非エルミート系の物理：
量子ウォークによるアプローチ
Non-Hermitian Physics:
An approach of quantum walks

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統計物理学懇談会　2022年3月10日

Importance of (discrete-time) quantum walks

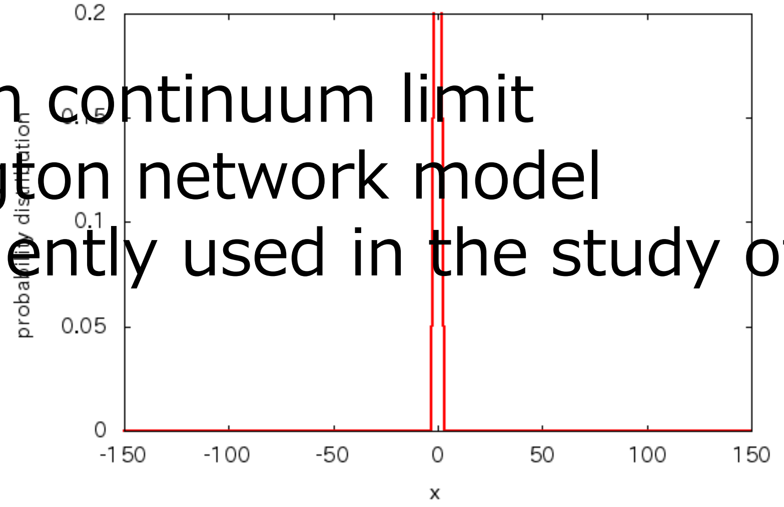
➡ Quantum walk defines a fundamental equation for discrete-time quantum dynamics.

Quantum walk:

- Quantum counterpart of classical random walks
- Faster propagations of probability
- Useful for quantum computations and information (Grover search).
- Interdisciplinary science; physics, math, quantum info, ⋯
- Various experimental setup (photon, cold atom, ion trap, ⋯)

- Dirac equation in continuum limit
- Chalker-Coddington network model

frequently used in the study of the Anderson localization



Importance of quantum walks for non-Hermitian physics

Quantum walk is an ideal platform for experiments of non-Hermitian systems.

Photonic quantum walks with entangled photons

- Quantitative control of loss of photons
- Quantumness is guaranteed by the entanglement
- Various physical phenomena can be reproduced due to high controllability of system parameters (quantum simulator).

nature physics
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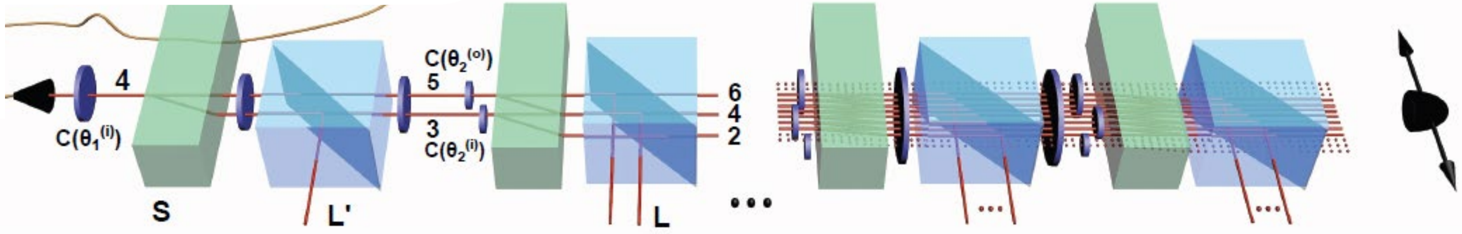
PUBLISHED ONLINE: 31 JULY 2017 | DOI: 10.1038/NPHYS4204

Observation of topological edge states in parity–time-symmetric quantum walks

L. Xiao[1], X. Zhan[1], Z. H. Bian[1], K. K. Wang[1], X. Zhang[1], X. P. Wang[1], J. Li[1], K. Mochizuki[2], D. Kim[2], N. Kawakami[3], W. Yi[4,5], H. Obuse[2], B. C. Sanders[5,6,7,8] and P. Xue[1,9]*

Nature Physics 13, 1117 (2017)



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[1] K. Mochizuki, D. Kim, and H. Obuse, Phys. Rev. A **93**, 062116 (2016).
[2] L. Xiao, X. Zhan, Z.H. Bian, K. K. Wang, X. Zhang, X.P. Wang, J.Li, K. Mochizuki, D. Kim, N. Kawakami,Y. Wi, H. Obuse, B. Sanders, P. Xue, Nature Phys. **13**, 1117 (2017).
[3] L. Xiao, X. Qin, K. Wang, Z. Bian, X. Zhan, H. Obuse, B.Sanders, W. Yi, P. Xue, Phys. Rev. A **98**, 063847 (2018).
[4] K. Mochizuki, D. Kim, N. Kawakami, and H. Obuse, Phys. Rev. A, **102**, 062202 (2020).
[5] M. Kawasaki, K. Mochizuki, N. Kawakami, and H. Obuse, Prog. Theor. Exp. Phys. **2020**, 12A105 (2020).
[6] N. Hatano and H. Obuse, Annals of Physics **435**, 168615 (2021).
[7] T. Bessho, K. Mochizuki, H. Obuse, and M. Sato, Phys. Rev. B (accepted).
[8] R. Okamoto, N. Kawakami, and H. Obuse (in preparation).

解説
- 「量子ウォークのトポロジカル相と光の振幅制御への応用」小布施、望月、金、川上、日本物理学会誌 **74**, 780 (2019)
- 「アンダーソン局在の臨界現象：最近の実験・理論の新展開」 小布施、日本物理学会誌 **70**, 14 (2015)

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- Manami Yamagishi (U. Tokyo)

■ Experiment :

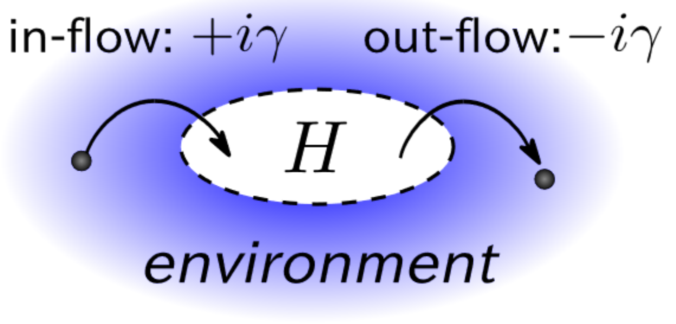
- Ryo Okamoto (Kyoto Univ.)
- Peng Xue group (BCSRC, China)

1.Non-Hermitian Systems

Non-Hermitian physics and open systems

Non-Hermitian Hamiltonian:

$$H \neq H^{\dagger}$$



Open quantum systems

(gain and/or loss, asymmetric hopping)

$H\psi = E\psi$ → Complex eigen-energy

$E \in \mathbb{C}$ Breakdown of conservation of probabilities

There are various phenomena which are peculiar to non-Hermitian systems.

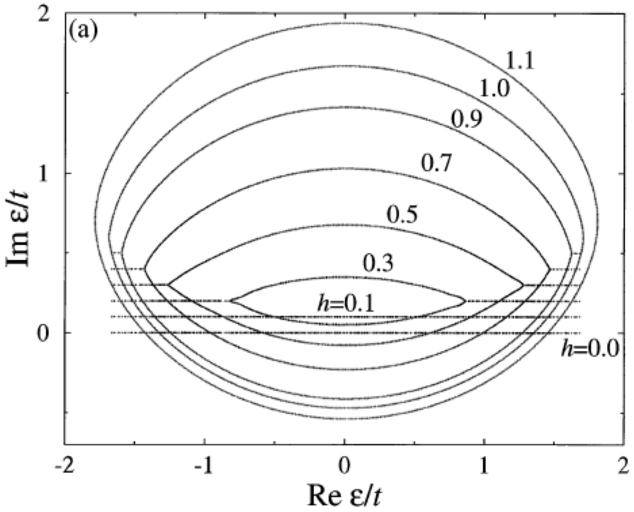
Anderson transition in 1D non-Hermitian systems

N. Hatano and D. R. Nelson, PRL **77**, 570 (1996)

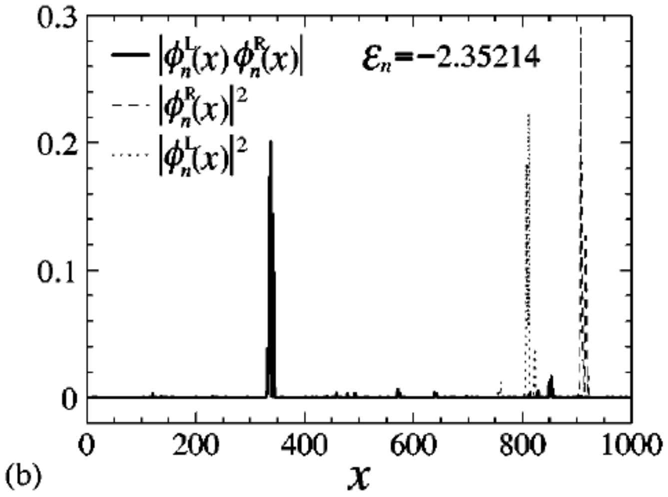
Hatano-Nelson model : asymmetric hopping with onsite disorder in 1D

$$H = \sum_x t(e^h c_{x+1}^\dagger c_x + e^{-h} c_{x-1}^\dagger c_x) + \varepsilon_x c_x^\dagger c_x$$

