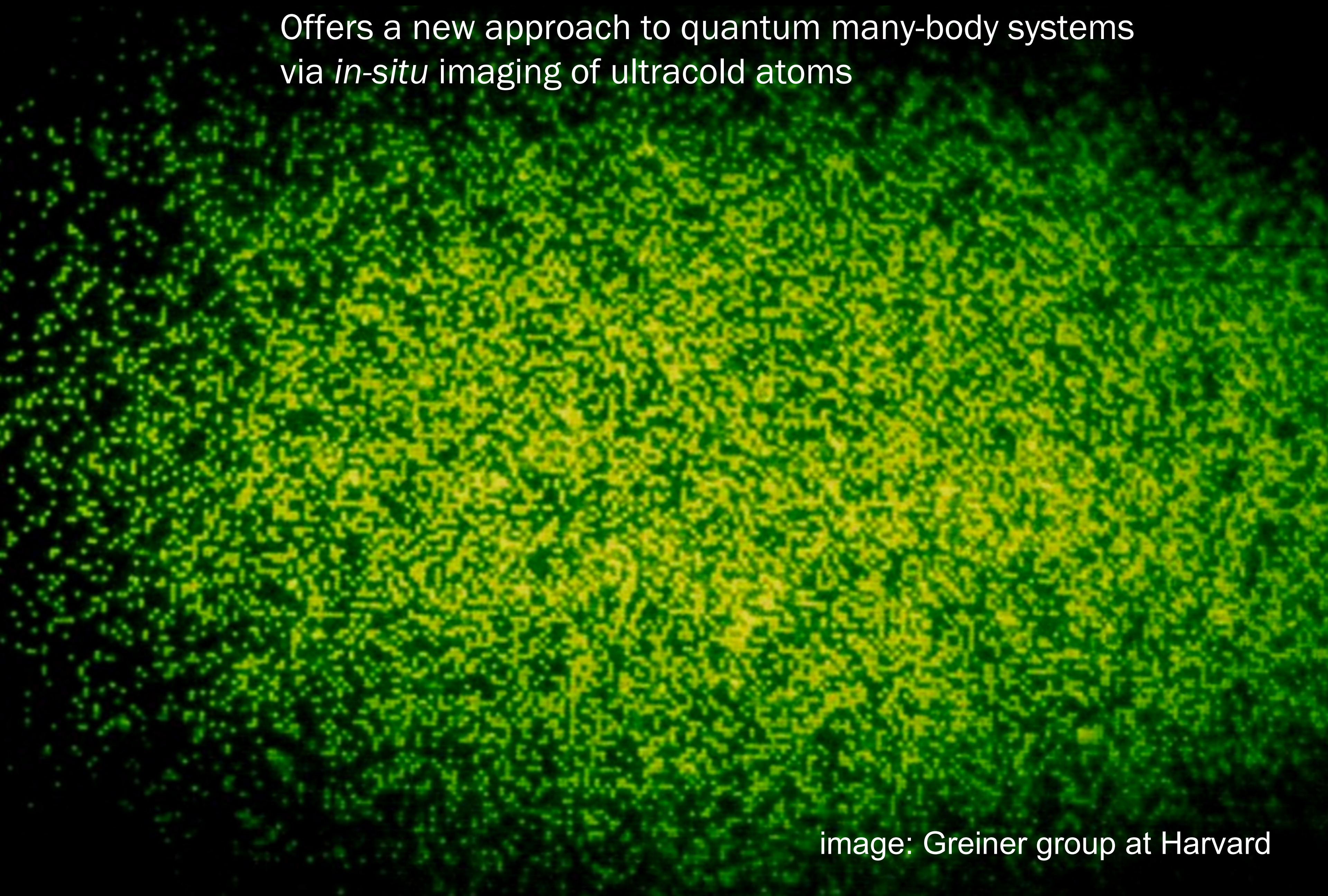
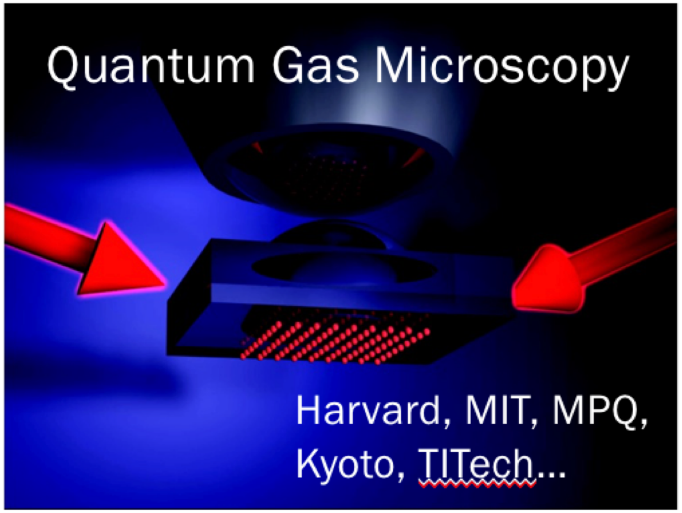
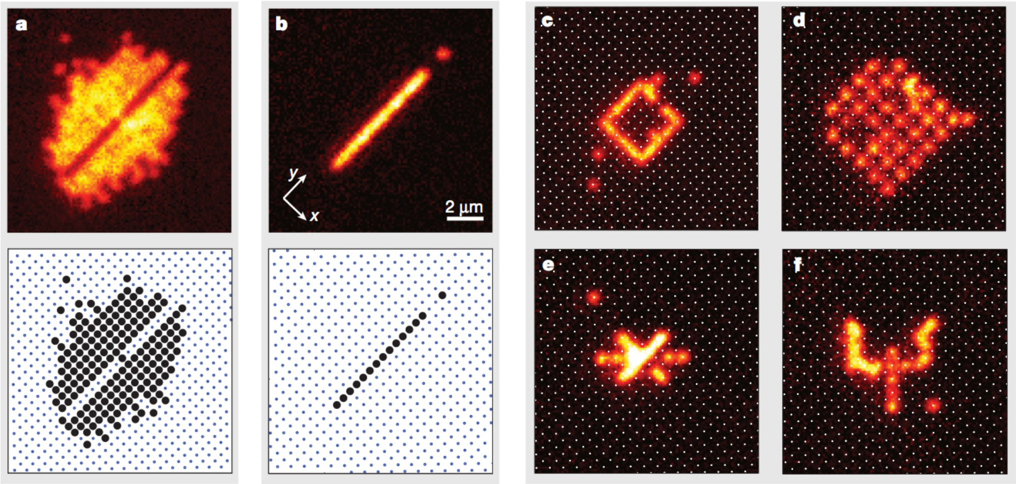
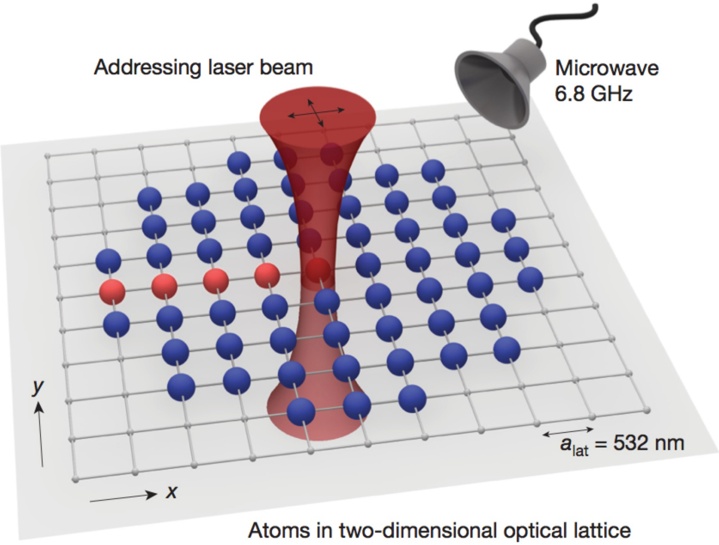


image: Greiner group at Harvard

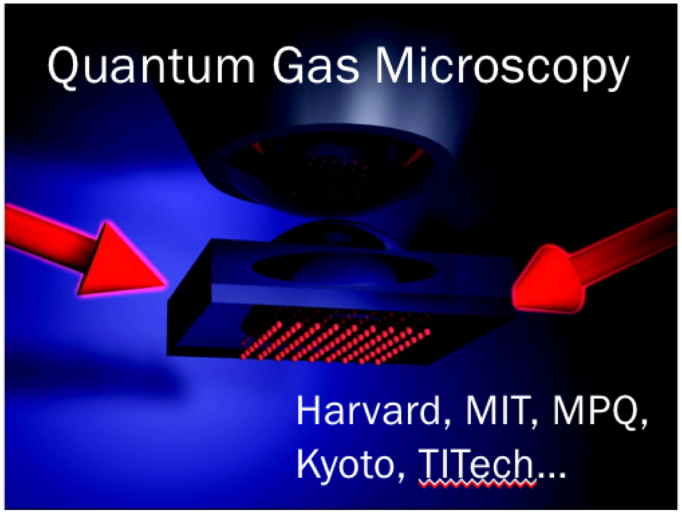
Developments in Quantum Gas Microscopy

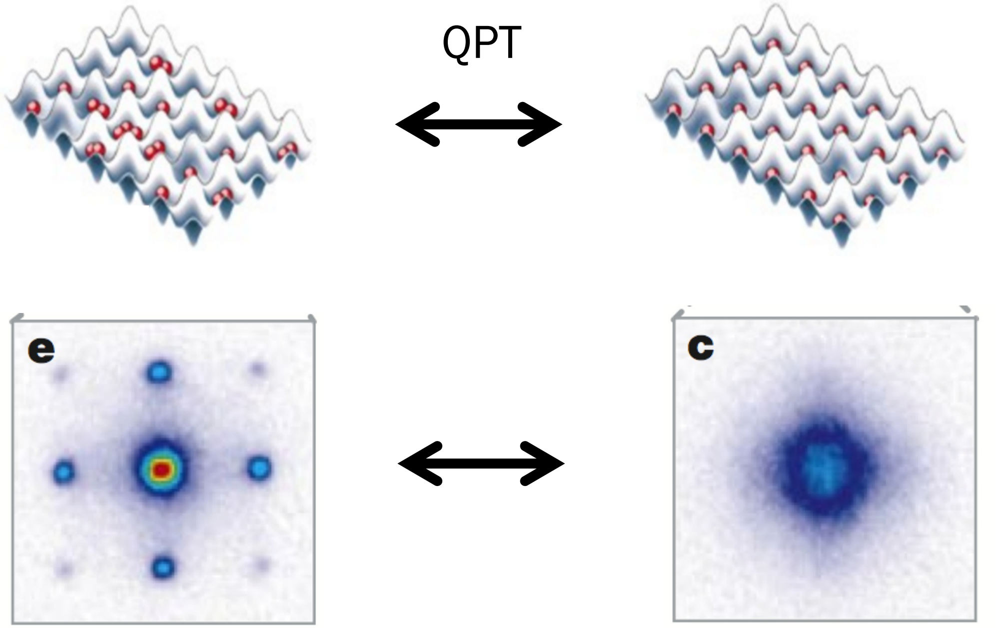






Developments in Quantum Gas Microscopy

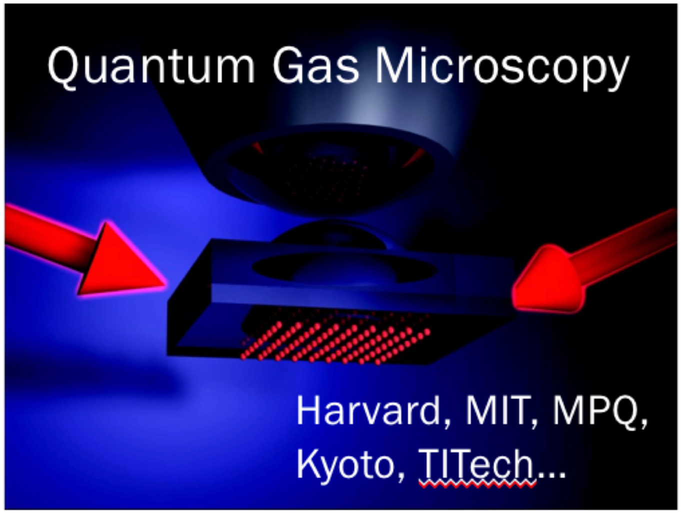



Superfluid

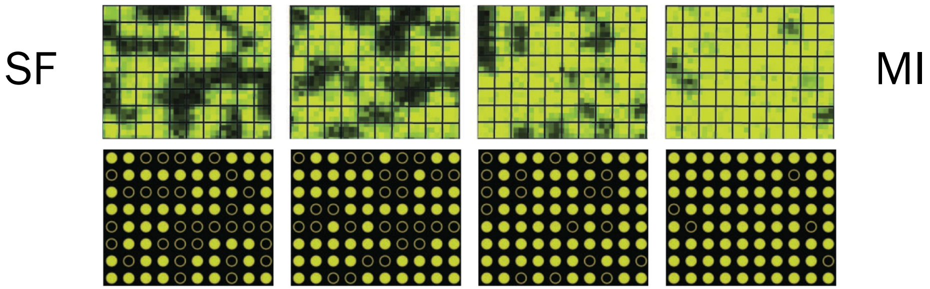
QPT

Mott insulator

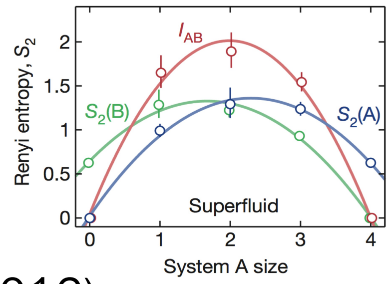
Developments in Quantum Gas Microscopy



Superfluid-to-Mott insulator transition has been observed at the **single-particle** level.



W. S. Bakr et al., Science 329, 547 (2010).



Recent breakthroughs:

Measurement of entanglement entropy:

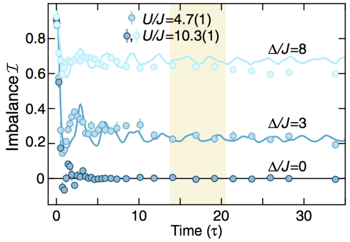
Islam *et al. Nature* 528 77 (2015).

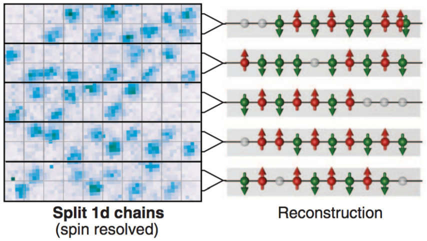
Observation of antiferromagnetic fermionic correlations:

Cheuk *et al*., Parsons *et al*., Boll *et al*., *Science* 353 1253-1260 (2016), Mazurenko *et al*., *Nature* 545 462 (2017).

Observation of many-body localization:

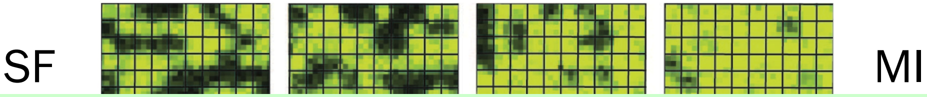
Schreiber *et al. Science* 349 842 (2015), J-y. Choi *et al. ibid* 352 1547 (2016)





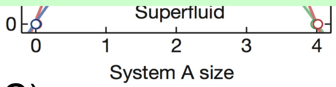
Developments in Quantum Gas Microscopy

Superfluid-to-Mott insulator transition has been observed at the **single-particle** level.



New direction: Continuous monitoring of many-body systems by QGM.

