(ガラスの)破壊現象が示す 非平衡臨界性

ゆらぎと応答のスケール分離？

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Mar. 27 2023, 第10回 統計物理懇談会

Agenda

- Introduction (general topics)

  What are glasses? Marginal stability?

- Glasses under External Field

  Non-eq. criticality governing structural failure

- Effect of Structural Failure: Mechanical Aspect

  Structural origin of a universal rheological law

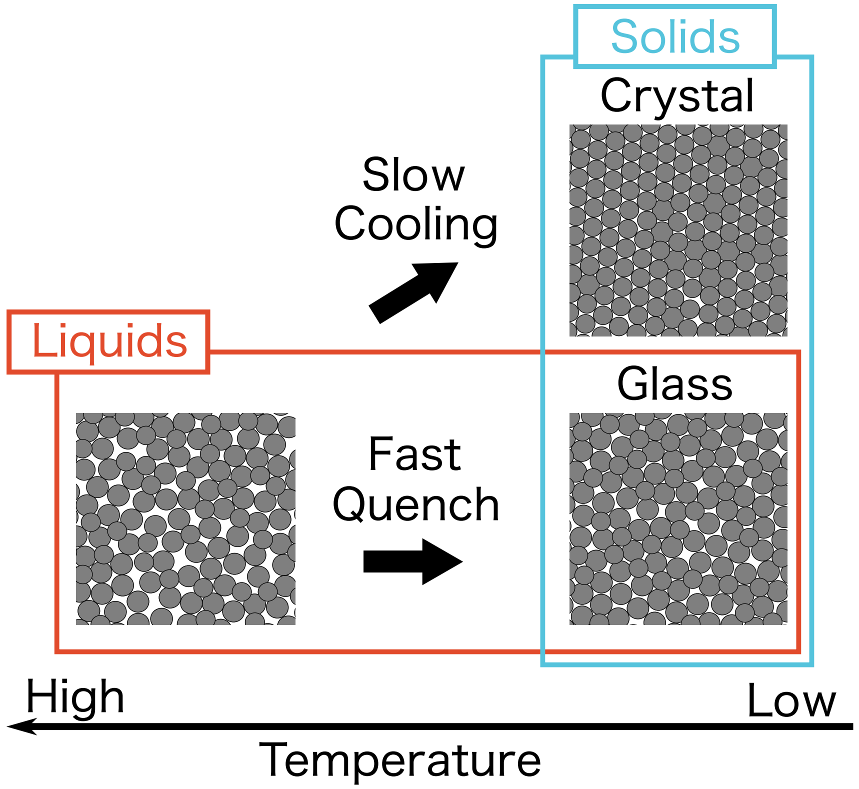
- Effect of Structural Failure: Dynamical Aspect

  Governed by a distinct correlation length?

Introduction

Glass Transition

- Emergence of universal "phase" of matters -



- Found in various soft matter systems universally
- Self-generated randomness: no randomness in Hamiltonian

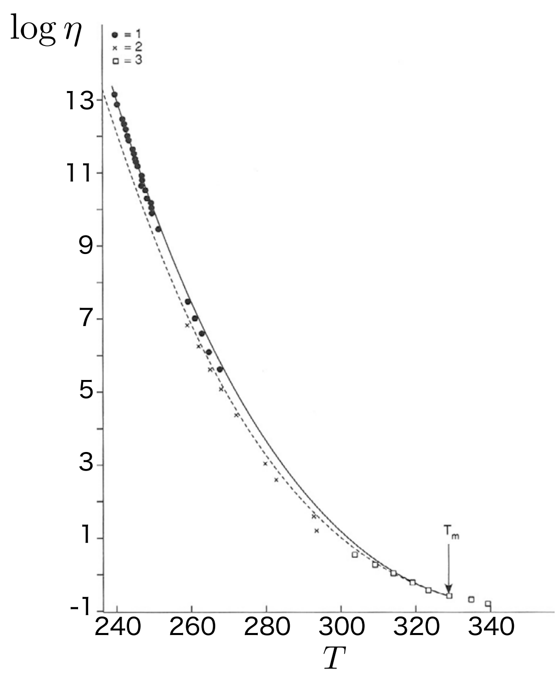
(simple liquids, alloys, colloids, polymer, emulsion, suspension, etc.)

- Glass "transition" = phase transition?
- Extremely viscous liquid? Solid with random structure?