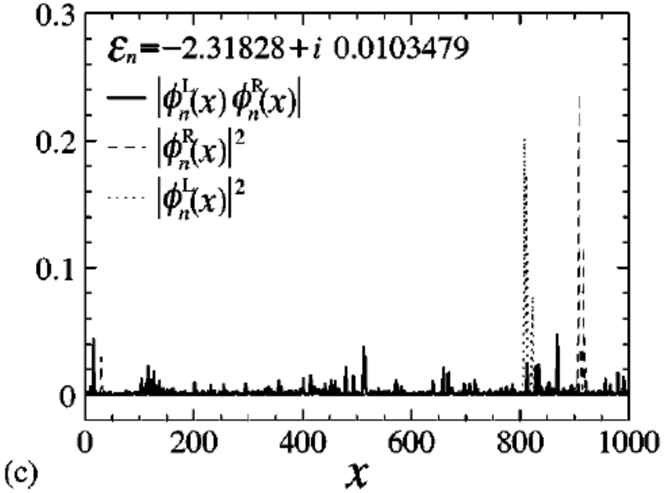
Spectrum on a complex plain



Real ε: localized states



Complex ε: extended states



In 1D non-Hermitian system the Anderson transition occurs,
while it never happens in Hermitian systems.

PT symmetric non-Hermitian quantum mechanics

C.M. Bender and S. Boettcher, PRL **80**, 5243 (1998)

Non-Hermitian systems with PT symmetry($H \neq H^\dagger$)

$\mathcal{P}$:Parity $\vec{x} \to -\vec{x}$

$\mathcal{T}$:Time-reversal symmetry $i \to -i$

$$\mathcal{PT}\, H\, (\mathcal{PT})^{-1} = H$$

$$\mathcal{PT}|\psi\rangle = e^{i\alpha}|\psi\rangle$$

$E = E^*$ ➡ $E \in \mathbb{R}$

➡ Real eigenenergy

Exceptional point :

the Hamiltonian is non-diagonalizable at a parameter space

Importance of symmetry in non-Hermitian systems.

Non-Hermitian topological phases

K. Esaki, M. Sato, K. Hasebe, M. Kohmoto, PRB **84**, 205128 (2011)
Z. Gong, Y. Ashida, K. Kawabata, K. Takasan, S. Higashikawa, M. Ueda, PRX **8**, 031079 (2018)
K. Kawabata, K. Shiozaki, M. Ueda, M. Sato, PRX **9**, 041015 (2019)

- **38 symmetry classes**

Ramification of symmetries

Time-reversal sym. (TRS):

$$TH^*(k)T^{-1} = H(-k)$$
$$TH^T(k)T^{-1} = H(-k)$$

Particle-hole sym. (PHS):

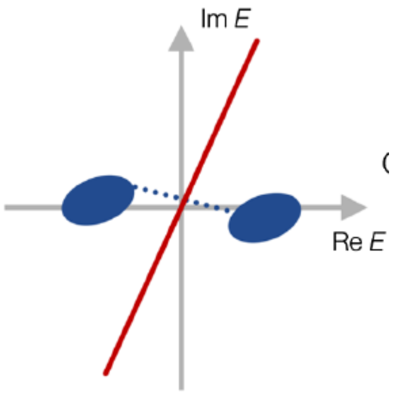
$$CH^*(k)C^{-1} = -H(-k)$$
$$CH^T(k)C^{-1} = -H(-k)$$

Chiral & sublattice symm.:

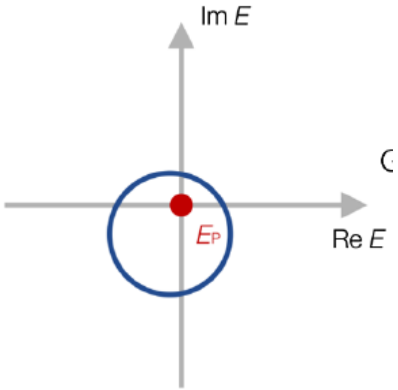
$$\Gamma H^\dagger(k)\Gamma^{-1} = -H(k)$$
$$SH(k)S^{-1} = -H(k)$$

- **Two kinds of energy gaps**

Line gap



Point gap



Hatano-Nelson model
Skin effect

Non-Hermitian systems possess richer topological phenomena.

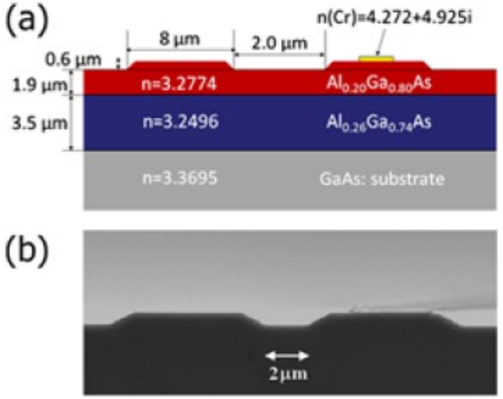
Experiment of non-Hermitian systems：classical optics

PRL 103, 093902 (2009) PHYSICAL REVIEW LETTERS week ending 28 AUGUST 2009

Observation of $\mathcal{PT}$-Symmetry Breaking in Complex Optical Potentials

Guo, Salamo, Duchesne, Morandotti, Ravat, Aimez, Siviloglou, Christodoulides, PRL **103**, 093902 (2009)

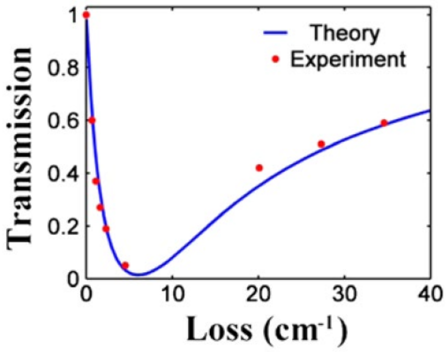
Deposition of Cr on one of a coupled wave guide ➡



Control of imaginary part of refractive index (loss)

$$i\frac{d}{dz}\begin{pmatrix}E_1\\E_2\end{pmatrix}=\begin{pmatrix}0&\kappa\\\kappa&-i\gamma\end{pmatrix}\begin{pmatrix}E_1\\E_2\end{pmatrix}$$

$$=\begin{pmatrix}+i\gamma/2&\kappa\\\kappa&-i\gamma/2\end{pmatrix}\begin{pmatrix}E_1\\E_2\end{pmatrix}-i\gamma/2\begin{pmatrix}E_1\\E_2\end{pmatrix}$$



Increase of transmission by increasing loss

➡ PT symmetry breaking

At the limit of $t^2 \ll \gamma^2$ $|\psi\rangle=\begin{pmatrix}1\\0\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$

light localizes at one of the coupled wave guide

No experiment of non-Hermitian quantum systems

No experimental verification of PT symmetric open quantum systems in 2014.

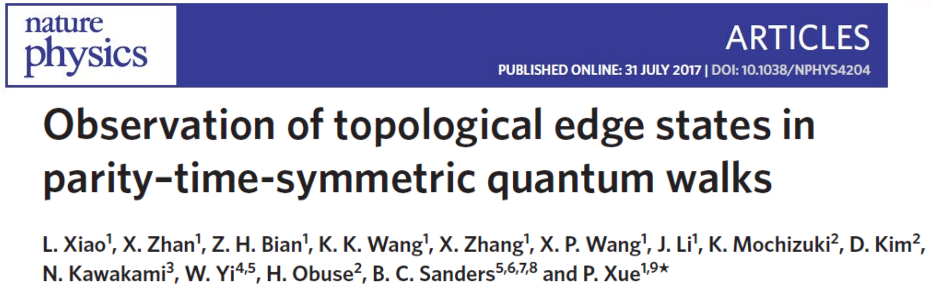
- Difficulty of controlling non-Hermitian effects (gain and/or loss, asymmetric hopping)
- How to guarantee quantumness
- Evidencing result

Importance of quantum walks for non-Hermitian physics

Quantum walk is an ideal platform for experiments of non-Hermitian systems.