Many-body physics ⟷ **Interplay** ⟷ **Measurement backaction**



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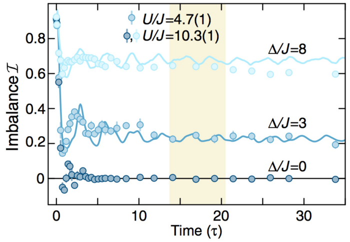
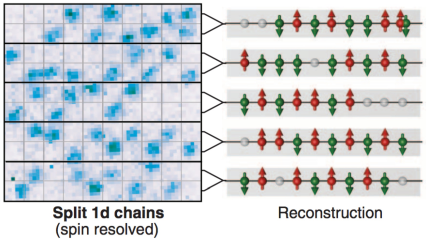
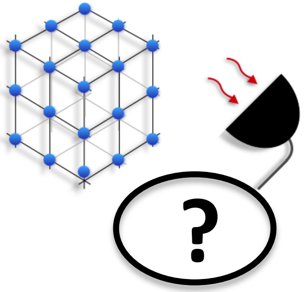




Overview of Part I : the ability to access quantum jumps

What does differentiate **continuous observation** from dissipation?

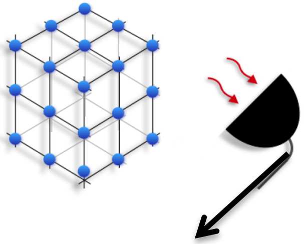
Dissipation: one is unable to access the information about measurement outcomes



Lindblad master equation

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H},\hat{\rho}] - \sum_{a}\left(\frac{1}{2}\hat{L}_a^{\dagger}\hat{L}_a\hat{\rho} + \frac{1}{2}\hat{\rho}\hat{L}_a^{\dagger}\hat{L}_a - \hat{L}_a\hat{\rho}\hat{L}_a^{\dagger}\right)$$

Continuous observation: one is able to access the information about measurement outcomes (i.e., quantum jumps)



measurement outcome

1. Complete information available:
   single-trajectory (pure-state) dynamics

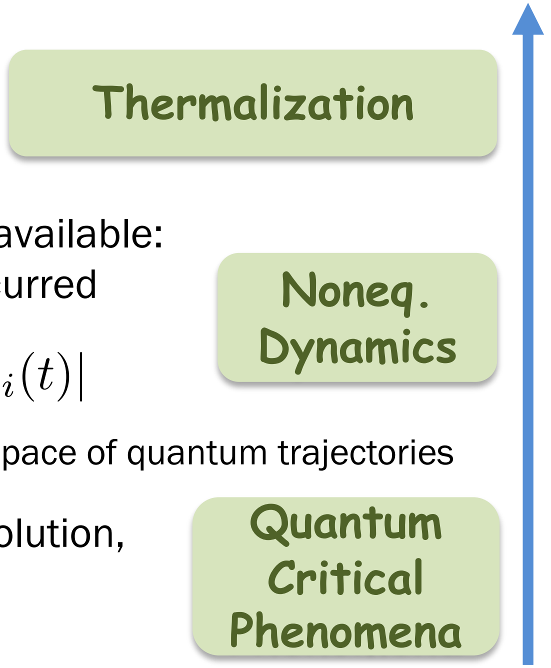
$$\hat{\rho}_{\mathrm{traj}}(t) = |\psi_{\mathrm{traj}}(t)\rangle\langle\psi_{\mathrm{traj}}(t)|$$

2. Partial (i.e., coarse-grained) information available:
   conditioned on the number of jumps occurred

$$\hat{\rho}_{\mathrm{post}}(t) = \sum_{i\in\mathcal{D}} |\psi_{\mathrm{traj},i}(t)\rangle\langle\psi_{\mathrm{traj},i}(t)|$$

$\mathcal{D}$: subspace of quantum trajectories

3. The simplest example: **non-Hermitian** evolution,
   conditioned on no-jump processes



Thermalization

Noneq. Dynamics

Quantum Critical Phenomena

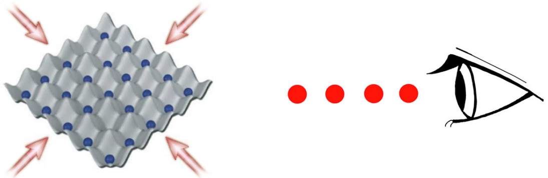
Part 1. Quantum many-body systems under continuous observation

Review of theory of continuous measurement of quantum systems

Quantum critical phenomena

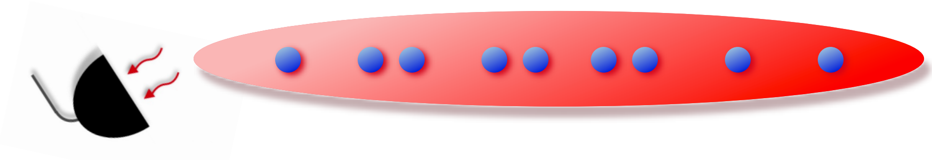
Out-of-equilibrium dynamics

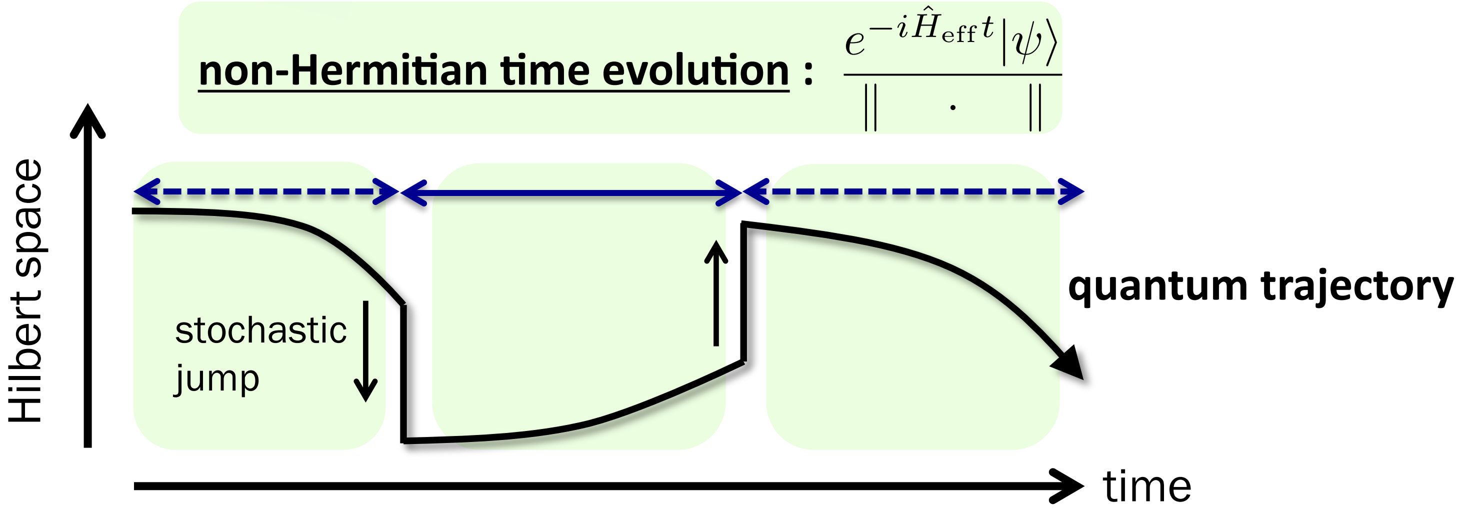
Thermalization



Quantum critical phenomena under continuous observation

Imagine that we **continuously monitor** a strongly correlated many-body system.





non-Hermitian time evolution : $\frac{e^{-i\hat{H}_{\mathrm{eff}}t}|\psi\rangle}{\|\quad\cdot\quad\|}$

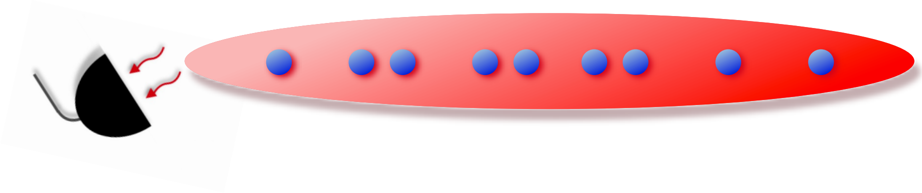
effective Hamiltonian: $\hat{H}_{\mathrm{eff}} = \hat{H} - \frac{i}{2}\sum_a \hat{L}_a^\dagger \hat{L}_a$ eigv: $E_\lambda - \frac{i\Gamma_\lambda}{2}$

effective ground state: $|\Psi_{\mathrm{GS}}\rangle$: eigenstate of $\hat{H}_{\mathrm{eff}}$ possessing the **lowest** E_λ

* $|\Psi_{\mathrm{GS}}\rangle$ also has (almost) the **minimal** Γ_λ (the **longest** lifetime).

Quantum critical phenomena under continuous observation

Q. Does such measurement backaction change universality class of quantum critical phenomena?



Idea:

(sine-Gordon model)

Quantum Phase Transition + **Spectral Singularity** → *New* universality class unique to open systems?

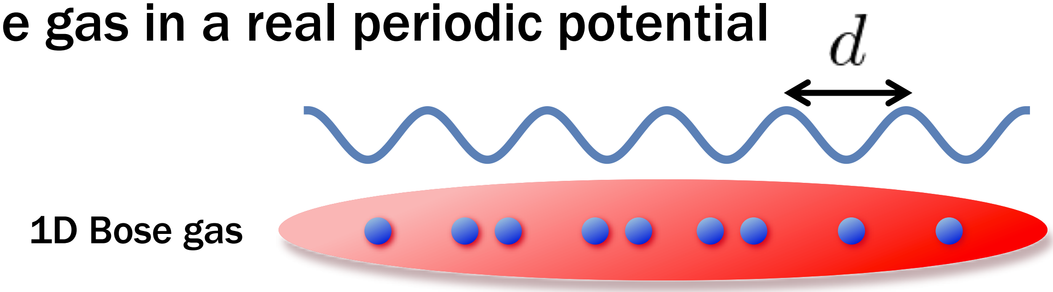
(PT-symmetry)

↑

Singularity due to **non-diagonalizability**
= ***unique*** to non-Hermitian systems

Many-body paradigm: BKT transition and sine-Gordon model

1D Bose gas in a real periodic potential



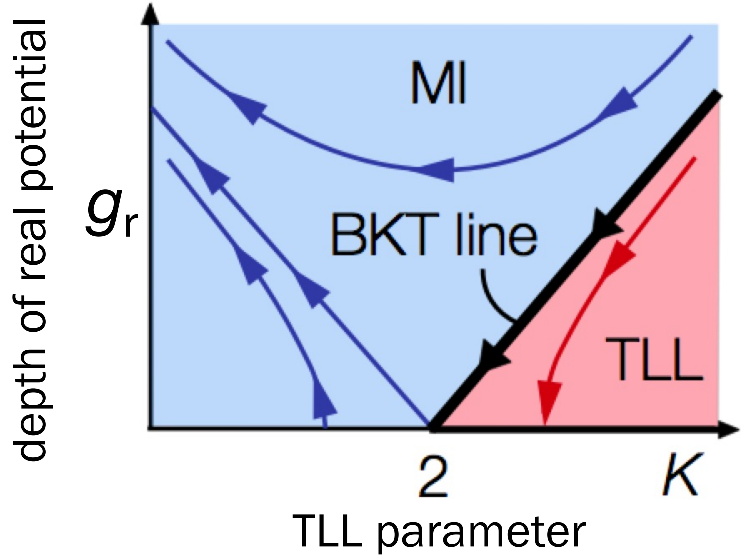
shallow periodic potential
(far-detuned light)

$$U(x) = U_{\mathrm{r}} \cos\left(\frac{2\pi x}{d}\right)$$

Low-energy Hamiltonian (sine-Gordon model):

$$\hat{H} = \int dx \left\{ \frac{\hbar v}{2\pi} \left[K(\partial_x \hat{\theta})^2 + \frac{1}{K}(\partial_x \hat{\phi})^2 \right] + V(\hat{\phi}) \right\}, \quad V(\hat{\phi}) = \frac{g_{\mathrm{r}}}{\pi} \cos(2\hat{\phi})$$

RG flows & Phase diagram:



gapped phase
(Mott insulator (MI))

↕ Berezinskii-Kosterlitz-Thouless transition

gapless quantum critical phase
(Tomonaga-Luttinger liquid (TLL))

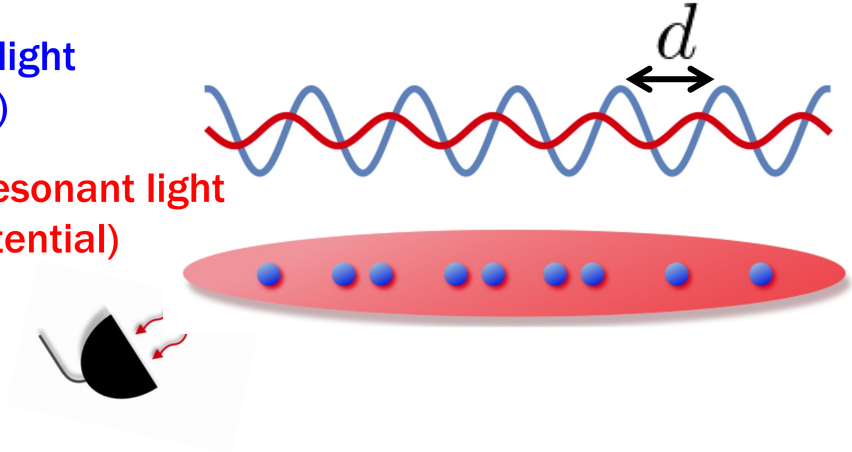
2016 Nobel Prize

Under continuous observation: Generalized sine-Gordon model

1D Bose gas subject to spatially modulated one-body loss

a far-detuned light
(real potential)

a weak near-resonant light
(imaginary potential)

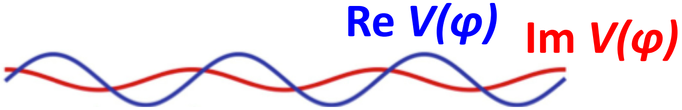


$$U(x) = \underline{U_{\mathrm{r}} \cos\left(\frac{2\pi x}{d}\right)} \underline{-iU_{\mathrm{i}} \sin\left(\frac{2\pi x}{d}\right)} - iU_{\mathrm{i}}$$

periodic gain-loss
structure

Low-energy Hamiltonian (Generalized sine-Gordon model):

$$\hat{H} = \int dx \left\{ \frac{\hbar v}{2\pi} \left[K(\partial_x \hat{\theta})^2 + \frac{1}{K}(\partial_x \hat{\phi})^2 \right] + V(\hat{\phi}) \right\}$$



$$V(\hat{\phi}) = \frac{g_{\mathrm{r}}}{\pi} \cos(2\hat{\phi}) \boxed{- \frac{i g_{\mathrm{i}}}{\pi} \sin(2\hat{\phi})}$$

g_{r} : depth of **real** potential

g_{i} : depth of **imaginary** potential

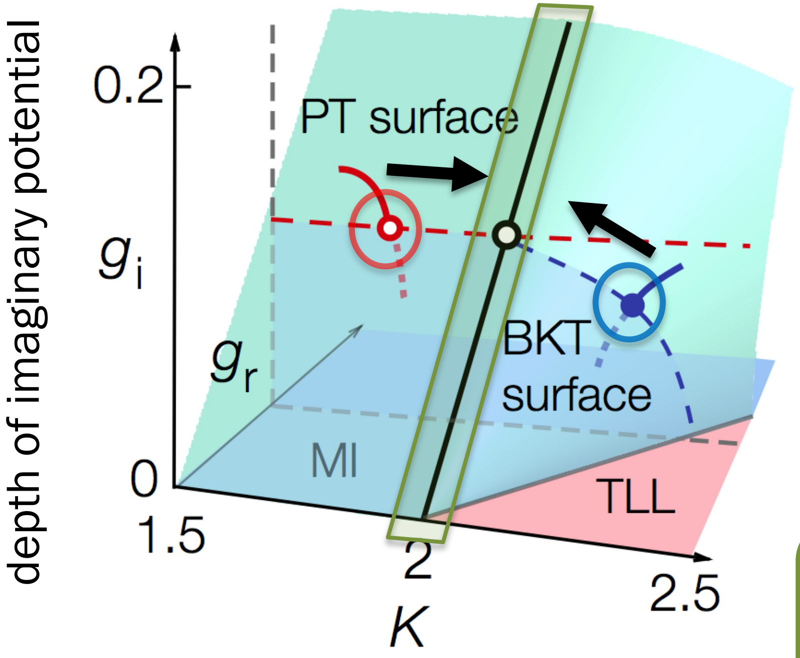
$$\hat{\mathcal{P}} \hat{\phi} \hat{\mathcal{P}} = -\hat{\phi} \quad \hat{\mathcal{T}} i \hat{\mathcal{T}} = -i \quad \rightarrow \quad [\hat{H}, \hat{\mathcal{P}}\hat{\mathcal{T}}] = 0$$