Stuctures vs. Dynamics

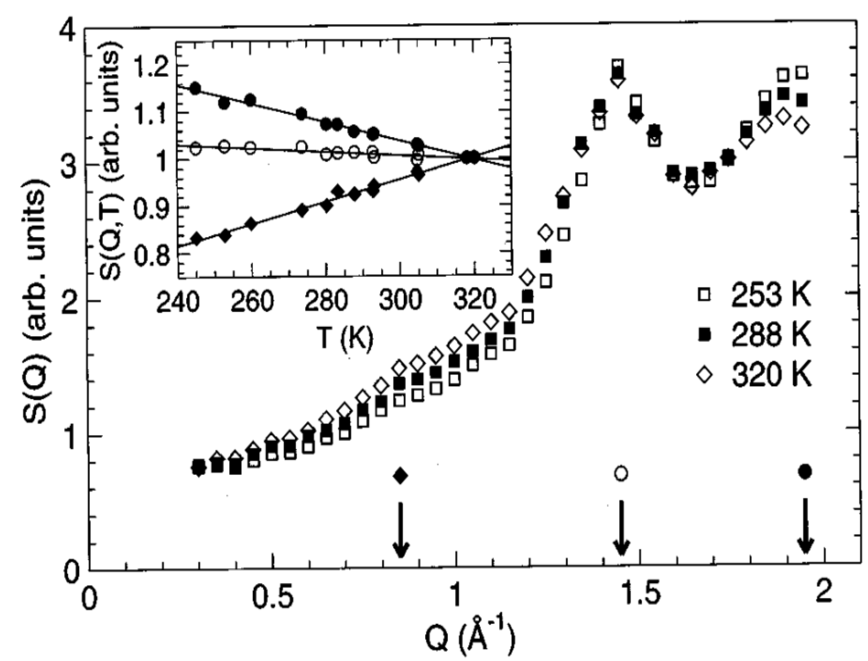
- Which is "the" essential factors of glass physics? -

Viscosity vs. temperature



[1] Alcoutlabi, J. Phys.: Condens. Matter (2005)

Static structure factor



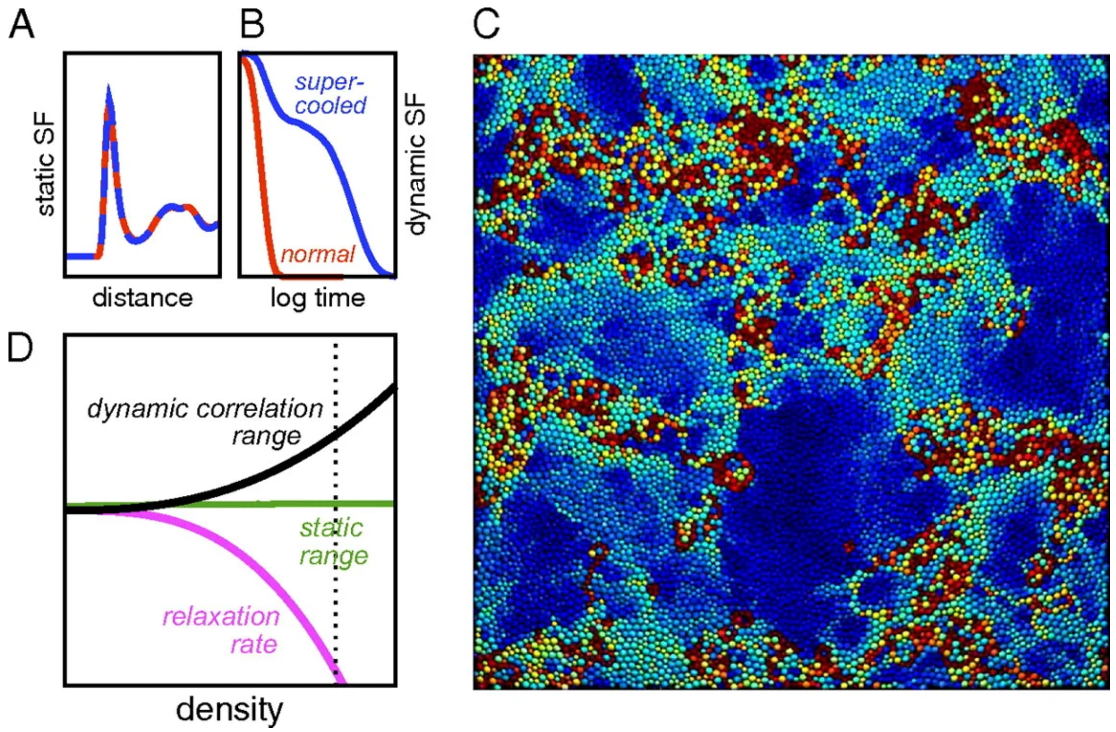
[2] Tölle, Phys. Rev. E (1997)