Photonic quantum walks with entangled photons

- Quantitative control of loss of photons
- Quantumness is guaranteed by the entanglement
- Various physical phenomena can be reproduced due to high controllability of system parameters (quantum simulator).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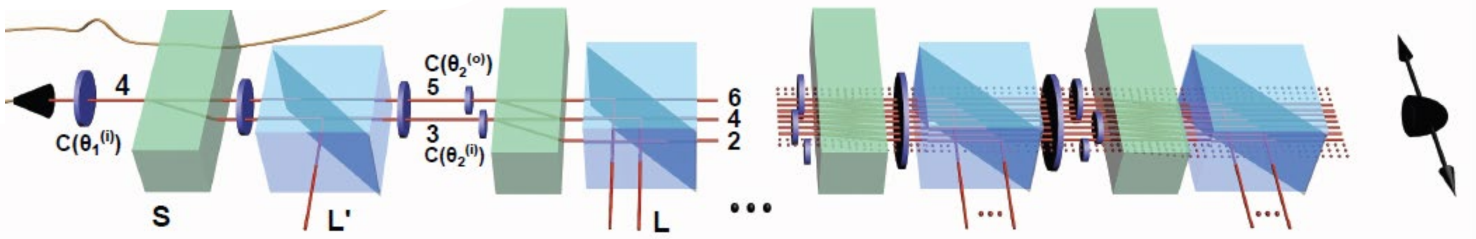
Nature Physics 13, 1117 (2017)



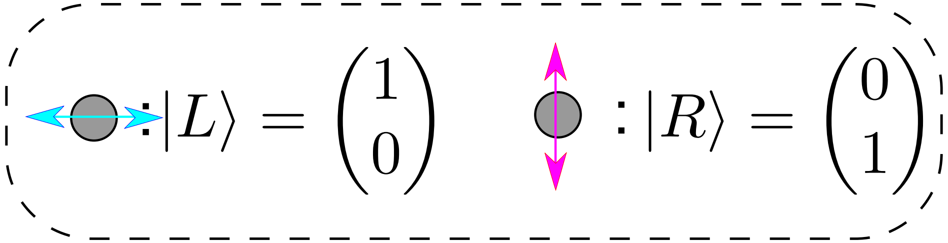
2.Quantum Walks

Quantum walk (QW) in 1D

1.Basis:

- position $\otimes$ internal states

$$|x\rangle \otimes |s\rangle \qquad x \in \mathbb{Z}, \quad s = L, R$$

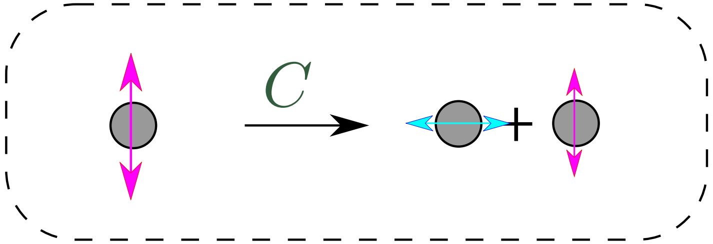


2. Basic operators:

- Coin operator: C

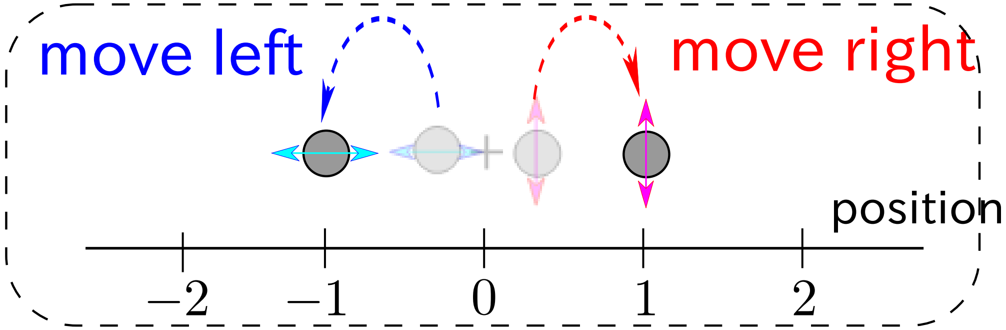
$$C[\theta(x)] = \sum_x |x\rangle\langle x| \otimes \mathcal{R}[\theta(x)]$$

$$\mathcal{R}(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$



- Shift operator : S

$$S = \sum_x \Big(|x+1\rangle\langle x| \otimes |R\rangle\langle R| + |x-1\rangle\langle x| \otimes |L\rangle\langle L| \Big)$$



Time-evolution operators

3. Time-evolution operator

A time evolution operator is defined by combining the basic operators.

E.g.)

Single step QW:

$$U = SC(\theta)$$

2 step QW:

$$U = SC(\theta_2)SC(\theta_1)$$

Continuum limit for position and time [Strauch PRA (2006)]

$$U = SC(\theta) \longrightarrow U = e^{-iHt}$$

Dirac equation

$$H = \hat{p}\sigma_z - \theta\sigma_y$$

Time-evolution

4. Time evolution

$$|\psi(t+1)\rangle = U^t|\psi_0\rangle \qquad t \in \mathbb{Z}$$

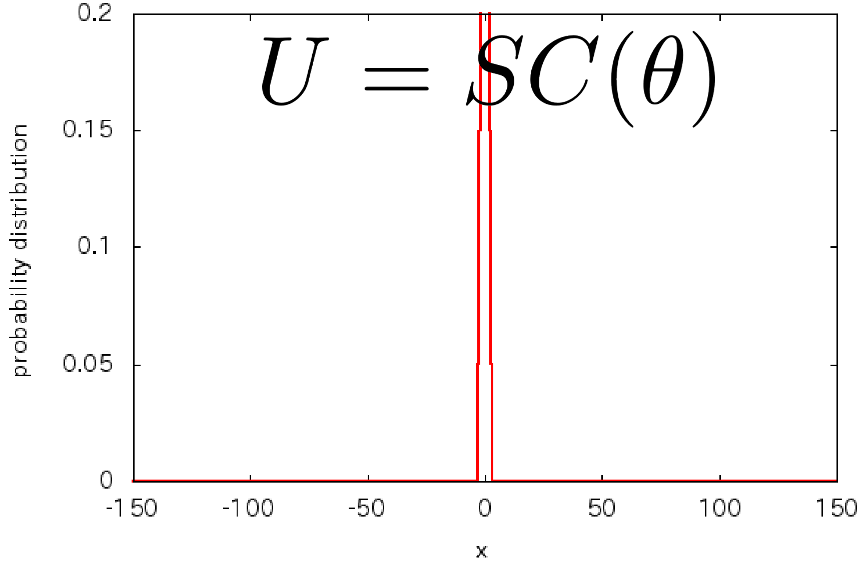
"Floquet system", "Floquet topological phase"

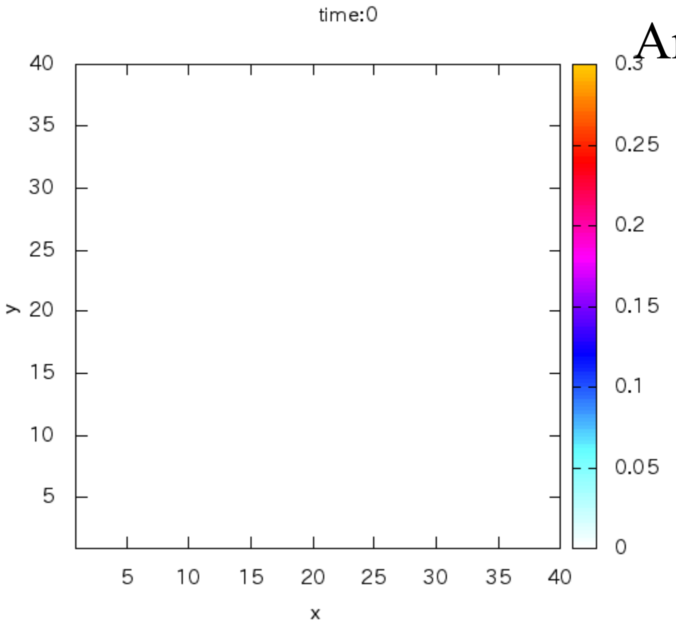
c.f. Bessho, Mochizuki, Obuse, Sato, PRB(accepted), arXiv:2112.03167

c.f. 2D Grover walk for quantum search

Ambainis, Kempe, Rivosh (2005)

$$U = SC(\theta)$$





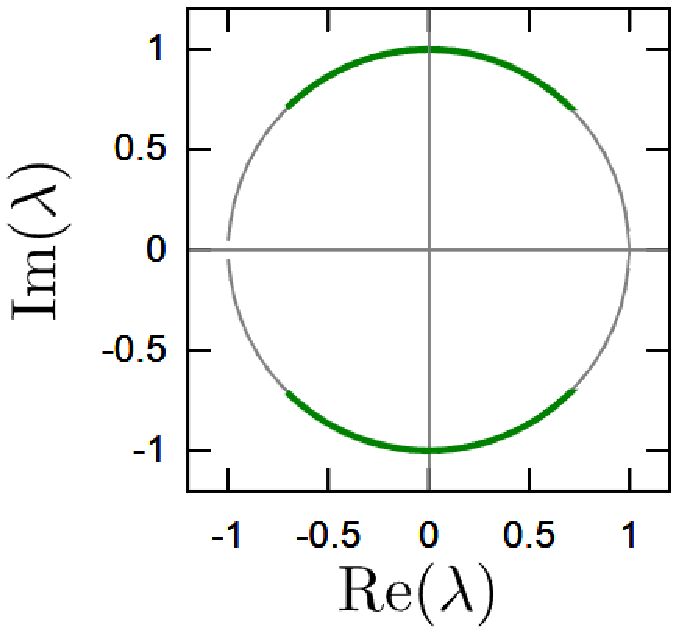
Effective Hamiltonian & Dispersion relation