PT symmetry

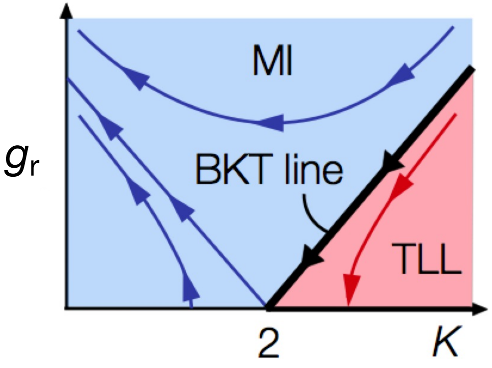
Bender & Boettcher PRL (1998)

Phase diagram of PT-symmetric sine-Gordon model

PT-symmetric sine-Gordon



*c.f.) conventional sine-Gordon



2D phase boundary:

- MI phase (below) and TLL phase (above)

Two phase transitions:

- BKT transition (K>2)
- **PT transition** (K<2)

↖ Measurement-induced QPT
Spectral Singularity

Merging line:

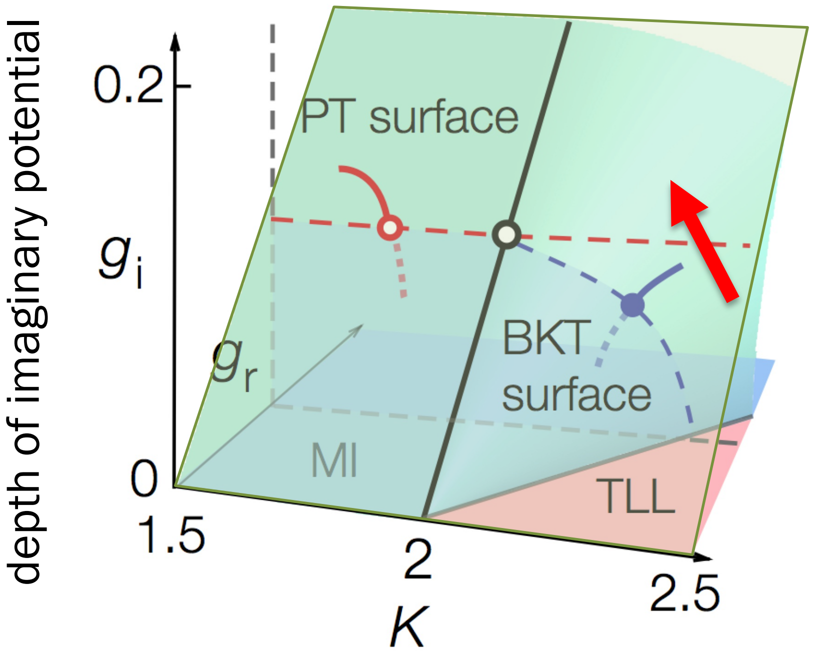
- Each point = RG **fixed** point.
- **Scale invariance** at K=2, g_s=g_c

Spectral singularity
+
quantum critical point
→ a nontrivial RG fixed point.

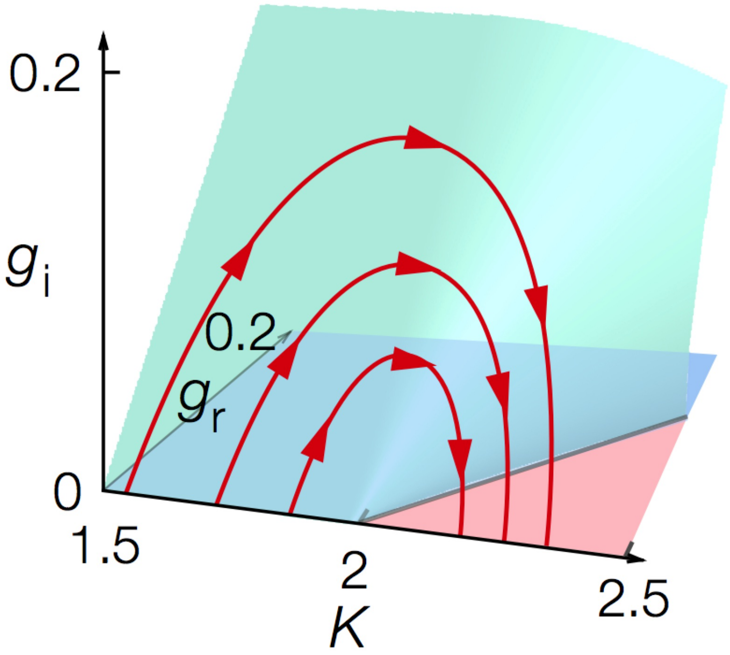
New universality class.
cf. non-unitary CFT [Ikhlef et al., PRL 116, 130601 (2016)].

New type of RG flows: Violation of c-theorem

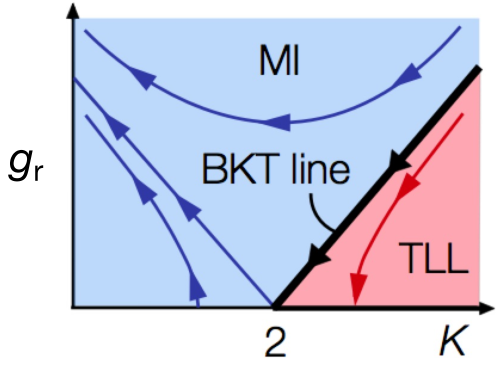
PT-symmetric sine-Gordon



PT broken (g_i>g_r)



*c.f.) conventional sine-Gordon



- **Semicircular** RG flows appear:

  → anomalous **increase** of TLL parameter K

  = **varying** critical exponent:

$$\langle \hat{\Psi}^\dagger(r)\hat{\Psi}(0)\rangle \propto (1/r)^{\frac{1}{2K}}$$

- **Violation of c-theorem**

  (Zamorodchikov, 1984)

$$\frac{dc}{dl} < 0$$

c : central charge l : RG scale

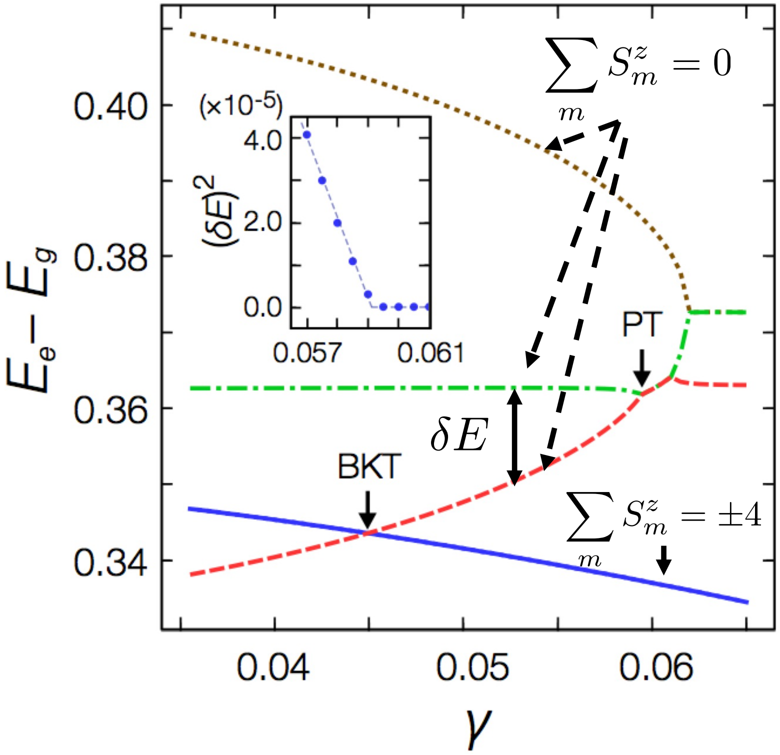
Numerical test of RG phase diagram (exact diagonalization)

PT-symmetric spin-chain model:

$$\hat{H}_{\mathrm{L}} = \sum_{m=1}^{N} \left[-(J+(-1)^m i\gamma)\left(\hat{S}^x_m \hat{S}^x_{m+1} + \hat{S}^y_m \hat{S}^y_{m+1}\right) + \Delta \hat{S}^z_m \hat{S}^z_{m+1} + (-1)^m h_s \hat{S}^z_m \right]$$

Correspondence to the effective field theory: $(-\Delta, h_s, \gamma) \Leftrightarrow (K, g_c, g_s)$

Typical low-energy excitation spectrum



BKT transition point:
crossing of appropriate energy levels
(level spectroscopy)

K. Nomura, J. Phys. A 28, 5451 (1995)

PT transition point:
merging of two energy levels
with square-root scaling
(characteristic of exceptional point)

T. Kato, *Perturbation theory for linear operators* (1966)

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

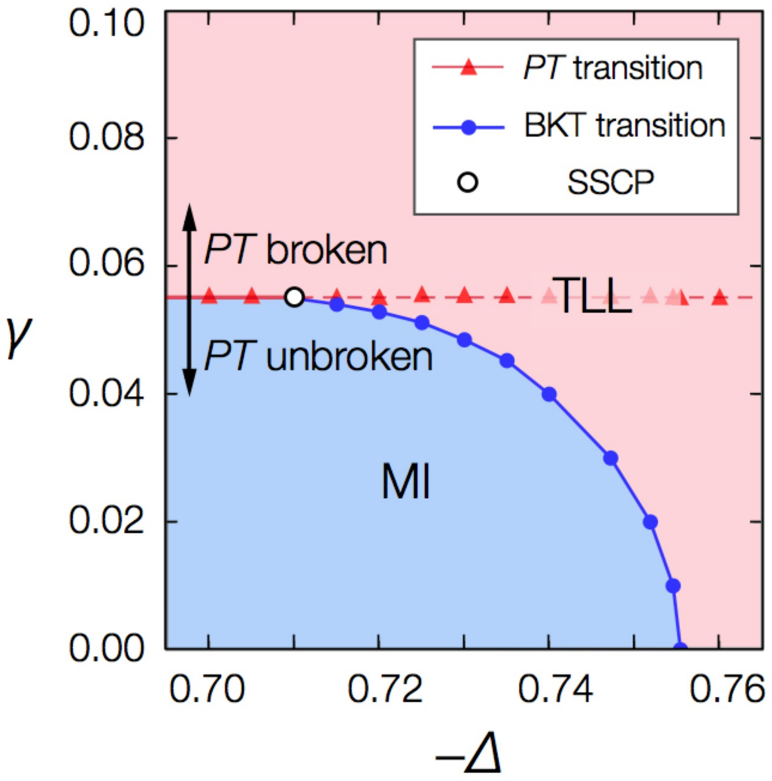
Numerical test of RG phase diagram (exact diagonalization)

PT-symmetric spin-chain model:

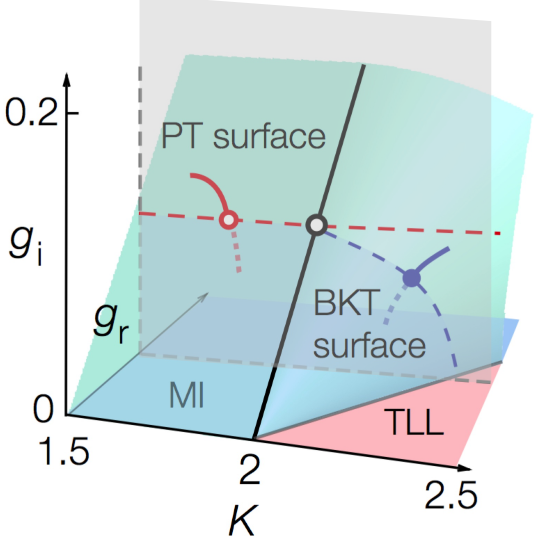
$$\hat{H}_{\mathrm{L}} = \sum_{m=1}^{N}\left[-\left(J+(-1)^m i\gamma\right)\left(\hat{S}^x_m\hat{S}^x_{m+1}+\hat{S}^y_m\hat{S}^y_{m+1}\right)+\Delta\hat{S}^z_m\hat{S}^z_{m+1}+(-1)^m h_s\hat{S}^z_m\right]$$

Correspondence to the effective field theory: $(-\Delta, h_s, \gamma) \Leftrightarrow (K, g_c, g_s)$

Phase diagram (fixed h_s)



fixed g_c



Numerical **supports** for RG phase diagram.

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

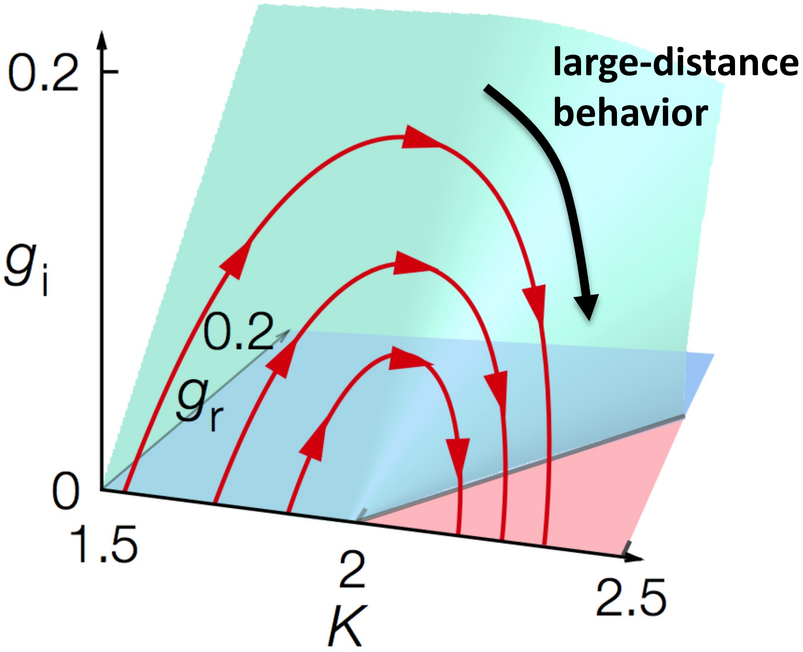
Numerical test of anomalous RG flows (iTEBD)

PT-symmetric spin-chain model:

$$\hat{H}_{\mathrm{L}} = \sum_{m=1}^{N} \left[-\left(J + (-1)^m i\gamma\right) \left(\hat{S}^x_m \hat{S}^x_{m+1} + \hat{S}^y_m \hat{S}^y_{m+1} \right) + \Delta \hat{S}^z_m \hat{S}^z_{m+1} + (-1)^m h_s \hat{S}^z_m \right]$$

Correspondence to the effective field theory: $(-\Delta, h_s, \gamma) \Leftrightarrow (K, g_c, g_s)$

PT broken (*g_i*>*g_r*)



Semicircular RG flows:

- **Varying** critical exponent in a **larger** distance
- **Violation of c-theorem**

Method:

infinite time-evolving block decimation (**iTEBD**) algorithm (G. Vidal, PRL 98, 070201 (2007))

imaginary-time evolution → ground state

$$\frac{\exp(-\hat{H}\tau)|\Psi_0\rangle}{\|\exp(-\hat{H}\tau)|\Psi_0\rangle\|}$$