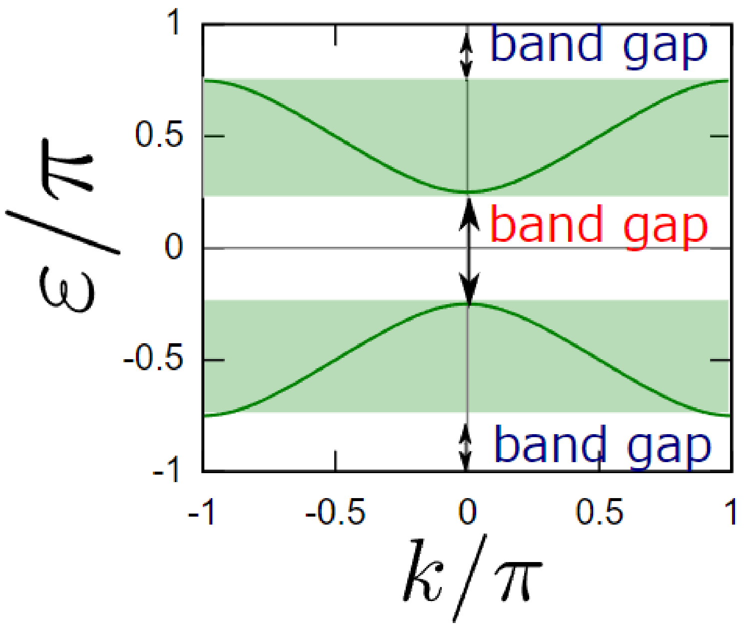
- Dispersion relation $U = e^{-iH_{\mathrm{eff}}t}$

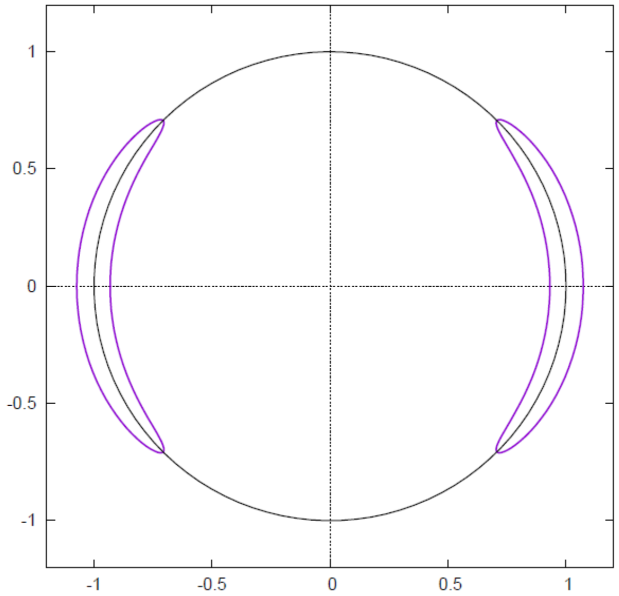
$U|\psi\rangle = \lambda|\psi\rangle$ $\lambda = e^{-i\varepsilon}$ ε : quasi-energy

Unitary operator: $\varepsilon \in \mathbb{R}$

Non-unitary operator: $\varepsilon \in \mathbb{C}$







Symmetry of quantum walks

Kitagawa, Rudner, Berg, Demler, PRA (2010)
Asboth and Obuse, PRB (2013)

TRS:

$$U = e^{-iHt}$$

$$TH^*(k)T^{-1} = H(-k) \quad \Leftrightarrow \quad TU^*(k)T^{-1} = U^\dagger(-k)$$

PHS:

$$CH^*(k)C^{-1} = -H(-k) \quad \Leftrightarrow \quad CU^*(k)C^{-1} = U(-k)$$

Chiral sym.:

$$\Gamma H(k)\Gamma^{-1} = -H(-k) \quad \Leftrightarrow \quad \Gamma U^*(k)\Gamma^{-1} = U^\dagger(k)$$

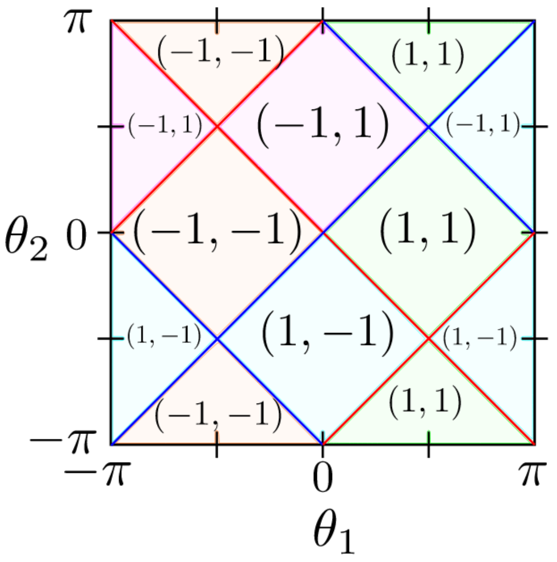
Topological phases of 1D quantum walks with chiral

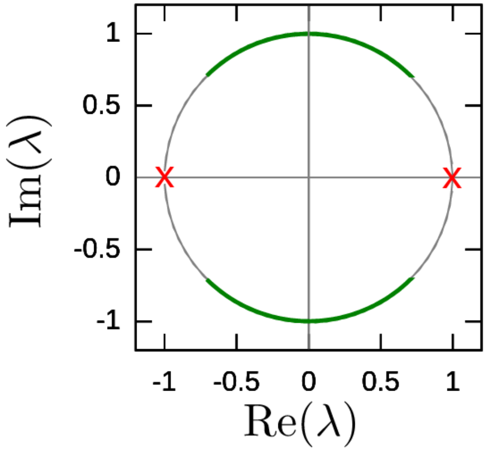
2 step QWs: $U = SC(\theta_2)SC(\theta_1)$

- Topological numbers (ν_0, ν_π) : two edge states at $\varepsilon = 0$ & π .

winding number $\nu' = \frac{1}{\pi i}\oint dk \langle\psi_-|\nabla_k|\psi_-\rangle$ Asbóth & HO, PRB ('13).

$$\nu_0 = \frac{\nu' + \nu''}{2}, \qquad \nu_\pi = \frac{\nu' - \nu''}{2},$$





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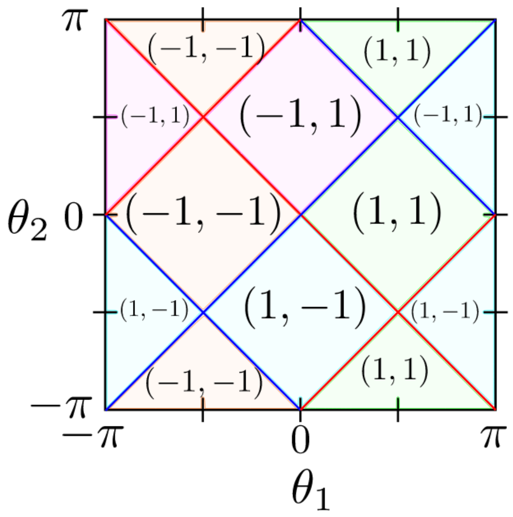
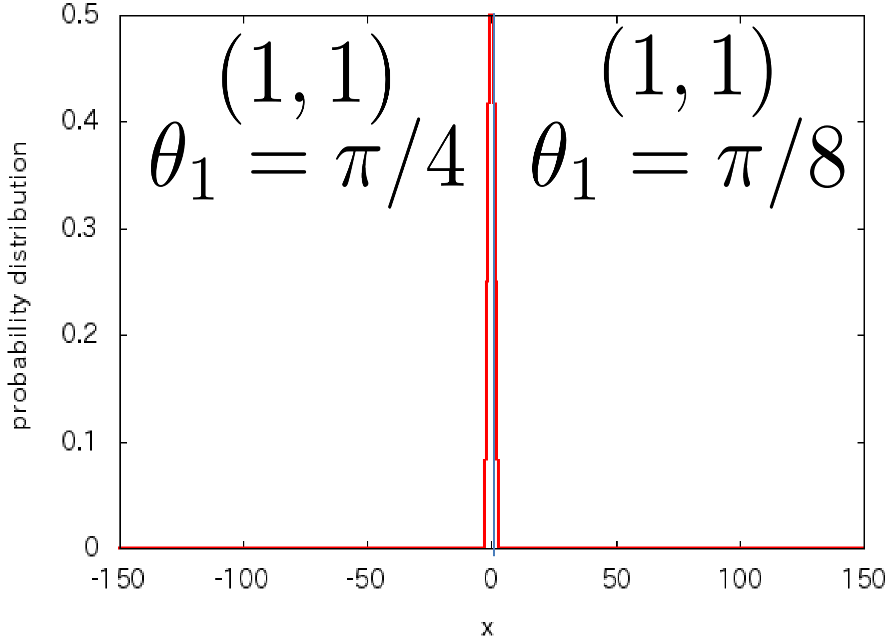
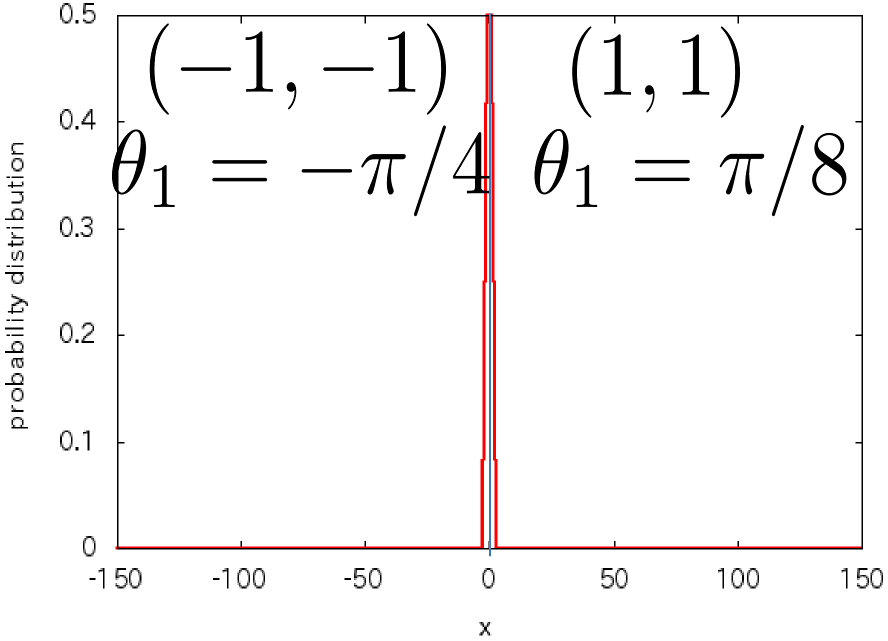


$$\theta_2 = \pi/5$$





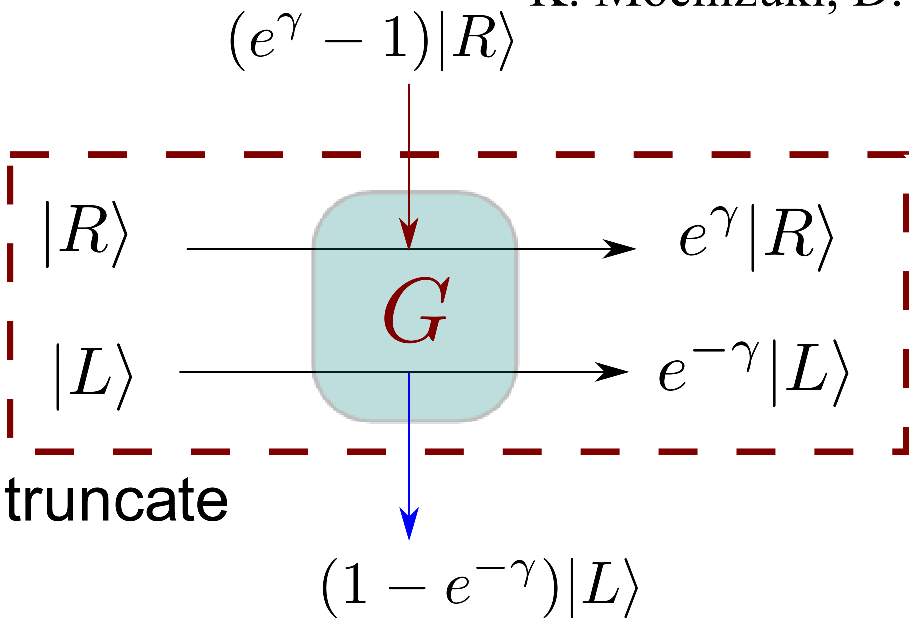
3.Non-unitary Quantum walks for Non-Hermitian Physic

Introduction of non-unitary operators

- Gain & loss operator:

K. Mochizuki, D. Kim, H. Obuse, PRA (2016)

$$G = \sum_x |x\rangle\langle x| \otimes \begin{pmatrix} e^{-\gamma} & 0 \\ 0 & e^{\gamma} \end{pmatrix}$$

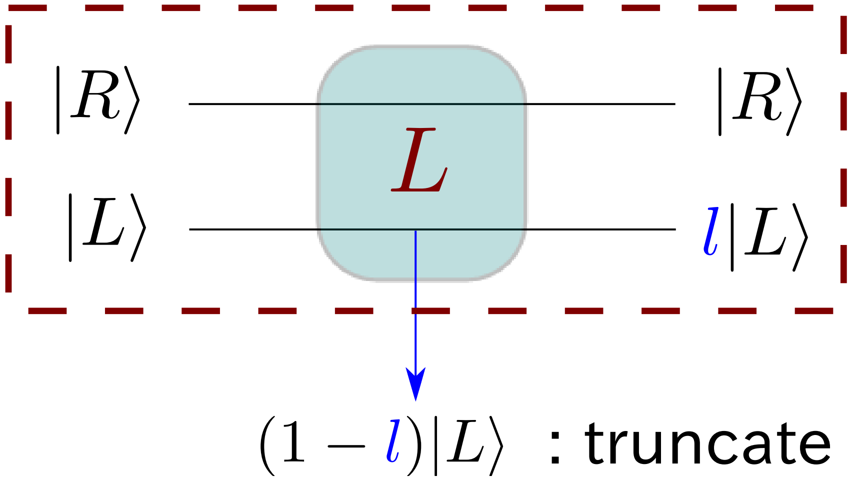


- Loss operators:

$$L = \sum_x |x\rangle\langle x| \otimes \begin{pmatrix} l & 0 \\ 0 & 1 \end{pmatrix}$$

$$L' = \sum_x |x\rangle\langle x| \otimes \begin{pmatrix} 1 & 0 \\ 0 & l \end{pmatrix}$$

$(l \leq 1)$



Non-unitary quantum walks

Non-unitary time evolution operator:

A time evolution operator is defined by combining the basic operators.

E.g.)

Single step QW:

$$U = GSC$$

2 step QW:

$$U = G^{-1}SC(\theta_2)GSC(\theta_1)$$

3 step QW:

$$U = G^{-1}SC(\theta_2)SC(\theta_2 + \delta)GSC(\theta_1)$$

Time evolution:

$$|\psi(t+1)\rangle = U^t|\psi_0\rangle$$

Non-Hermitian topological phases

K. Esaki, M. Sato, K. Hasebe, M. Kohmoto, PRB **84**, 205128 (2011)
Z. Gong, Y. Ashida, K. Kawabata, K. Takasan, S. Higashikawa, M. Ueda, PRX **8**, 031079 (2018)
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- **38 symmetry classes**

Ramification of symmetries

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$$TH^*(k)T^{-1} = H(-k)$$
$$TH^T(k)T^{-1} = H(-k)$$

Particle-hole sym. (PHS):

$$CH^*(k)C^{-1} = -H(-k)$$
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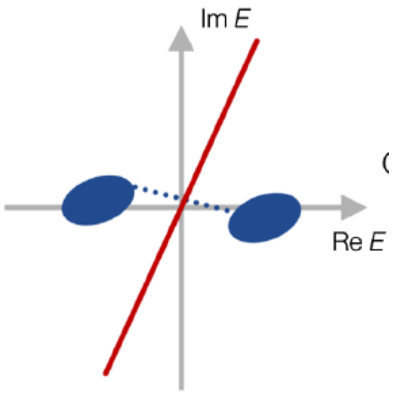
Chiral & sublattice sym.:

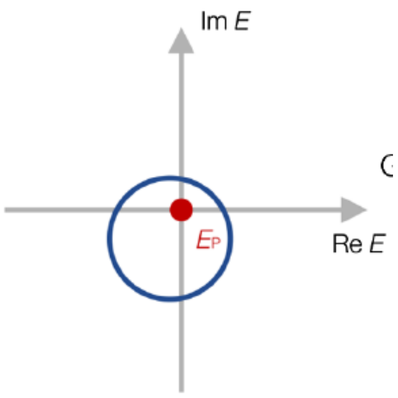
$$\Gamma H^\dagger(k)\Gamma^{-1} = -H(k)$$
$$SH(k)S^{-1} = -H(k)$$

- **Two kinds of energy gaps**

Line gap



Point gap



Hatano-Nelson model
Skin effect

Non-Hermitian systems possess richer topological phenomena.