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

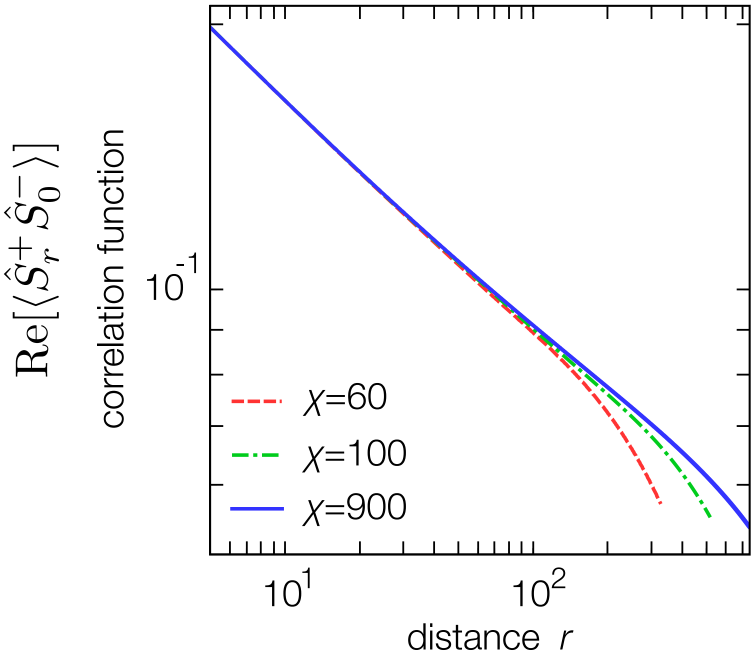
Numerical test of anomalous RG flows (iTEBD)

PT-symmetric spin-chain model:

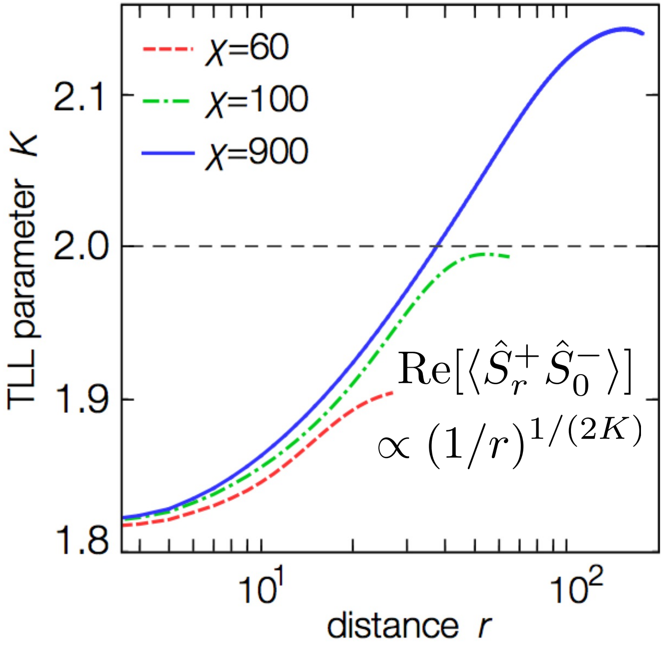
$$\hat{H}_{\mathrm{L}} = \sum_{m=1}^{N} \left[-(J + (-1)^m i\gamma) \left(\hat{S}_m^x \hat{S}_{m+1}^x + \hat{S}_m^y \hat{S}_{m+1}^y \right) + \Delta \hat{S}_m^z \hat{S}_{m+1}^z + (-1)^m h_s \hat{S}_m^z \right]$$

Correspondence to the effective field theory: $(-\Delta, h_s, \gamma) \Leftrightarrow (K, g_c, g_s)$

Critical decay in PT broken phase



Varying TLL parameter *K*



YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

Numerical test of anomalous RG flows (iTEBD)

PT-symmetric spin-chain model:

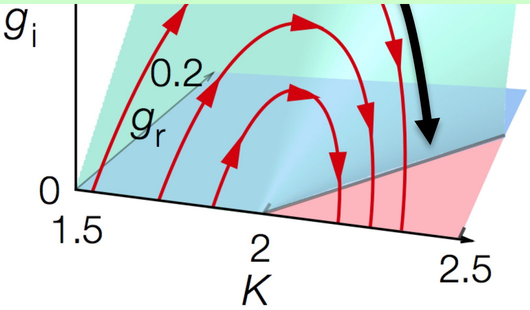
$$\hat{H}_{\mathrm{L}} = \sum_{m=1}^{N} \left[-\left(J + (-1)^m i\gamma\right) \left(\hat{S}^x_m \hat{S}^x_{m+1} + \hat{S}^y_m \hat{S}^y_{m+1} \right) + \Delta \hat{S}^z_m \hat{S}^z_{m+1} + (-1)^m h_s \hat{S}^z_m \right]$$

Correspondence to the effective field theory: $(-\Delta, h_s, \gamma) \Leftrightarrow (K, g_c, g_s)$

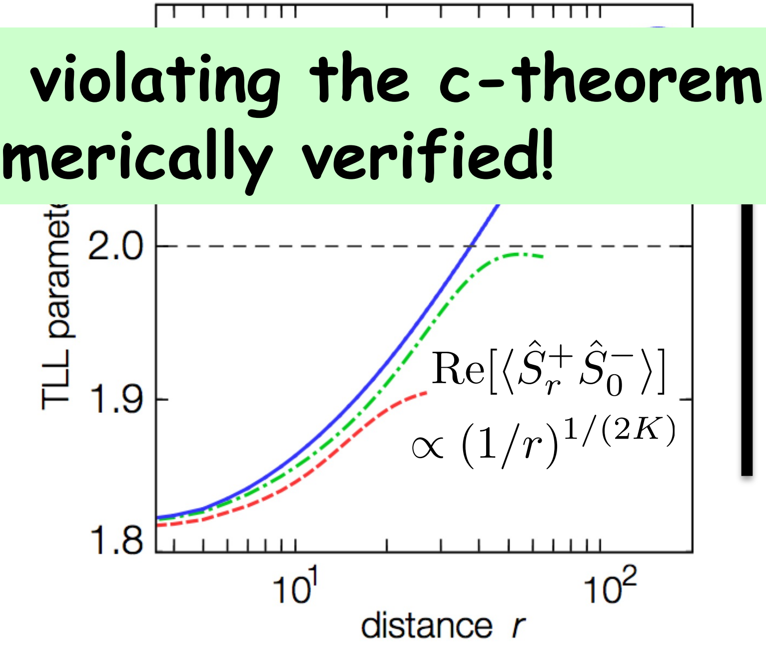
RG analysis

Varying TLL parameter *K*

Anomalous RG flows violating the c-theorem have been numerically verified!



Semicircular RG flows
= **Enhancement** of TLL parameter
in a **larger** distance



YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

Part 1. Quantum many-body systems under continuous observation

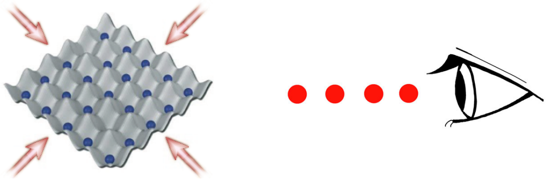
Review of theory of continuous measurement of quantum systems

Quantum critical phenomena

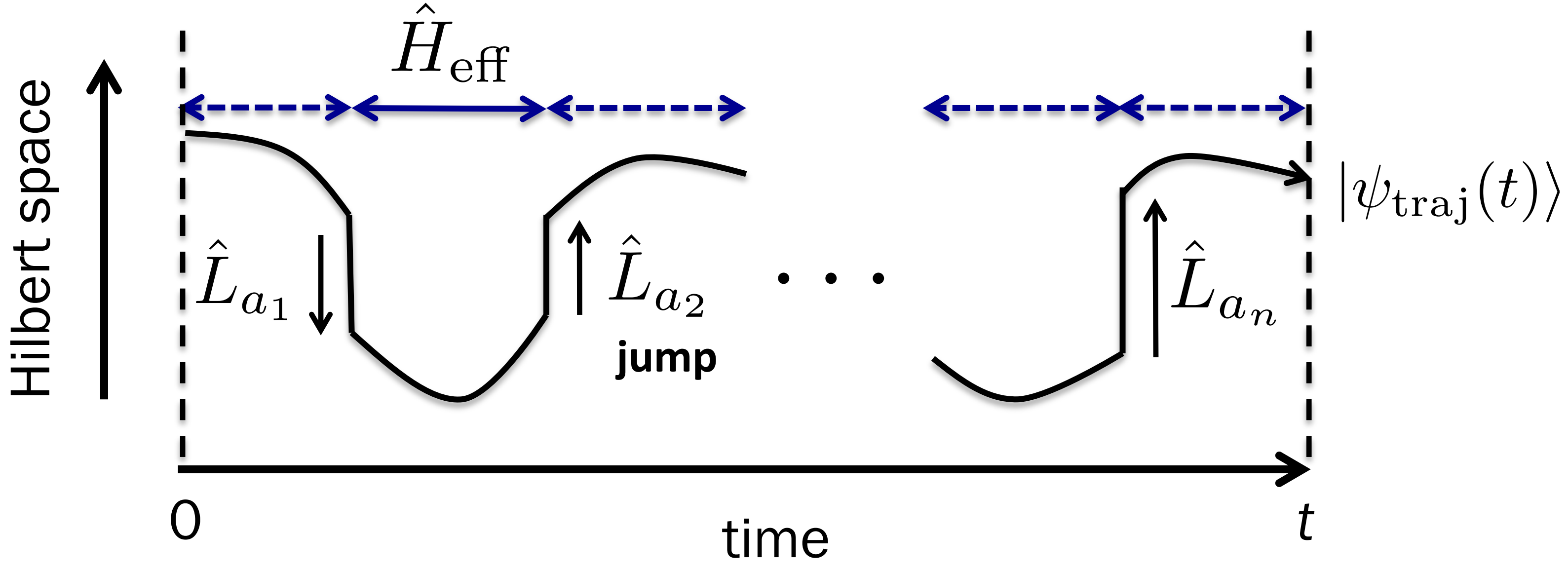
Short Summary

Out-of-equilibrium dynamics

Thermalization



Out-of-equilibrium dynamics under continuous observation



How does continuous observation alter thermalization dynamics in generic (nonintegrable) many-body systems?

Introduction: Thermalization in quantum systems

1. Quantum systems in contact with thermal bath.

(Phenomenological) Master equation:

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] - \sum_a \left(\frac{1}{2}\hat{L}_a^\dagger \hat{L}_a \hat{\rho} + \frac{1}{2}\hat{\rho}\hat{L}_a^\dagger \hat{L}_a - \hat{L}_a \hat{\rho} \hat{L}_a^\dagger \right)$$

The detailed balanced condition for $\hat{L}_a$ → The Gibbs ensemble $\hat{\rho}_\beta = e^{-\beta \hat{H}}/Z$ is ensured to be a steady state solution.

β : bath temperature

E. B. Davies, Comm. Math. Phys. 39, 91 (1974).
H. Spohn and J. L. Lebowitz, Adv. Chem. Phys. 109 (1978).

2. Isolated quantum many-body systems.

"Generic (typically, nonintegrable) many-body systems will thermalize under unitary dynamics."

Possible mechanism: **the eigenstate thermalization hypothesis (ETH)**

✓ Numerically verified

$$\langle E_a|\hat{O}|E_a\rangle \underset{\uparrow}{\simeq} \mathrm{Tr}[\hat{O}\hat{\rho}_\beta]$$

equality in the thermodynamic limit

M. Srednicki, Phys. Rev. E 50, 888 (1994).
J. M. Deutch, Phys. Rev. A 43, 2046 (1991).
H. Tasaki, Phys. Rev. Lett. 80, 1373 (1998).
M. Rigol et al., Nature 452, 854 (2008).

$|E_a\rangle$: eigenstate of $\hat{H}$ with energy E_a

$\hat{O}$: observable

β is determined from $E_a = \mathrm{Tr}[\hat{H}\hat{\rho}_\beta]$

3. Our aim: many-body systems under continuous observation
= open systems without detailed balance condition
(i.e. no a priori bath temperature)

Many-body systems under continuous observation

Our assumptions:

(i) Equilibrium initial state: $\hat{\rho}(0) = \hat{\rho}_{\mathrm{eq}}$

(ii) ETH on the system Hamiltonian; $\langle E_a|\hat{O}|E_a\rangle \simeq \mathrm{Tr}[\hat{O}\hat{\rho}_\beta]$

(iii) Minimally destructive observation; taking $\gamma \to 0$ with γt finite

* Physically, the condition (iii) ensures that
- A waiting time of quantum jump is longer than the equilibration time.
- Finite quantum jump → Not heat up to infinite temperature.

*NESS: Shirai & Mori, arXiv:1812.09713

Matrix-vector product ensemble (MVPE):

$$\hat{\rho}_{\mathcal{M}} \propto \sum_a [\mathcal{V}_{m_n} \cdots \mathcal{V}_{m_1} p_{\mathrm{eq}}]_a \hat{P}_a$$

$\mathcal{M} = (m_1, \ldots m_n)$: sequence of outcomes

$(p_{\mathrm{eq}})_a = \langle E_a|\hat{\rho}_{\mathrm{eq}}|E_a\rangle$: initial vector

$(\mathcal{V}_m)_{ab} = |\langle E_a|\hat{L}_m|E_b\rangle|^2$: matrix of jump operator

$\hat{P}_a = |E_a\rangle\langle E_a|$: projector on energy eigenstate

*Energy fluctuation of $\hat{\rho}_{\mathcal{M}}$ is **subextensive**

Thermalization at each moment of open many-body dynamics under continuous observation.

YA, K. Saito and M. Ueda, Phys. Rev. Lett. (2018).

Numerical simulations on nonintegrable hard-core bosons

Nonintegrable model of hard-core bosons:

Hamiltonian: $\hat{H} = \hat{K} + \hat{U}$

kinetic energy: $\hat{K} = -\sum_l (t_{\mathrm{h}} \hat{b}_l^\dagger \hat{b}_{l+1} + t'_{\mathrm{h}} \hat{b}_l^\dagger \hat{b}_{l+2} + \mathrm{H.c.})$

interaction energy: $\hat{U} = \sum_l (U \hat{n}_l \hat{n}_{l+1} + U' \hat{n}_l \hat{n}_{l+2})$

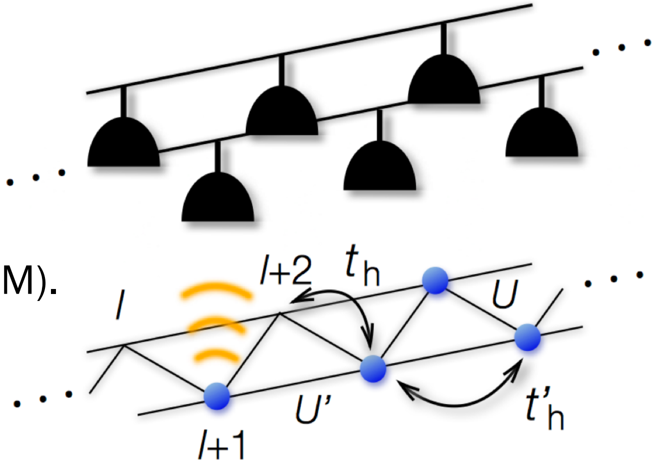
initial state: energy eigenstate at finite temperature

Measurement: site-resolved occupation number

$$\hat{L}_l = \hat{n}_l = \hat{b}_l^\dagger \hat{b}_l$$

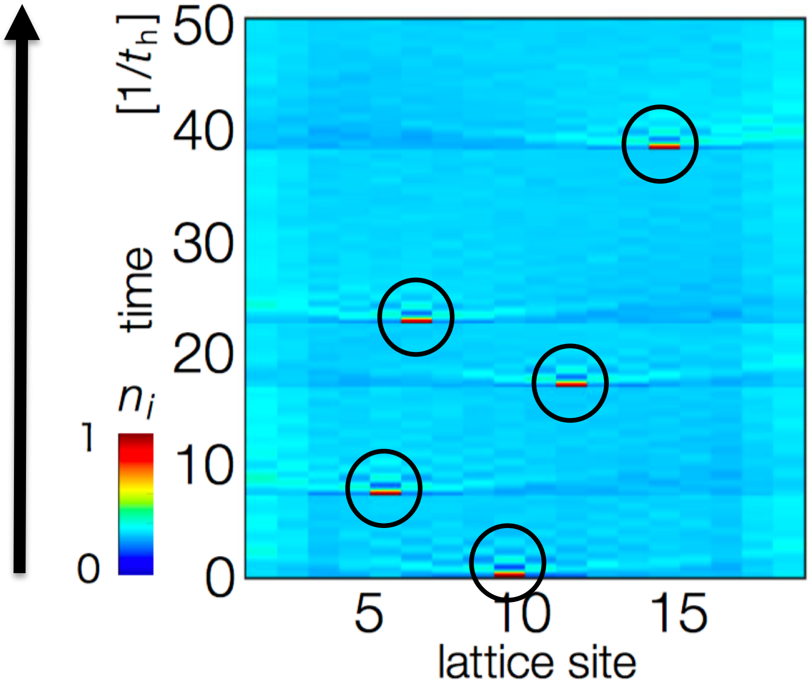
*Physically, this can be realized by light scattering (e.g., in QGM).