Topological phases for real line gaps

2step non-unitary quantum walk

K. Mochizuki, D. Kim, and H. Obuse, Phys. Rev. A **93**, 062116 (2016).
K. Mochizuki, D. Kim, N. Kawakami, and H. Obuse, Phys. Rev. A, **102**, 062202 (2020).

$$U_{gl} = G^{-1} S C(\theta_2) G S C(\theta_1)$$

$$U'_{gl} = AB, \quad A = C(\theta_1/2) S G^{-1} C(\theta_2/2), \quad B = C(\theta_2/2) S G C(\theta_1/2)$$

$$U|\psi\rangle = \lambda|\psi\rangle$$

$$\lambda = e^{-i\varepsilon}$$

Chiral symmetry

$$\Gamma A \Gamma^{-1} = B^\dagger \qquad \Gamma = \sum |x\rangle\langle x| \otimes \sigma_1$$

Particle-hole symmetry†

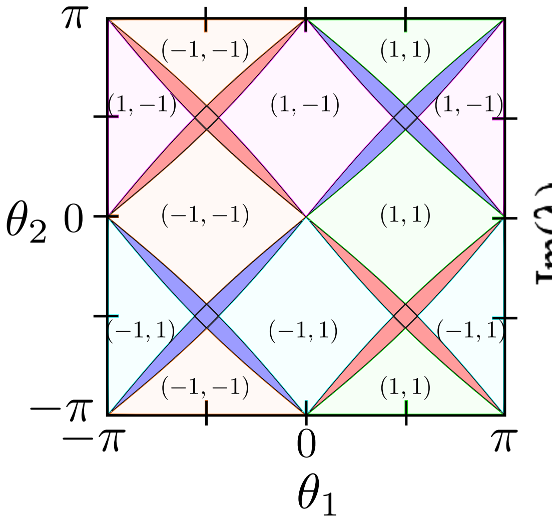
$$\Xi U'_{gl} \Xi^{-1} = U'^{*}_{gl} \qquad \Xi = \sum |x\rangle\langle x| \otimes \sigma_0$$

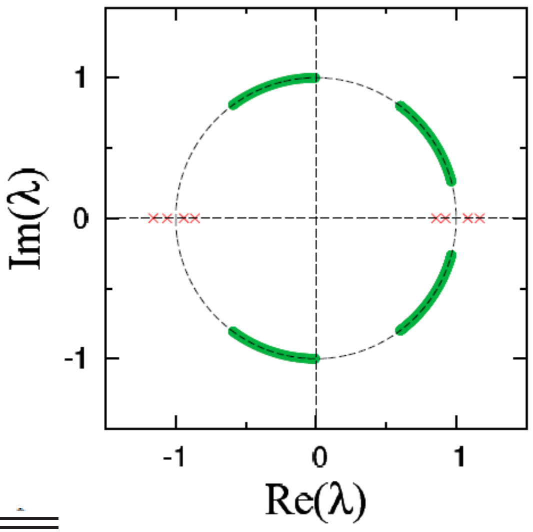
Time-reversal symmetry†

$$T A T^{-1} = B^T \qquad T = \Gamma\, \Xi$$

→ class BDI†

topological numbers (ν_0, ν_π)



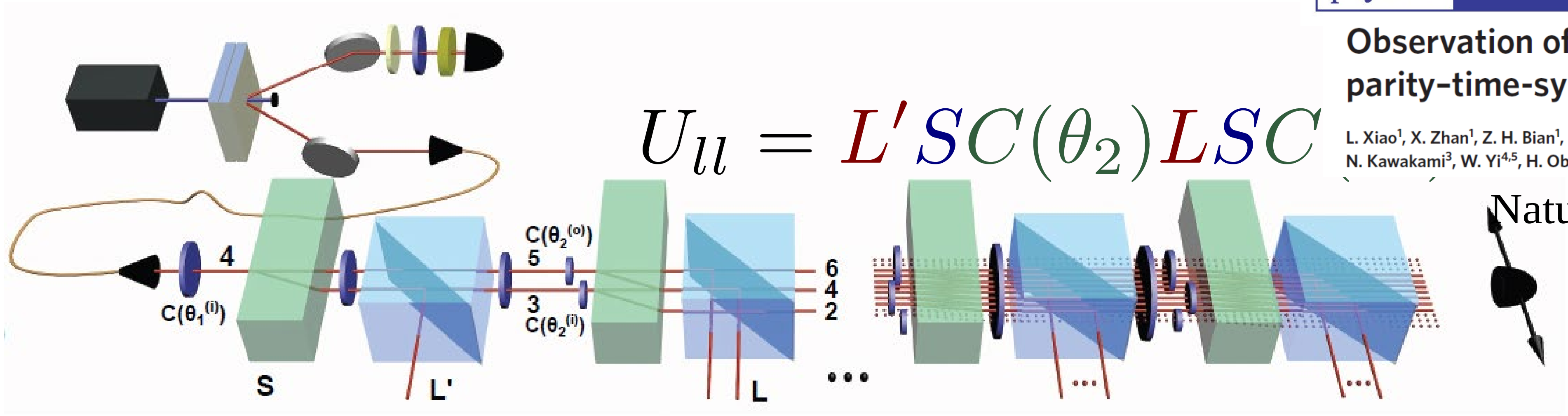


Z topological phases for real line gaps in 1D

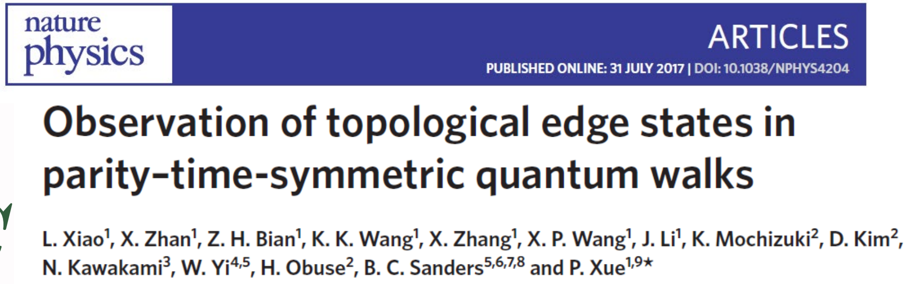
AZ† class	Gap	Classifying space	$d=0$	$d=1$	$d=2$	$d=3$
BDI†	P	$\mathcal{R}_0$	$\mathbb{Z}$	0	0	0
	L_r	$\mathcal{R}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	L_i	$\mathcal{R}_0 \times \mathcal{R}_0$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0