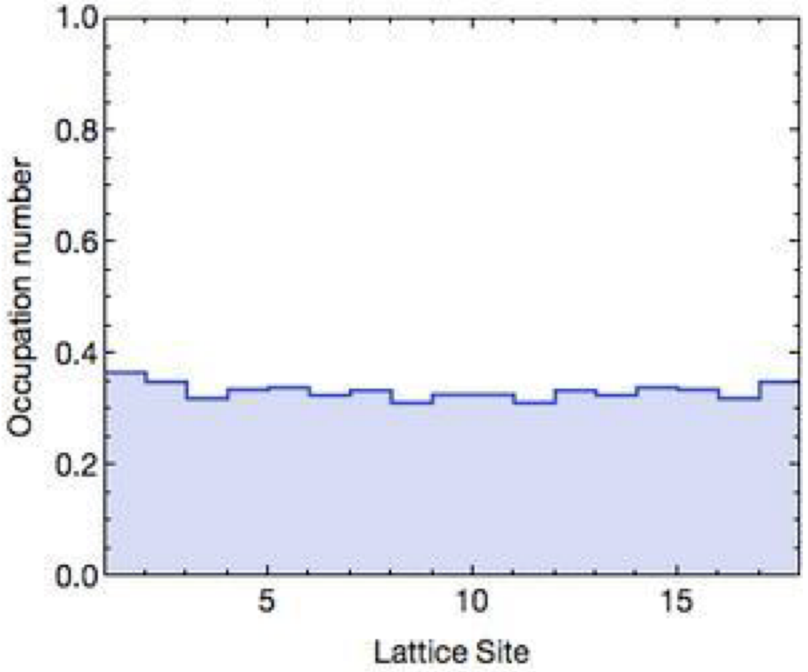
(c.f. **YA** and M. Ueda, PRL 2015)



Numerical simulations with the local measurement

Typical trajectory dynamics: time evolution of occupation number at each site.



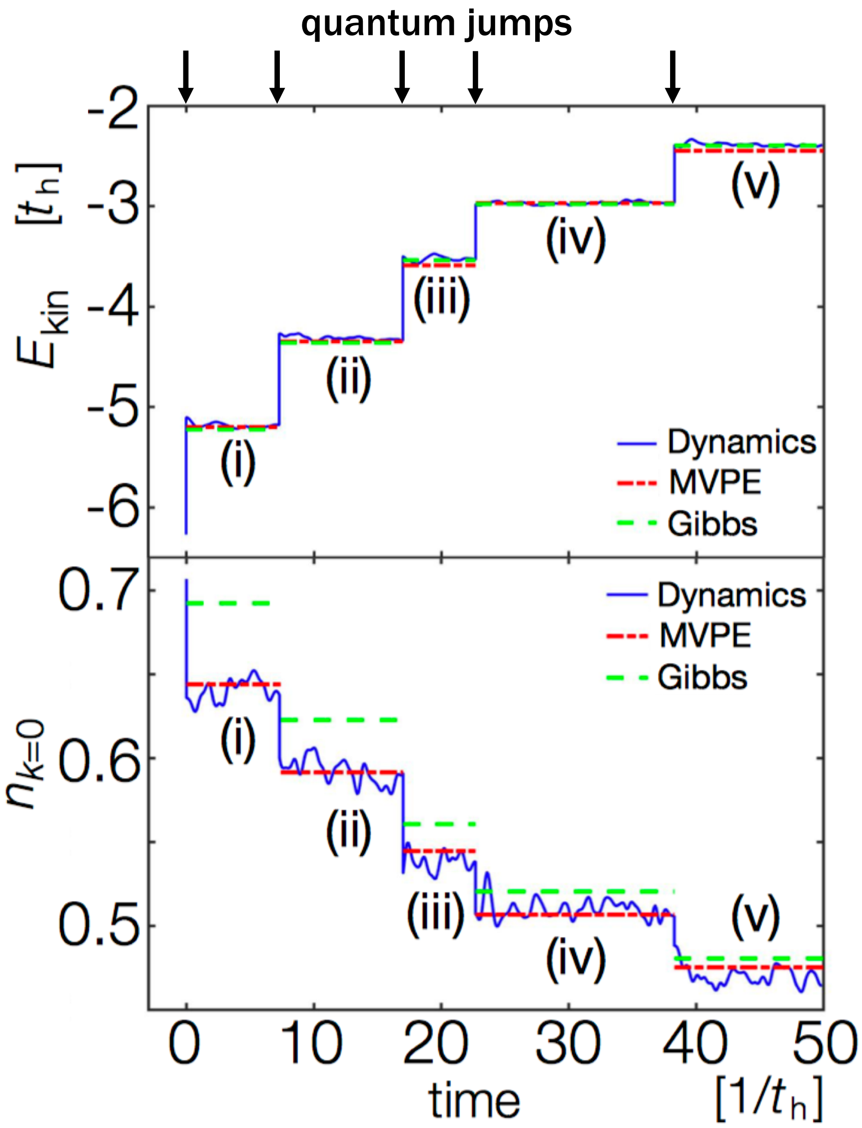


- Quantum jump = Detection of an atom
- Wavefunction localization due to measurement backaction
- **Quick relaxation to the equilibrium density distribution.**

YA, K. Saito and M. Ueda, Phys. Rev. Lett. (2018).

Numerical simulations on nonintegrable hard-core bosons

Typical trajectory dynamics: time evolution of kinetic energy and occupation at *k*=0.



- MVPE (red dashed line):

$$\hat{\rho}_{\mathcal{M}} \propto \sum_a \left[\mathcal{V}_{m_n} \cdots \mathcal{V}_{m_1} p_{\mathrm{eq}}\right]_a \hat{P}_a$$

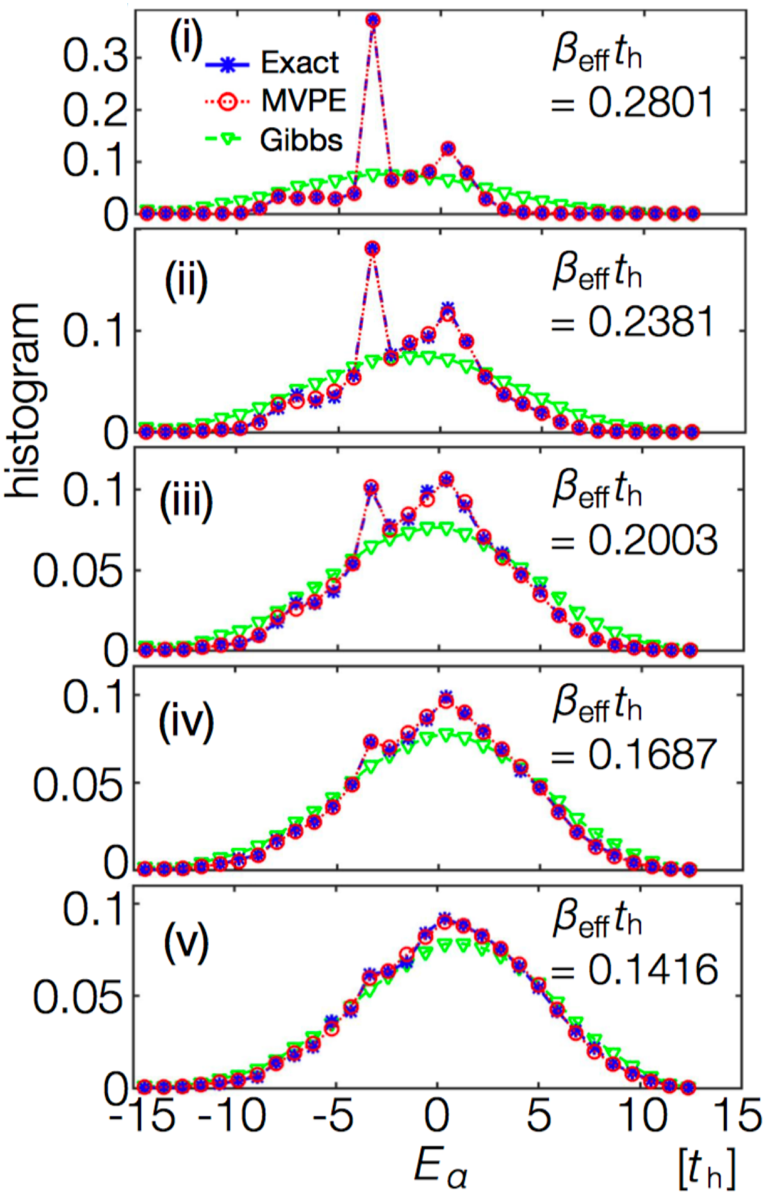
- The Gibbs ensemble at an effective temp. (green dashed line): $\hat{\rho}_{\beta^{\mathcal{M}}_{\mathrm{eff}}}$
- **The dynamics agrees with the MVPE predictions.**

*Discrepancy from the Gibbs ensemble can be attributed to finite-size effects.

A generic (nonintegrable) many-body system under continuous observation thermalizes at a single trajectory level.

Numerical simulations on nonintegrable hard-core bosons

Energy distributions after each jump



- **After a few jumps, the distribution is almost indistinguishable from that of the corresponding Gibbs ensemble.**

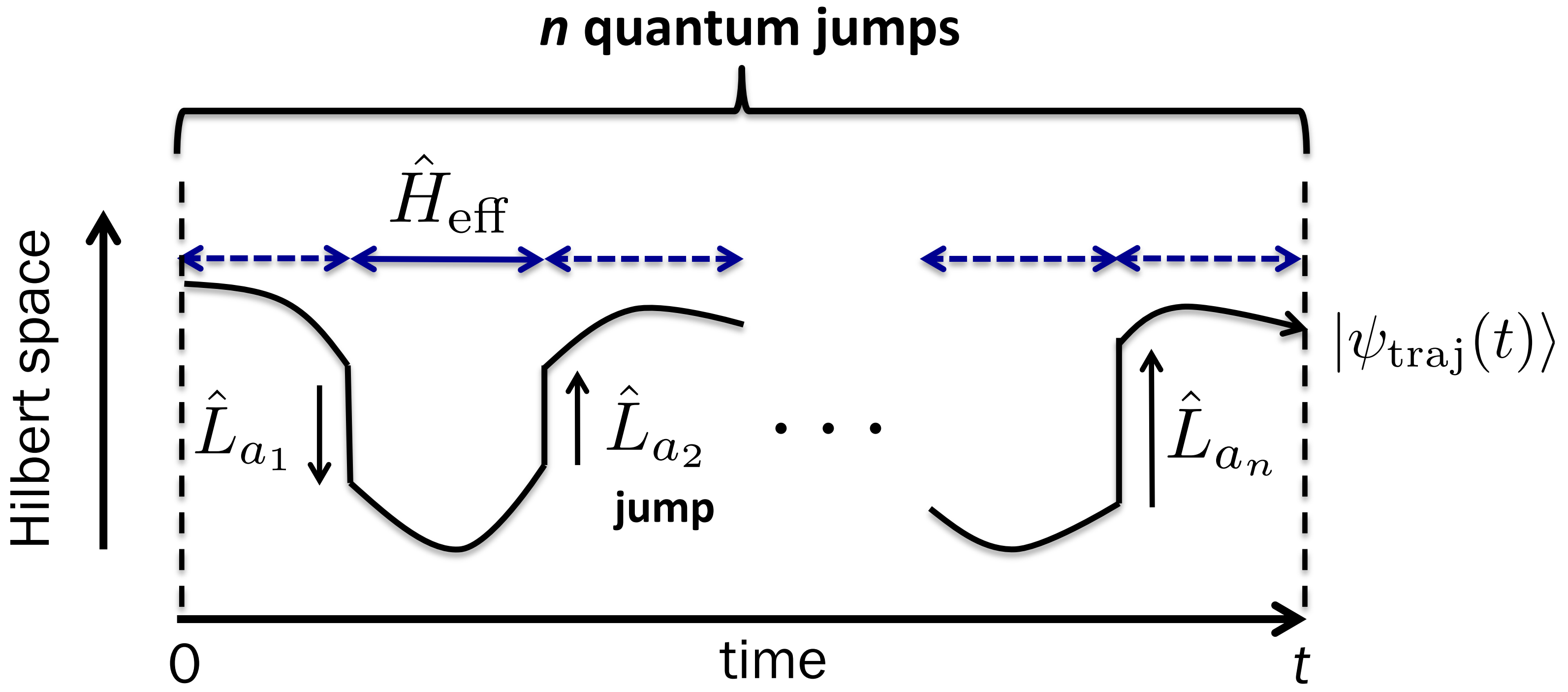
* Application: efficient simulation of Lindblad equation without taking ensemble average.

* We have demonstrated the same conclusions also for a global measurement: $\hat{L} = \sum_l (-1)^l \hat{n}_l$

A generic (nonintegrable) many-body system under continuous observation thermalizes at a single trajectory level.

Full-counting "dynamics" under continuous observation

What happens if only **incomplete information** about measurement outcomes is available?



Post-measurement density matrix conditioned on the number of jumps *n* :

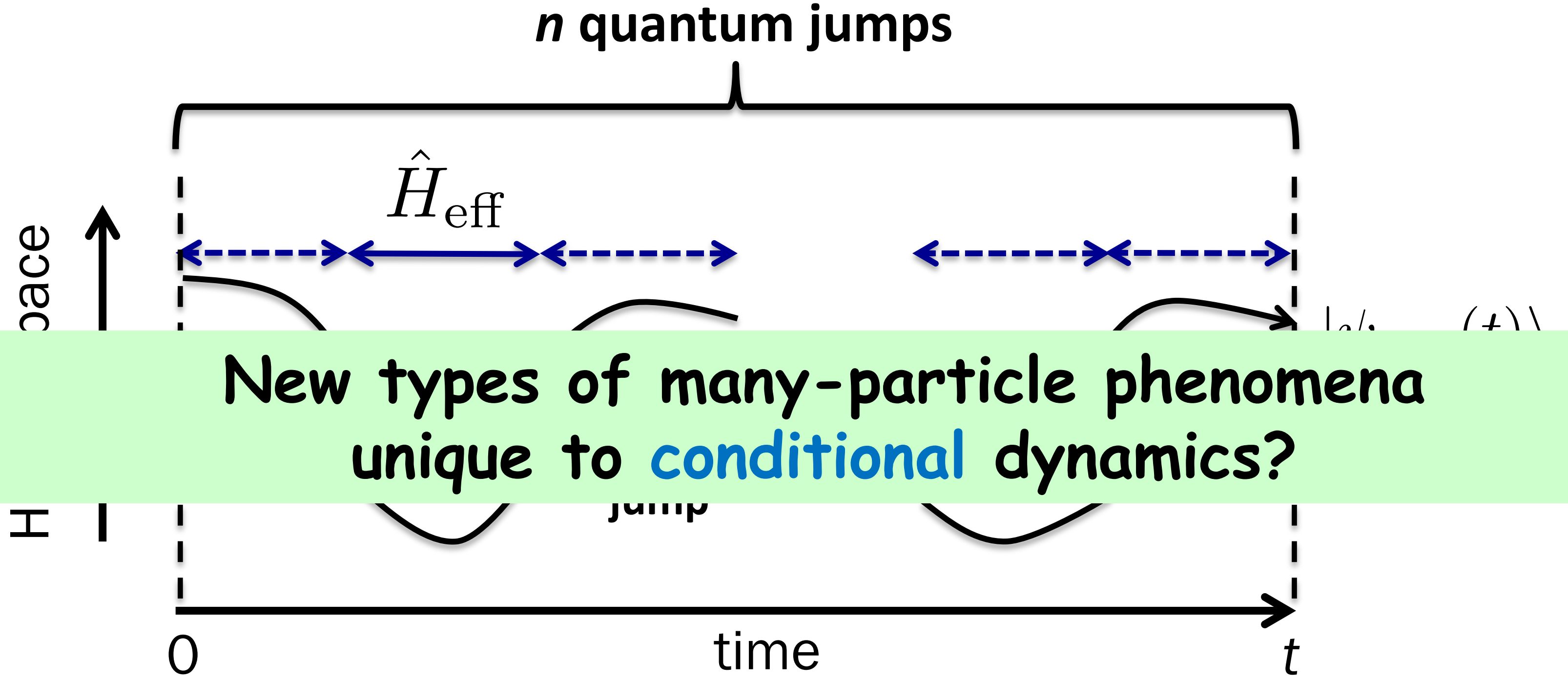
$$\hat{\rho}_{\mathrm{post}}^{(n)}(t) \propto \sum_{i \in \mathcal{D}_n} |\psi_{\mathrm{traj},i}(t)\rangle\langle\psi_{\mathrm{traj},i}(t)|$$

$\mathcal{D}_n$: subspace of quantum trajectories having *n* jumps

***different** from the unconditional dynamics (= Lindblad dynamics)

Full-counting "dynamics" under continuous observation

What happens if only **incomplete information** about measurement outcomes is available?



New types of many-particle phenomena unique to conditional dynamics?

Post-measurement density matrix conditioned on the number of jumps *n* :

$$\hat{\rho}^{(n)}_{\rm post}(t) \propto \sum_{i \in \mathcal{D}_n} |\psi_{{\rm traj},i}(t)\rangle\langle\psi_{{\rm traj},i}(t)|$$

$\mathcal{D}_n$: subspace of quantum trajectories having *n* jumps

***different** from the unconditional dynamics (= Lindblad dynamics)

Propagation beyond the Lieb-Robinson bound (summary)

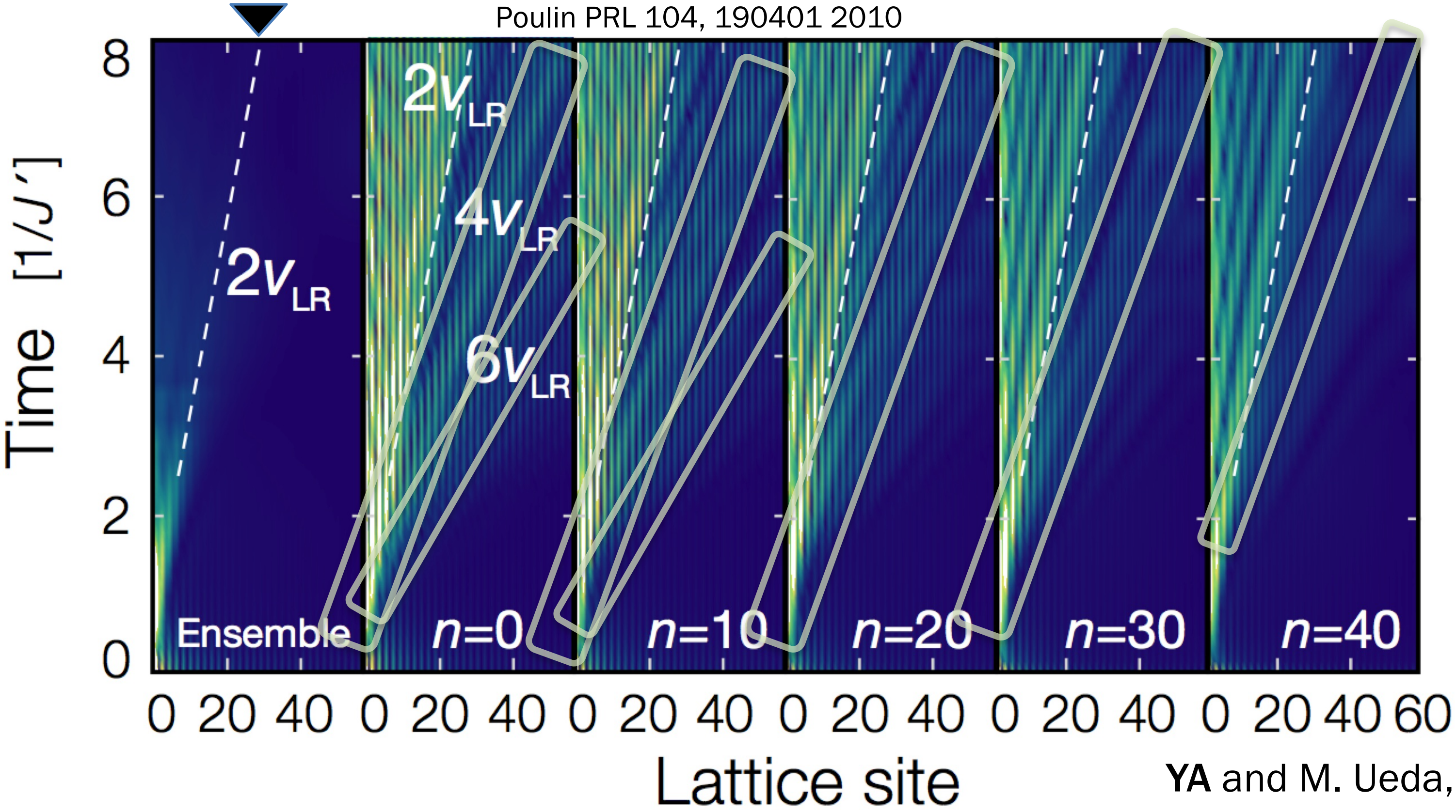
Exact calculations of quench dynamics for solvable model

Lindblad dynamics: $C(l,t)=\mathrm{Tr}[\hat{\rho}(t)\hat{c}_l^\dagger\hat{c}_0]$ $\quad\hat{\rho}(t)=\sum_n P_n(t)\hat{\rho}^{(n)}(t)$

conditional dynamics: $C^{(n)}(l,t)=\mathrm{Tr}[\hat{\rho}^{(n)}(t)\hat{c}_l^\dagger\hat{c}_0]$

Lieb-Robinson (LR) bound for Lindblad dynamics

Poulin PRL 104, 190401 2010



YA and M. Ueda, PRL (2018).

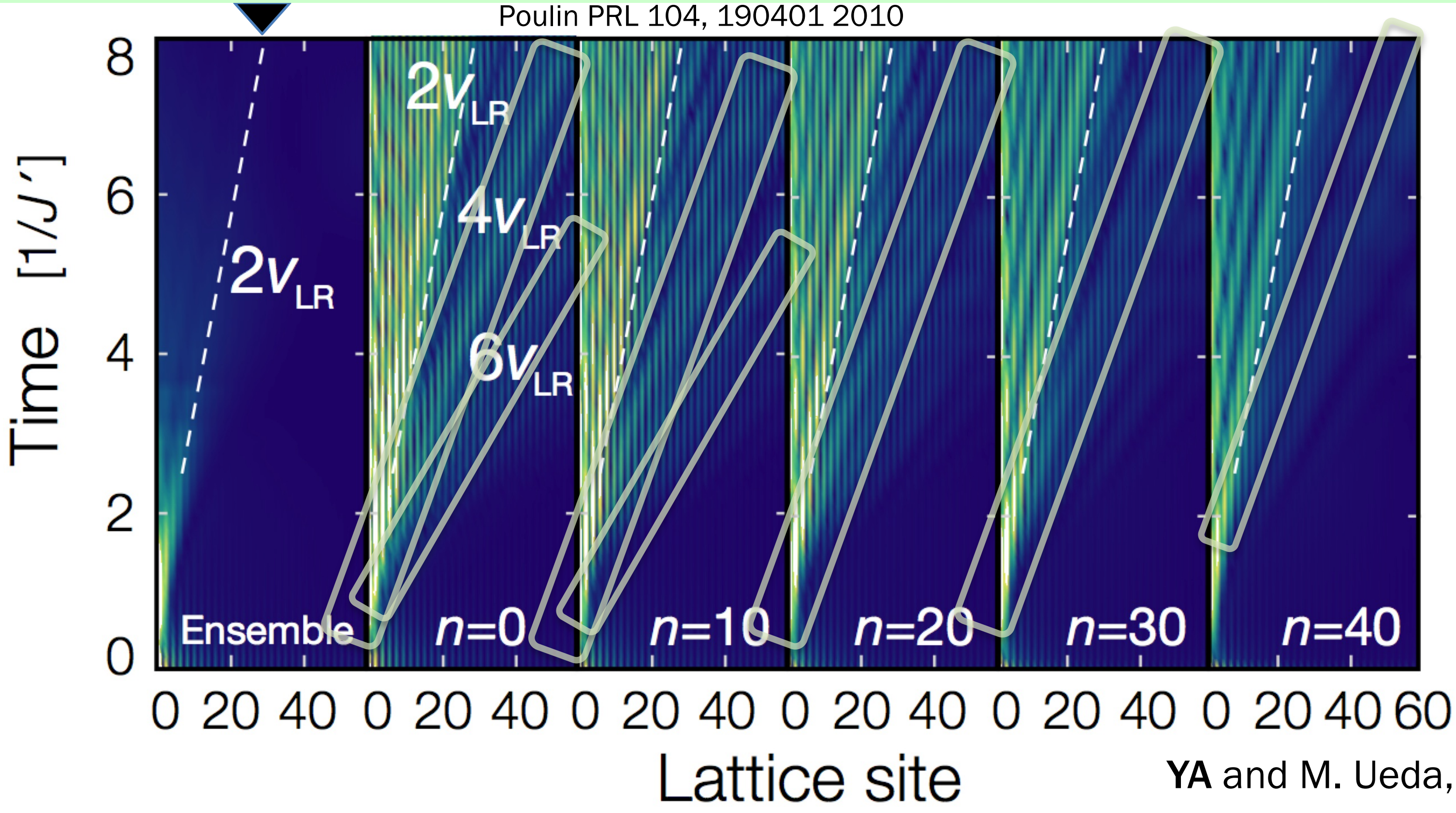
Propagation beyond the Lieb-Robinson bound (summary)

Exact calculations of quench dynamics for solvable model

Lindblad dynamics: $C(l,t) = \mathrm{Tr}[\hat{\rho}(t)\hat{c}_l^\dagger \hat{c}_0]$ $\hat{\rho}(t) = \sum_n P_n(t)\hat{\rho}^{(n)}(t)$

conditional dynamics: $C^{(n)}(l,t) = \mathrm{Tr}[\hat{\rho}^{(n)}(t)\hat{c}_l^\dagger \hat{c}_0]$

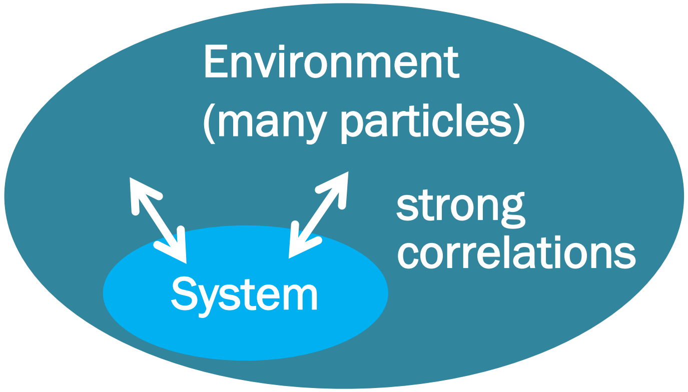
Propagation beyond the LR bound can (probabilistically) appear as a result of the global postselection.

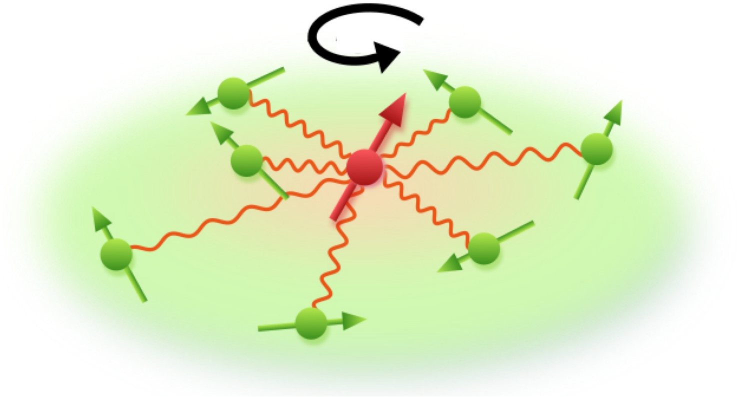


Poulin PRL 104, 190401 2010

YA and M. Ueda, PRL (2018).

II. Quantum systems strongly correlated with environment: Nonequilibrium quantum impurities (short summary)





* Intrinsically non-Markovian open systems due to strong correlations.
→ Theoretical framework beyond the first part required.

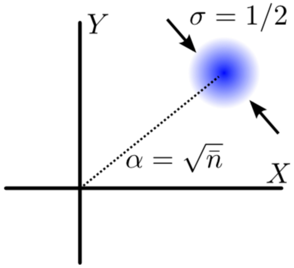
Introduction: Solving quantum many-body problems

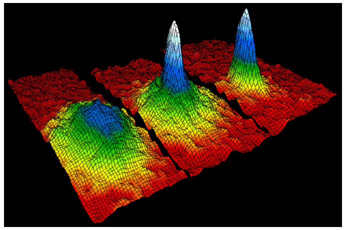
Difficulty: exponentially large Hilbert space

Guiding principle:
Design a family of variational states $\{|\Psi_X\rangle\}$ that can capture essential physics behind a problem in an efficient way.

Bose-Einstein Condensate:

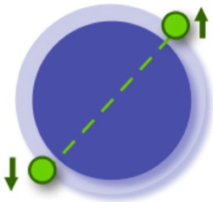
Coherent state





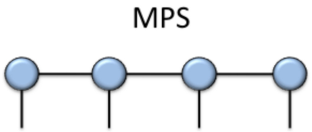
Quadratic Hamiltonian (noninteracting systems)

Gaussian states



Low-temperature physics of 1D systems

Matrix-product states



Introduction: Solving quantum many-body problems

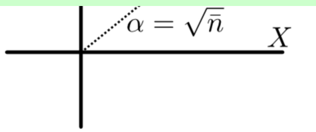
Difficulty: exponentially large Hilbert space

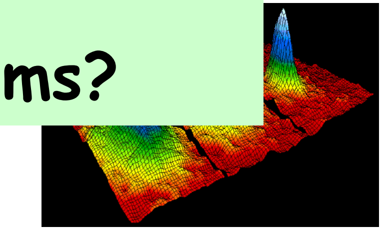
Guiding principle:
Design a family of variational states $\{|\Psi_X\rangle\}$ that can capture essential physics behind a problem in an efficient way.

Bo

What is an efficient ansatz for generic quantum spin-impurity systems?