Topological phases for real line gaps : experiment

a pair of entangled photons



$$U_{ll} = L'SC(\theta_2)LSC$$



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ARTICLES
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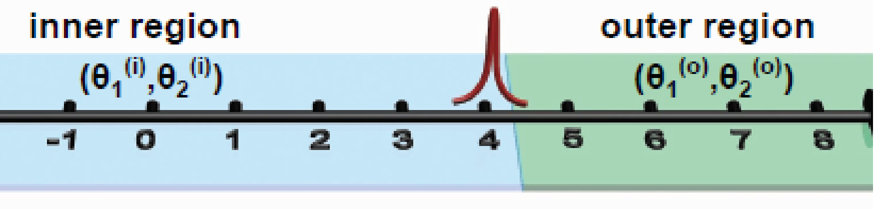
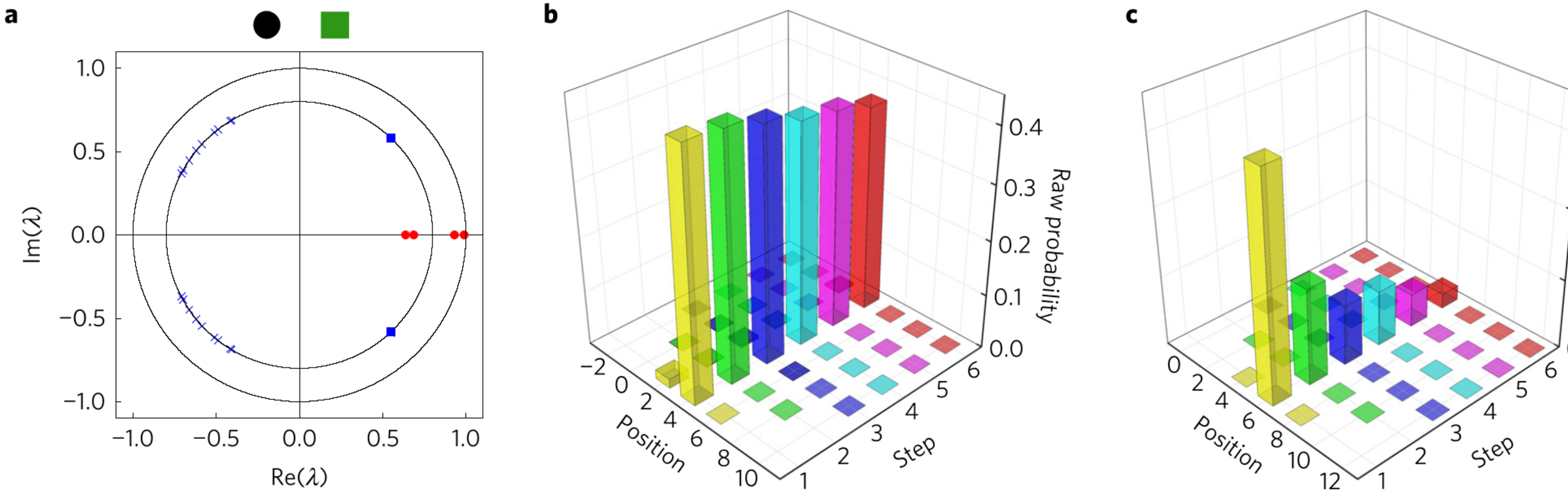
Observation of topological edge states in parity-time-symmetric quantum walks

L. Xiao[1], X. Zhan[1], Z. H. Bian[1], K. K. Wang[1], X. Zhang[1], X. P. Wang[1], J. Li[1], K. Mochizuki[2], D. Kim[2], N. Kawakami[3], W. Yi[4,5], H. Obuse[2], B. C. Sanders[5,6,7,8] and P. Xue[1,9]*

Nature Physics 13, 1117 (2017)





Observation of edge states for a real line gaps

Breakdown of bulk-edge correspondence

M. Kawasaki, K. Mochizuki, N. Kawakami, and H. Obuse,
Prog. Theor. Exp. Phys. 2020, 12A105 (2020).

3step non-unitary quantum walk
with chiral symmetry breaking term

$$U_\delta = G^{-1} SC(\theta_2) SC(\theta_2 + \delta) GSC(\theta_1)$$

AZ† class	Gap	Classifying space	$d=0$	$d=1$
D†	P	$\mathcal{R}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$
	$\mathrm{L_r}$	$\mathcal{R}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
	$\mathrm{L_i}$	$\mathcal{R}_0$	$\mathbb{Z}$	0

When $\delta \neq 0$

Chiral symmetry

$$\Gamma U'_\delta \Gamma^{-1} \neq U'^{\dagger}_\delta$$

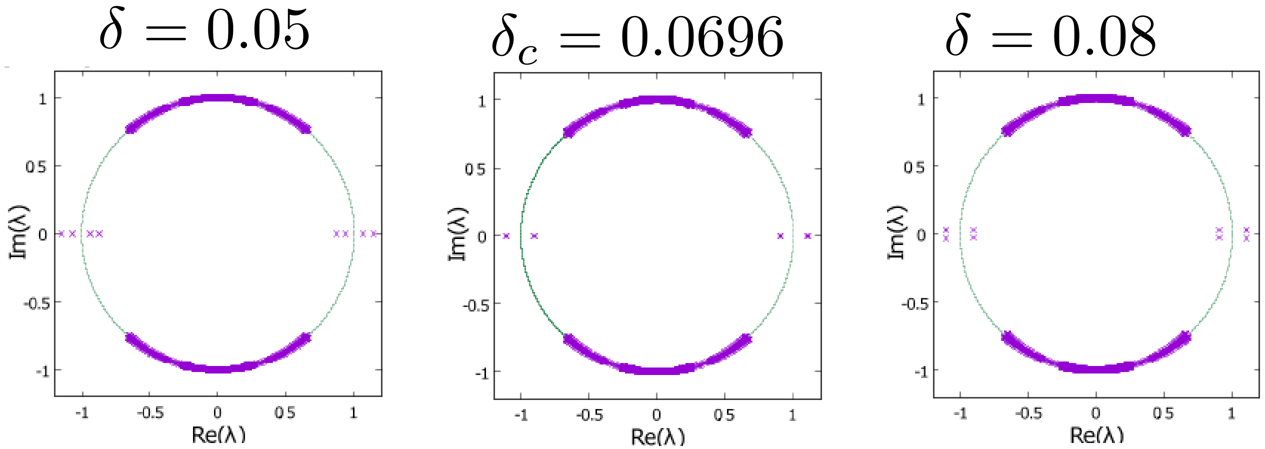
Particle-hole symmetry†

$$\Xi U'_\delta \Xi^{-1} = U'_\delta$$

$$\Xi = \sum |x\rangle\langle x| \otimes \sigma_0 K$$

$\mathrm{BDI}^\dagger(\mathbb{Z})$ ➡ $\mathrm{D}^\dagger(\mathbb{Z}_2)$

- Eigenvalue: difference of topo# is 2 at $\delta = 0$



Edge states with $\mathrm{Re}(\varepsilon) = 0$ survives up to δ_c even in class $\mathrm{D}^\dagger(\mathbb{Z}_2)$

Breakdown of bulk-edge correspondence

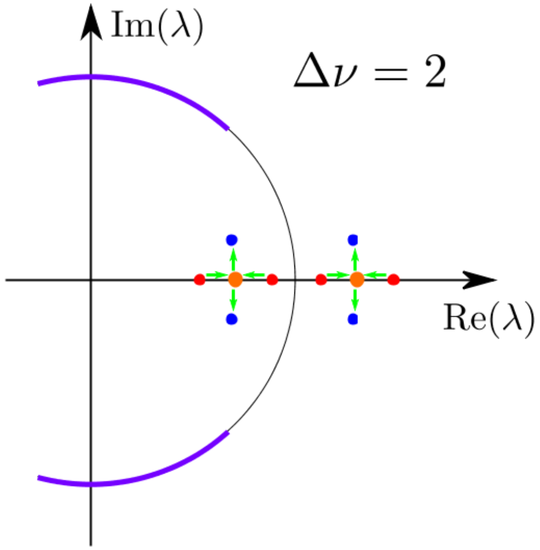
M. Kawasaki, K. Mochizuki, N. Kawakami, and H. Obuse, Prog. Theor. Exp. Phys. 2020, 12A105 (2020).

Particle-hole symmetry† $\Xi U^* \Xi^{-1} = U$

$U = H$

Time-reversal symmetry $\Xi H^* \Xi = H$

Eigenvalues remain real unless exceptional points appear!



A new kind of the breakdown of bulk-edge correspondence!

Cf. K. Sone, Y. Ashida, T. Sagawa, Nat. Commun. 11, 5745 (2020).

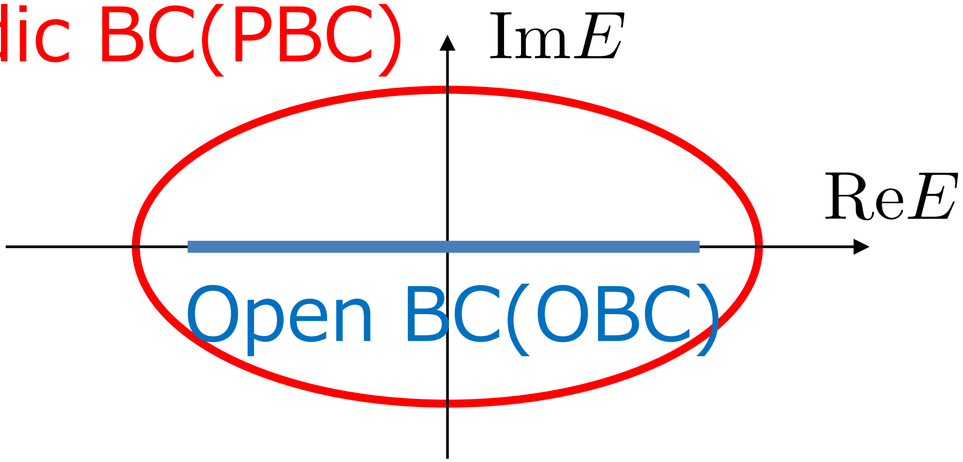
Skin effects in non-Hermitian systems

Non-Hermitian system with the asymmetric hopping

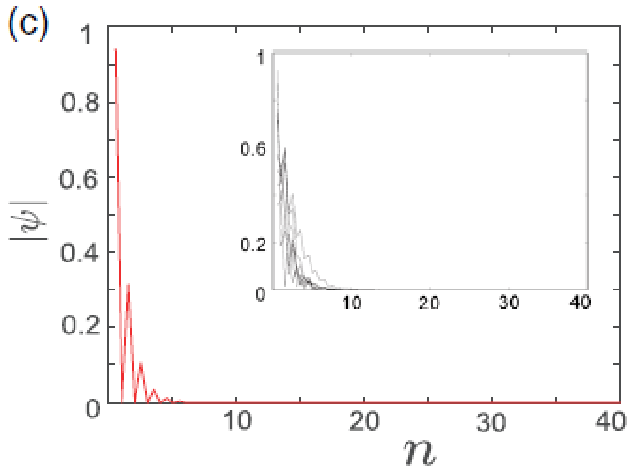
Hatano-Nelson model (w/o disorder) N. Hatano and D. R. Nelson, PRL **77**, 570 (1996)

$$H = \sum_x t(e^{\gamma} c^{\dagger}_{x+1} c_x + e^{-\gamma} c^{\dagger}_{x-1} c_x) \qquad \gamma \in \mathbb{R}$$