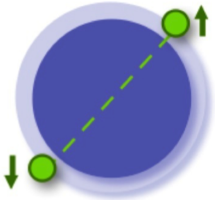
Coherent state





Quadratic Hamiltonian (noninteracting systems)

Gaussian states



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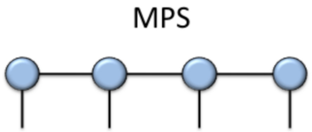
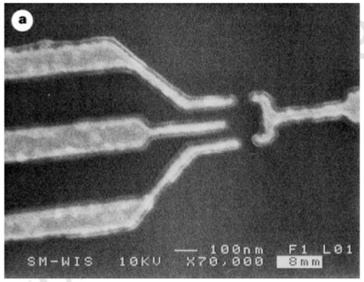


Quantum impurity: a paradigm in many-body physics

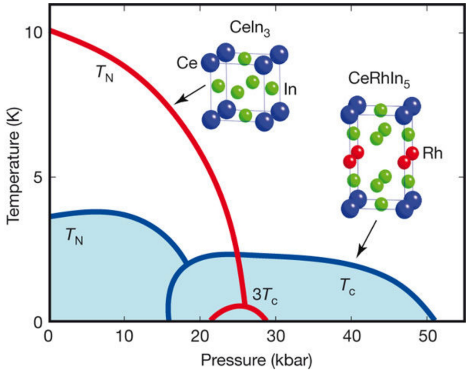
- **Relevance to a variety of physical systems:**

quantum dots



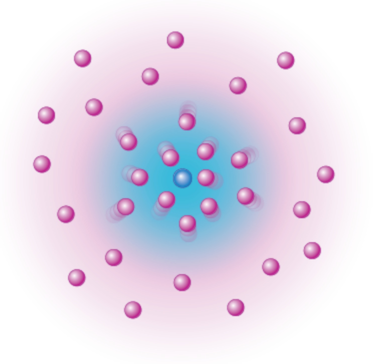
Goldhaber-Gordon et al.,
Nature 391, 156 (1998).

heavy electron materials



Monthoux et al.,
Nature 450, 1177 (2007).

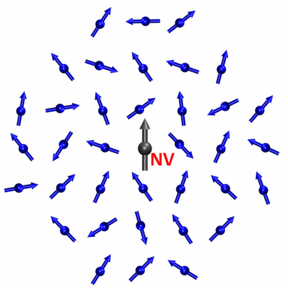
ultracold atoms



Physics 9, 86 (2016).

- **Prototypical open system**

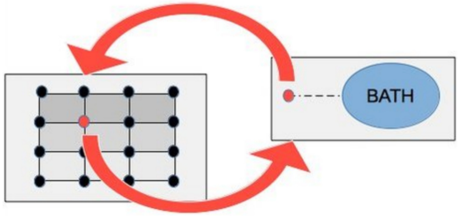
decoherence



Shin et al., PRB 88, 161412 (2013).

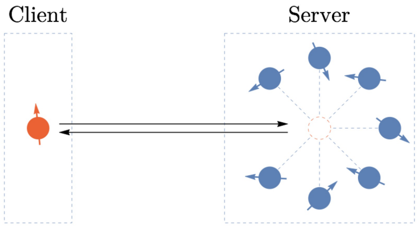
- **Numerical method**

DMFT solver



From Web page of LMU
theoretical nanophysics.

- **Universal quantum computation**



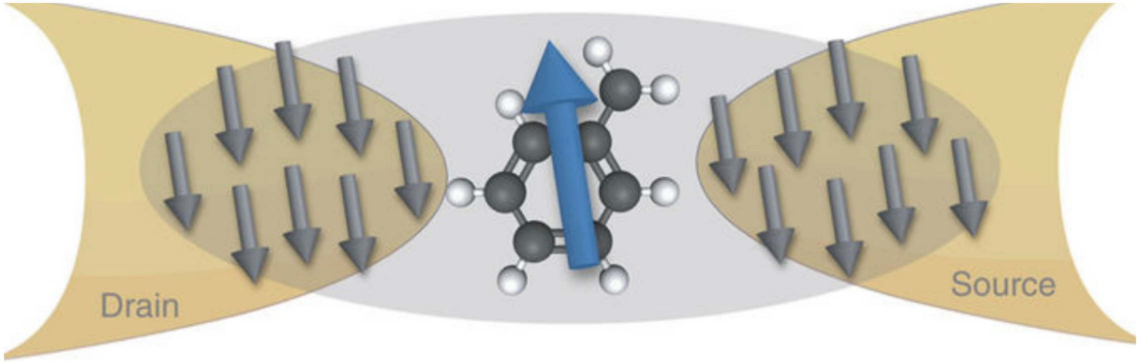
M. C. Tran & J. M. Taylor,
arXiv:1801.04006

New “disentangling” canonical transformation

Essential feature of spin-impurity systems:

strong entanglement between impurity and bath

(*this strong correlation invalidates the Markov approximation).



Kondo singlet

impurity
spin 1/2

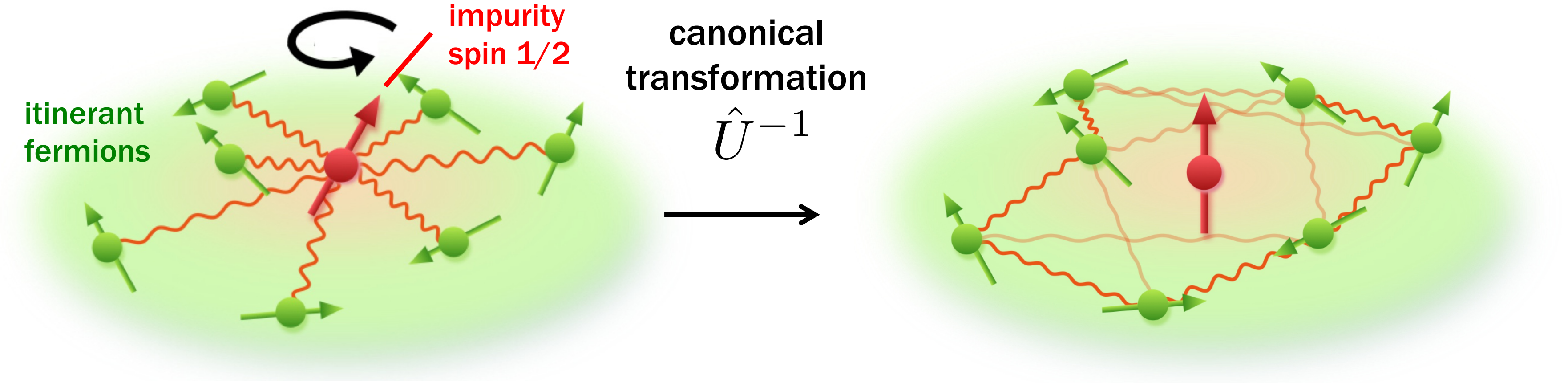
itinerant fermions

Nat. Commun. 8, 15210 (2017)

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{\text{imp}}|\Psi_{\downarrow}\rangle - |\downarrow\rangle_{\text{imp}}|\Psi_{\uparrow}\rangle\right)$$

New “disentangling” canonical transformation

Construct a **“disentangling”** canonical transformation U



Original frame

canonical transformation $\hat{U}^{-1}$

“Corotating” frame

$$|\Psi_{\mathrm{tot}}\rangle$$

Strong entanglement

$$\hat{U}^{-1}|\Psi_{\mathrm{tot}}\rangle = |\Psi_{\mathrm{imp}}\rangle|\Psi_{\mathrm{bath}}\rangle$$

Impurity spin is **decoupled** from bath!

Efficient variational states: $|\Psi_{\mathrm{var}}\rangle = \hat{U}|\Psi_{\mathrm{imp}}\rangle\underset{\uparrow}{|\Psi_{\mathrm{bath}}\rangle}$

Gaussian states

YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, Phys. Rev. Lett. (2018).

YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, Phys. Rev. B (2018).

New “disentangling” canonical transformation

1. Notice **parity** symmetry in a Hamiltonian of a generic spin-impurity system:

$$\hat{H} = \sum_{nl\alpha} h_{nl}\hat{\Psi}^{\dagger}_{n\alpha}\hat{\Psi}_{l\alpha} + \frac{1}{4}\sum_{\gamma=x,y,z}\hat{\sigma}^{\gamma}_{\text{imp}}\cdot\sum_{n\alpha\beta} g^{\gamma}_{n}\hat{\Psi}^{\dagger}_{n\alpha}\sigma^{\gamma}_{\alpha\beta}\hat{\Psi}_{n\beta} - \frac{h_{\text{i}}}{2}\hat{\sigma}^{z}_{\text{imp}}$$

invariance under π rotation of impurity and bath spins around *z* axis:

$$\hat{\sigma}^{x,y} \to \hat{\mathbb{P}}^{-1}\hat{\sigma}^{x,y}\hat{\mathbb{P}} = -\hat{\sigma}^{x,y} \longrightarrow \boxed{[\hat{H},\hat{\mathbb{P}}]=0}$$

parity operator: $$\hat{\mathbb{P}} = e^{i\pi\hat{\sigma}^{z}_{\text{imp}}/2}\underbrace{e^{i\pi(\sum_n \hat{\sigma}^{z}_{n}+\hat{N})/2}}_{\text{bath parity: }\hat{\mathbb{P}}_{\text{bath}}}$$

spin operator of bath mode *n*: $\hat{\sigma}^{z}_{n}$

2. Find the unitary transformation *U* mapping the parity to the impurity spin-1/2:

$$\hat{U}^{\dagger}\hat{\mathbb{P}}\hat{U} = \hat{\sigma}^{x}_{\text{imp}}$$ ⟶ impurity is **decoupled** ! $$\boxed{[\hat{\tilde{H}},\hat{\sigma}^{x}_{\text{imp}}]=0}$$

Transformed Hamiltonian

$$\hat{\tilde{H}} = \hat{U}^{\dagger}\hat{H}\hat{U} \longleftarrow \hat{U} = \frac{1}{\sqrt{2}}\left(1 + i\hat{\sigma}^{y}_{\text{imp}}\hat{\mathbb{P}}_{\text{bath}}\right)$$

effective bath-bath interaction mediated by the impurity

$$= \sum_{lm\alpha} h_{lm}\hat{\Psi}^{\dagger}_{l\alpha}\hat{\Psi}_{m\alpha} + \frac{1}{4}\sum_{l}\left[g^{x}_{l}\sigma^{x}_{\text{imp}}\hat{\sigma}^{x}_{l} + \underline{\hat{\mathbb{P}}_{\text{bath}}\left(-ig^{y}_{l}\hat{\sigma}^{y}_{l} + g^{z}_{l}\sigma^{x}_{\text{imp}}\hat{\sigma}^{z}_{l}\right)}\right]$$

$\sigma^{x}_{\text{imp}} = \pm 1$ is c-number.

Efficient variational states for quantum spin impurity

Approximate the fermionic bath in the "corotating" frame by **Gaussian states:**

$$|\Psi_{\rm var}\rangle = \hat{U}|\pm_x\rangle|\Psi_{\rm bath}\rangle$$

fully characterized by the **covariance matrix** ↑

$$(\Gamma)_{\xi l\alpha,\eta m\beta} = \frac{i}{2}\langle\Psi_{\rm bath}|[\hat{\psi}_{\xi,l\alpha},\hat{\psi}_{\eta,m\beta}]|\Psi_{\rm bath}\rangle,$$

Majorana operators: $\hat{\psi}_{1,l\alpha} = \hat{\Psi}^{\dagger}_{l\alpha} + \hat{\Psi}_{l\alpha},\ \hat{\psi}_{2,l\alpha} = i(\hat{\Psi}^{\dagger}_{l\alpha} - \hat{\Psi}_{l\alpha})$

Time-dependent variational equations:

Imaginary-time evolution: $$\frac{d\Gamma}{d\tau} = -\mathcal{H} - \Gamma\mathcal{H}\Gamma\,, \qquad \mathcal{H} = 4\delta\langle\hat{H}\rangle/\delta\Gamma$$

Ground-state properties in $\tau \to \infty$

Real-time evolution: $$\frac{d\Gamma}{dt} = \mathcal{H}\Gamma - \Gamma\mathcal{H}$$

Out-of-equilibrium dynamics

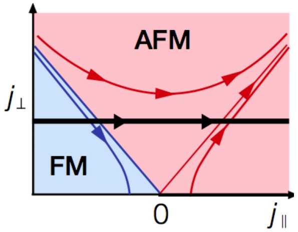
Benchmark tests with MPS results in and out of equilibrium

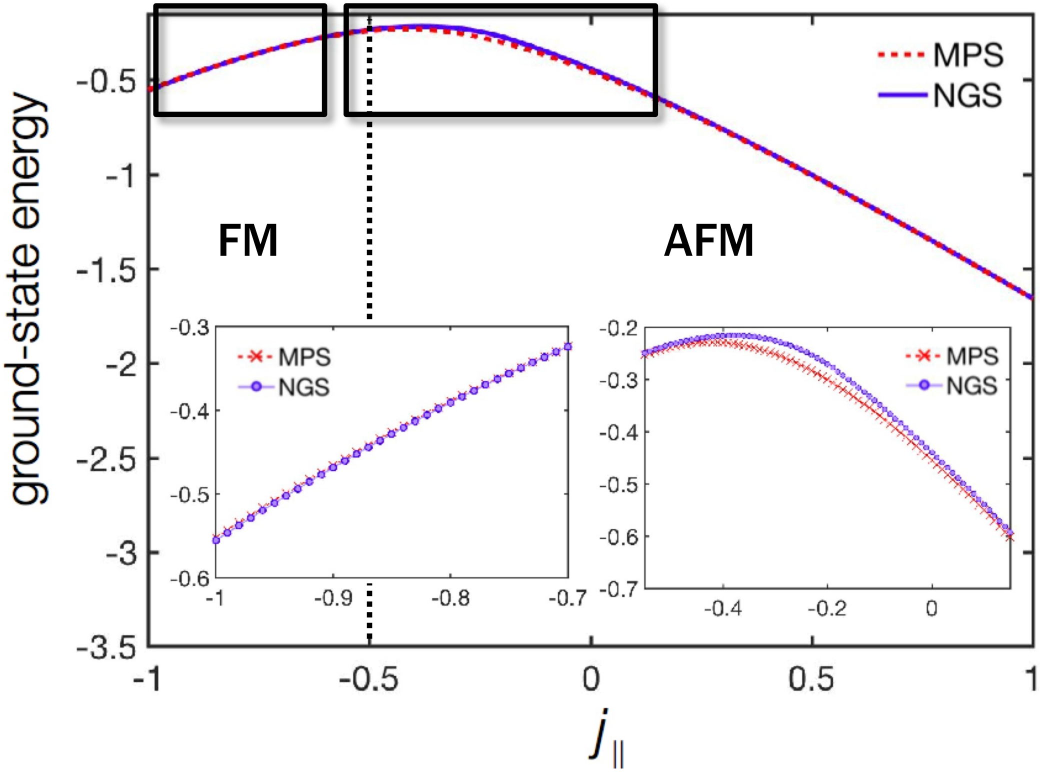
Benchmark tests for the anisotropic Kondo model

Ground-state energies (L=200)

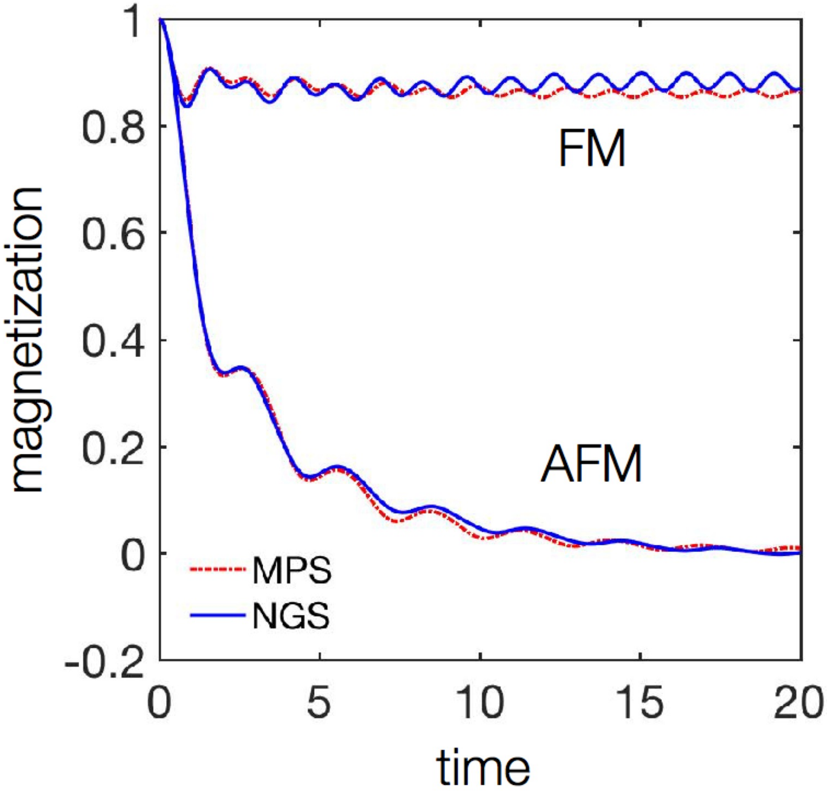
red: MPS (D=280)

blue: our variational states





Quench dynamics of the impurity



YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, Phys. Rev. Lett. (2018).

YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, Phys. Rev. B (2018).

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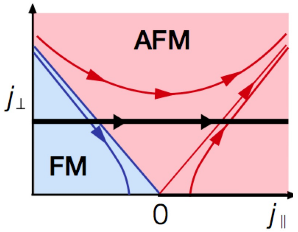
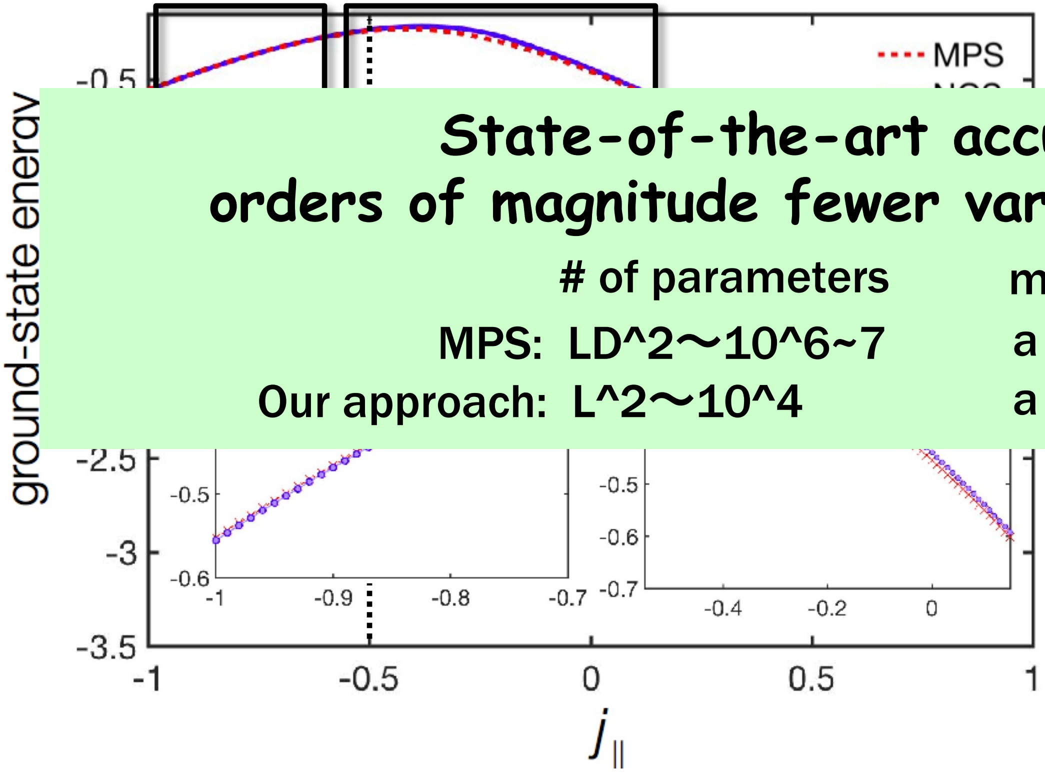
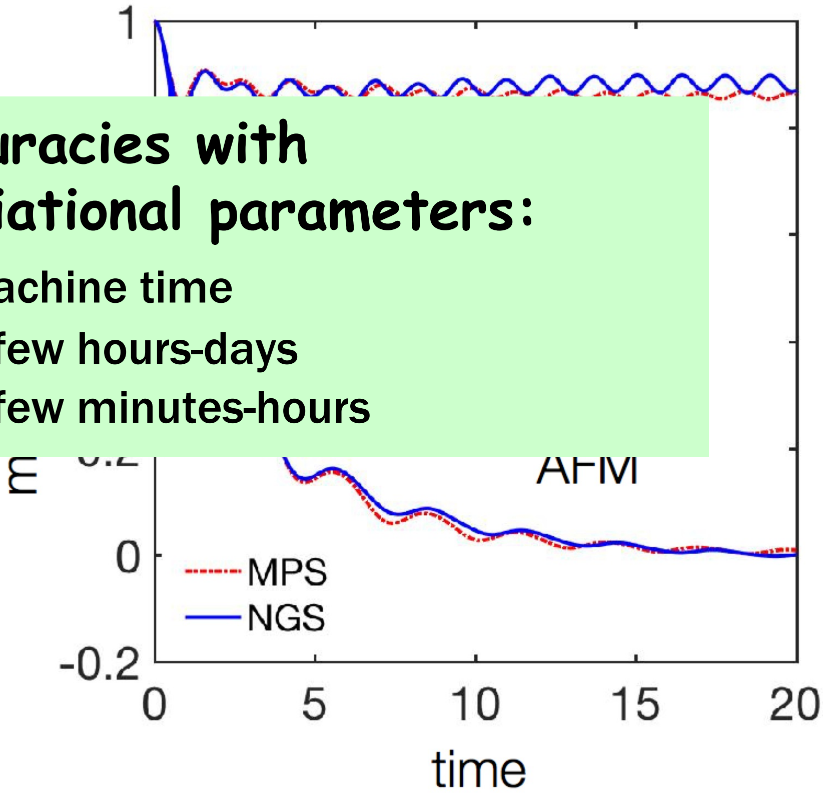


Quench dynamics of the impurity





State-of-the-art accuracies with orders of magnitude fewer variational parameters:

	# of parameters	machine time
MPS:	LD^2〜10^6~7	a few hours-days
Our approach:	L^2〜10^4	a few minutes-hours

YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, Phys. Rev. Lett. (2018).

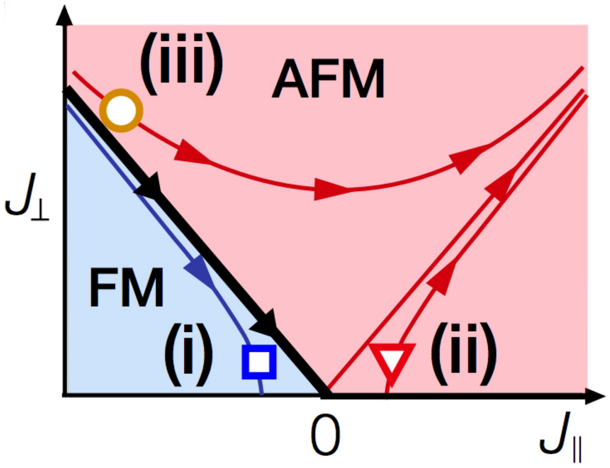
YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, Phys. Rev. B (2018).

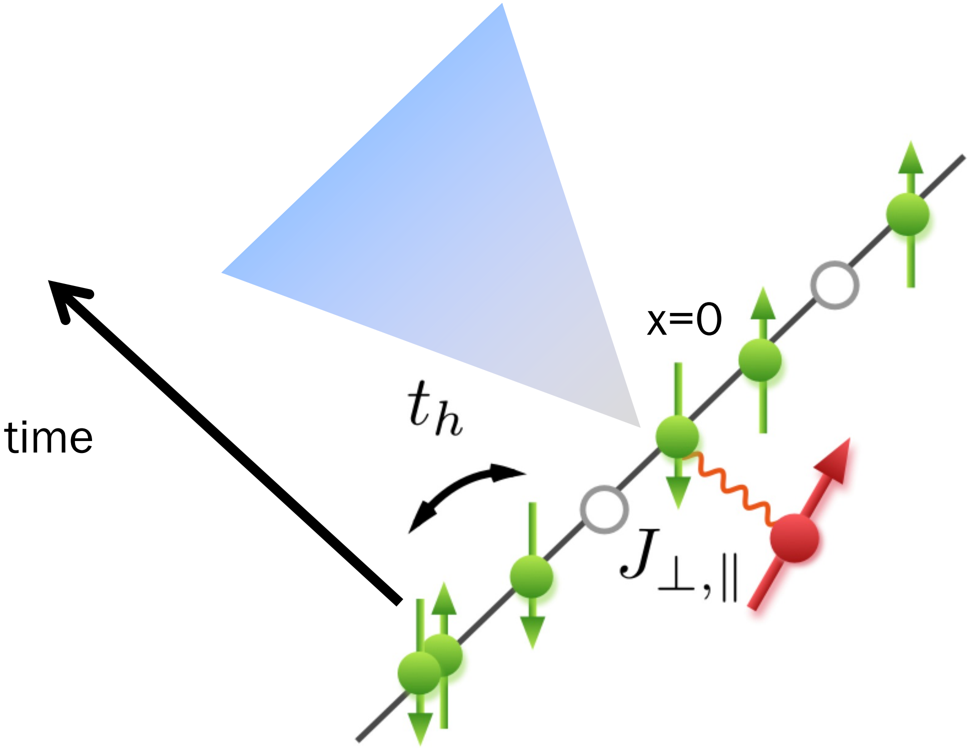
Spatiotemporal dynamics of correlations after quench

initial state: $|\Psi_0\rangle = |\uparrow\rangle_{\mathrm{imp}}|\mathrm{Fermi\ sea}\rangle$

Quench the impurity-bath interaction at site *x*=0.

Real-time formation and spread of the impurity-bath correlation?





Spatiotemporal dynamics of correlations after quench

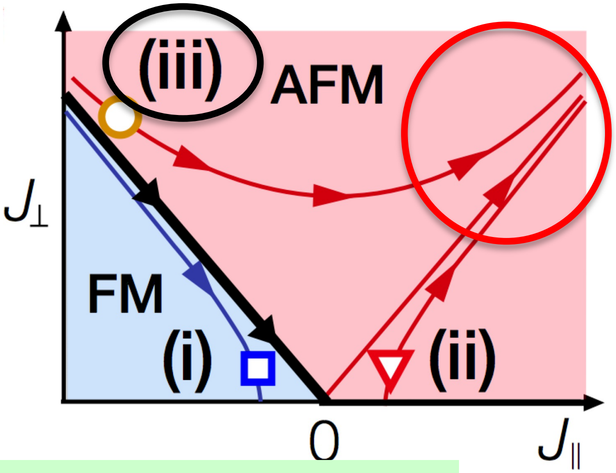
(iii) **FM→AFM crossover : mimicking RG flow in time**

short-time = high-energy physics → FM (Jz<0)

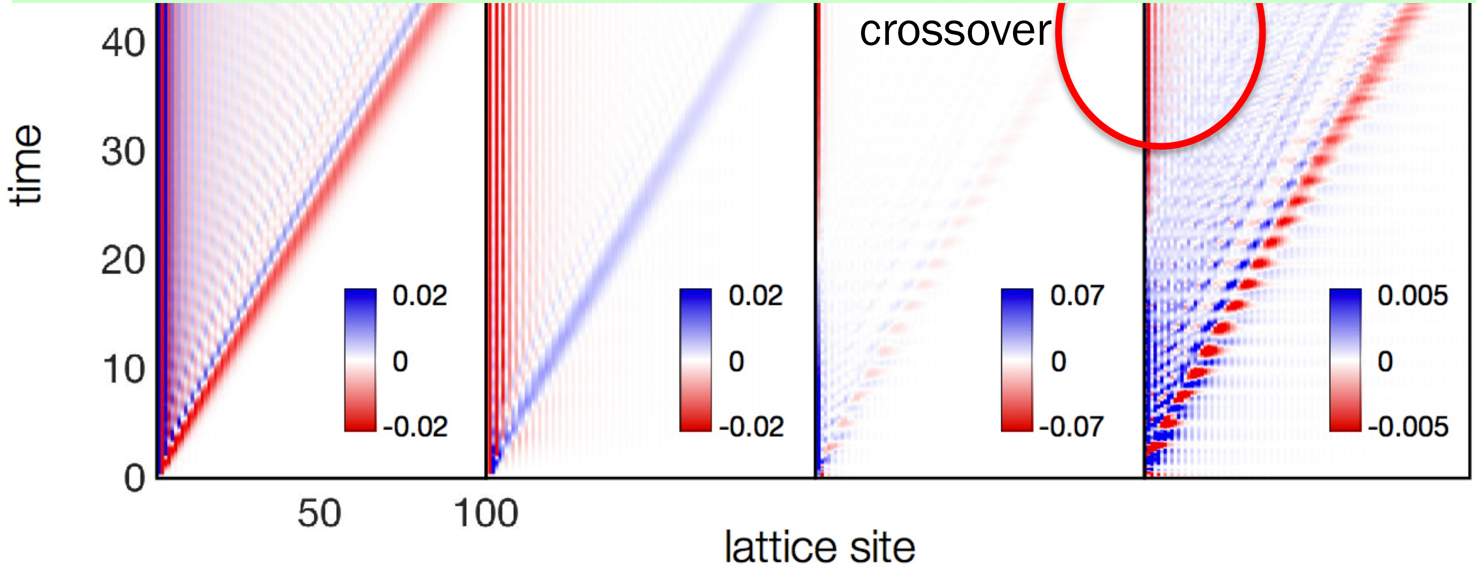
long-time = low-energy physics → AFM (J>0)

Effective temperature after quench: $T_{\mathrm{eff}} \simeq \dfrac{2}{\pi t}$

Nordlander et al., PRL 83, 808 (1999)



Long-time many-body dynamics in previously unexplored regime.



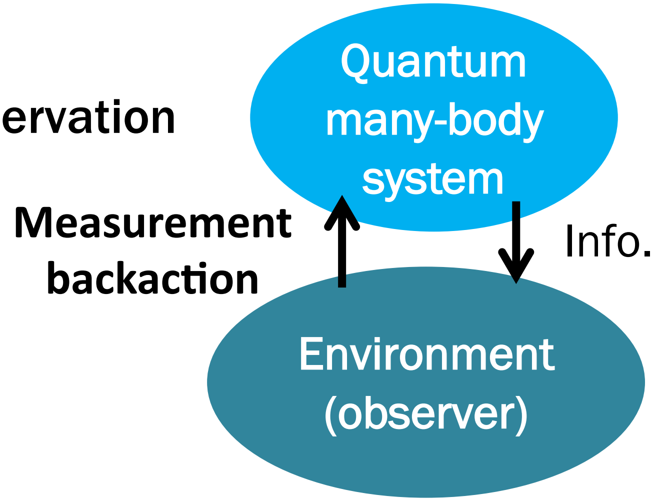
Summary: Quantum Many-Body Physics in Open Systems

Main theme: New frontier of quantum many-body physics pioneered with the ability to measure and manipulate single quanta of many-body systems.

(Measurement)

I: Quantum many-body physics under continuous observation

- New types of RG flows, fixed points and phase transitions in quantum critical phenomena.
- Unique out-of-equilibrium dynamics due to backaction from continuous observation.



Quantum many-body system

Measurement backaction

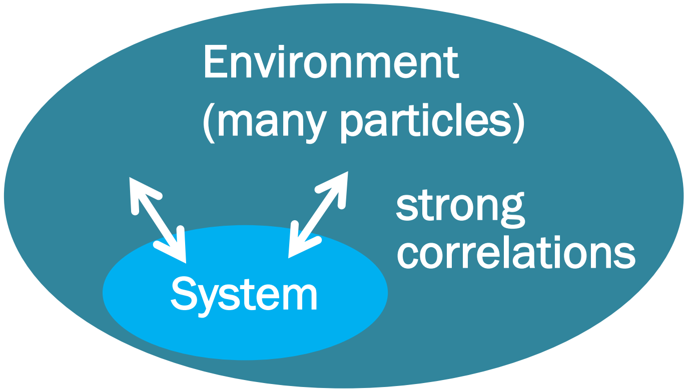
Info.

Environment (observer)

(Manipulation)

II: Quantum systems strongly correlated with environment

- Versatile and efficient theoretical approach to generic spin-impurity systems.
- Allows one to analyze out-of-equilibrium many-body dynamics in previously challenging regimes.



Environment (many particles)

strong correlations

System