Periodic BC(PBC)



Point gap → #topo 1



OBC

S. Yao et al., PRL (2018)
Z. Gong *et al.*, PRX (2018)
K. Yokomizo *et al.*, PRL (2019)
N. Okuma *et al.*, PRL (2020)

Skin effect :
All eigenstates localize at the edge.

Unique topological phenomena in the non-Hermitian systems

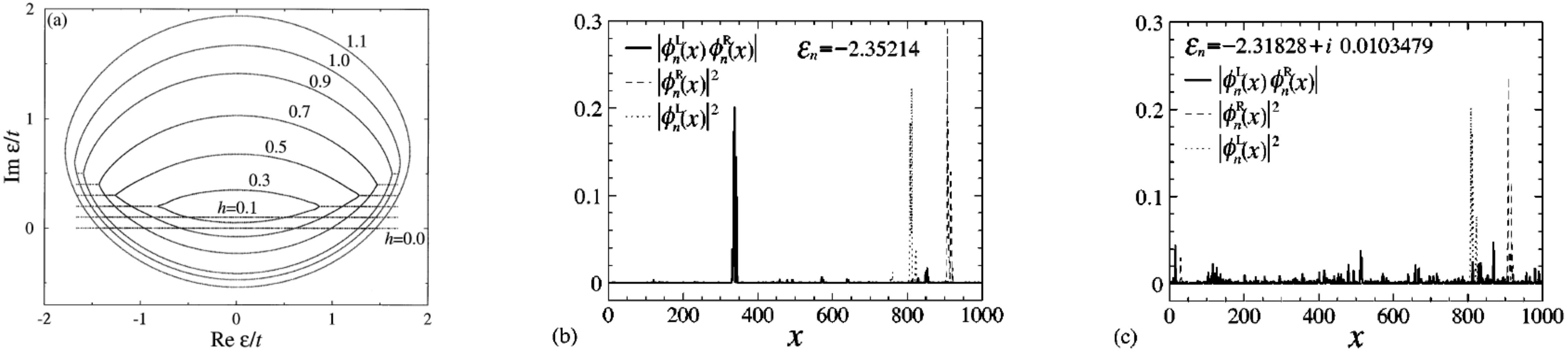
Anderson transition 1D by non-unitary QW

Hatano-Nelson model : asymmetric hopping with onsite disorder in 1D

N. Hatano and D. R. Nelson, PRL **77**, 570 (1996)

$$H = \sum_x t(e^{\gamma} c^\dagger_{x+1} c_x + e^{-\gamma} c^\dagger_{x-1} c_x) + \varepsilon_x c^\dagger_x c_x$$

Spectrum on a complex plain | **Real ε: localized states** | **Complex ε: extended states**



The Anderson transition in 1D non-Hermitian system occurs.

c.f. Kawabata and Ryu, PRL **126**, 166801 (2021)
Luo, Xiao, Kawabata, Ohtsuki, Shindou, arXiv:2105.02514

Model & Results

N. Hatano and H. Obuse, Annals of Physics (2021).

$$U = GSC^{\mathrm{rnd}}$$

$$C^{\mathrm{rnd}} = \sum_x |x\rangle\langle x| \otimes c_x^{\mathrm{rnd}},$$

$$c_x^{\mathrm{rnd}} := e^{i\phi} \begin{pmatrix} e^{i\alpha}\cos\vartheta & -e^{i\beta}\sin\vartheta \\ e^{-i\beta}\sin\vartheta & e^{-i\alpha}\cos\vartheta \end{pmatrix}.$$

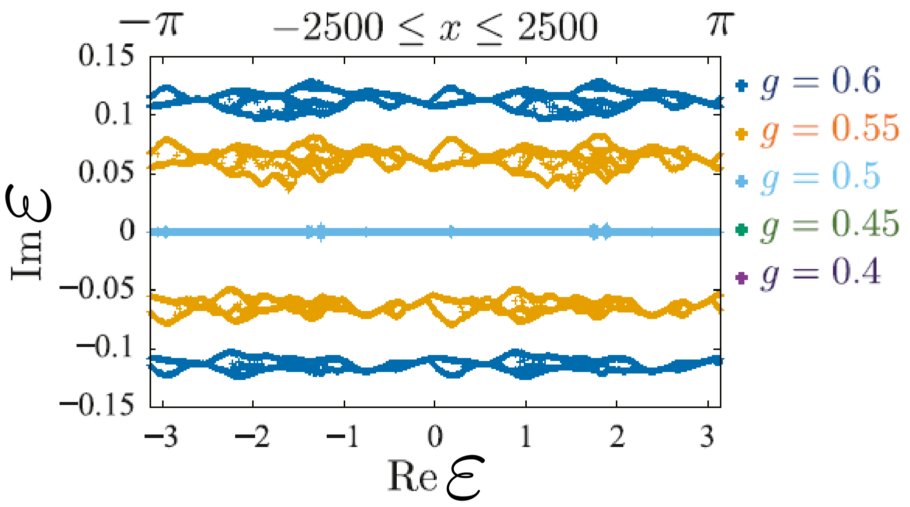
:

$\alpha, \beta, \phi \quad [0, 2\pi]$

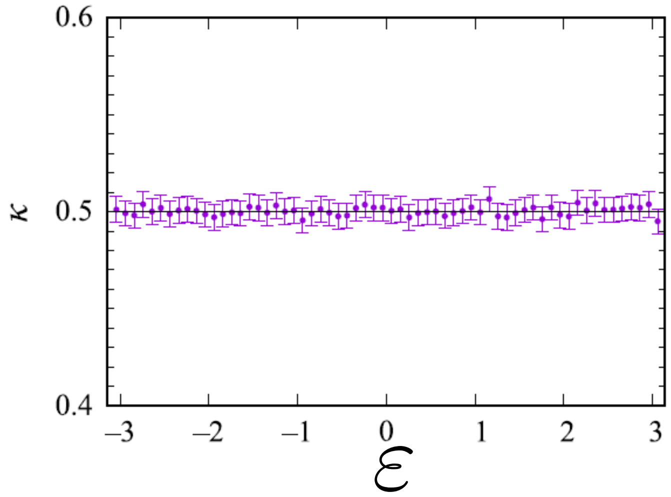
$P(\vartheta)d\vartheta = \sin(2\vartheta)d\vartheta$

No symmetry → class A (the same with the original Hatano-Nelson model)

Spectrum:



Localization length κ:



No symmetry & No band gap due to 2π periodicity → All states simultaneously delocalize at $g = 0.5$.

Summary

Topological edge states:

- **We have demonstrated the non-Hermitian quantum systems by using the photonic quantum walk first time.**

Skin effects:

- **We have shown the ballistic behaviors in the system exhibiting the skin effects by using the quantum walk, which can be realized by using the photonic quantum walk with fine control of loss.**

Anderson transition in 1D non-unitary QW:

- **All states simultaneously delocalize due to no symmetry and band gap originating from 2π periodicity of quasi-energy.**

The quantum walk is an ideal platform to study non-Hermitian quantum physics.

国際会議 Localisation 2022
https://2022.localisation.cloud

不規則性に伴う局在現象に関する国際会議(ハイブリッド形式)
LT29(8/18～24, 札幌)のサテライト

期間：2022年8月25～30日

場所：北海道大学 鈴木章ホール, Zoom

4月から口頭発表・ポスター発表の募集を開始予定

Co-chairs: Stefan Kettemann, Hideaki Obuse, Keith Slevin
Organizers: Tomi Ohtsuki, Dragana Popovic, Satoshi Tanda, Kousuke Yakubo