- Viscosity changes by more thatn 10 orders between $250 - 320K$
- Static structure barely changes between these temperatures

Stuctures vs. Dynamics

- Which is "the" essential factors of glass physics? -

Dynamic heterogeneity



Garrahan, PNAS (2011)

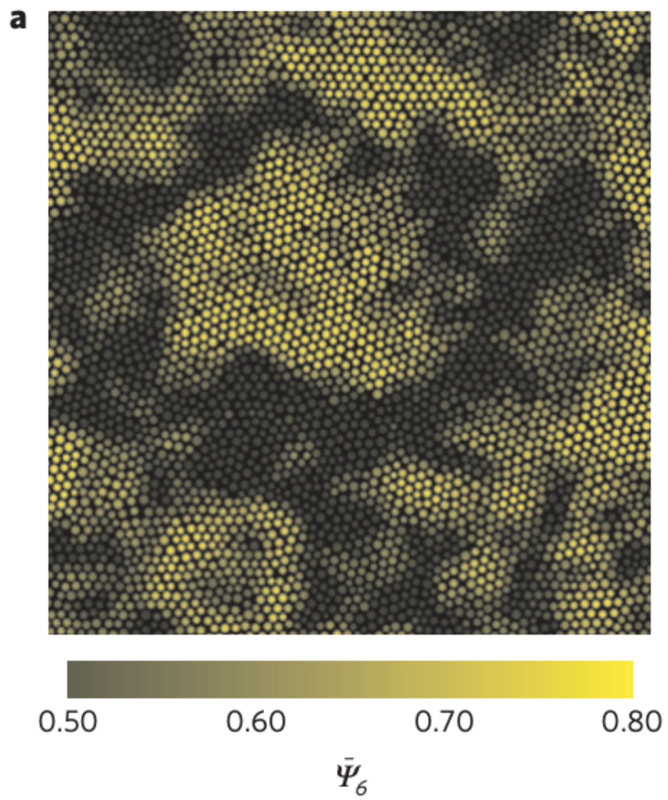
- Heterogeneous dynamics exhibit "correlated structures"
- Dynamic correlation length grows as temperature decreases
- Relaxation time: critical phenomenon-like scaling

Karmakar et al, PNAS **106**(10), 3675 (2009)

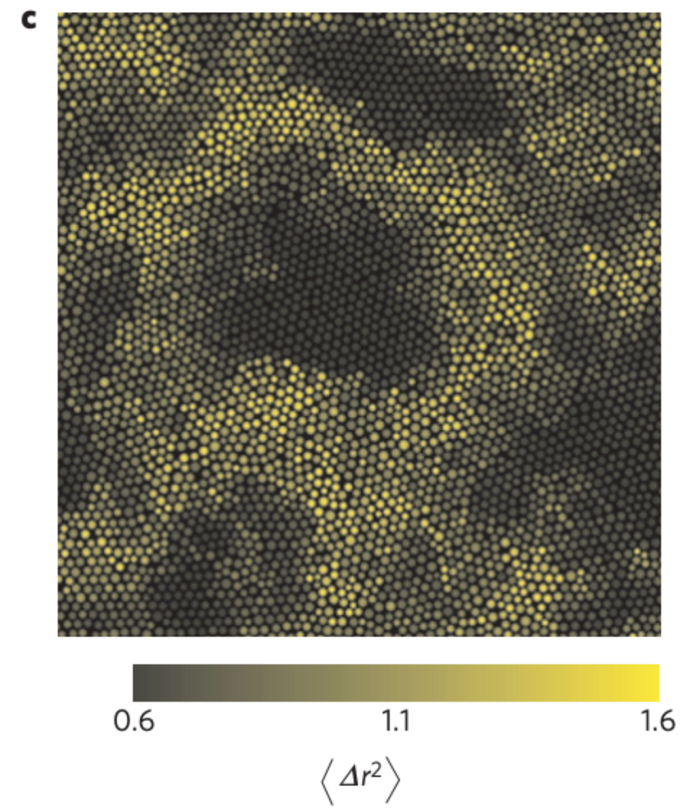
Structural Order Parameters

- System dependent, tailor-made indicators -

Structural indicator



Dynamics



Tanaka, Nature Materials (2010)

- Structural "order" strongly corrrelated with dynamics
- Initially,heuristic finding in a tailor-made manner
- Machine learning approches to find universal one

Bapst, Nat. Phys. (2020), Boattini, Nat. Commun. (2020), Shiba, J. Chem. Phys. (2023), Oyama, Front. Phys. (2023) etc.

Glasses As...

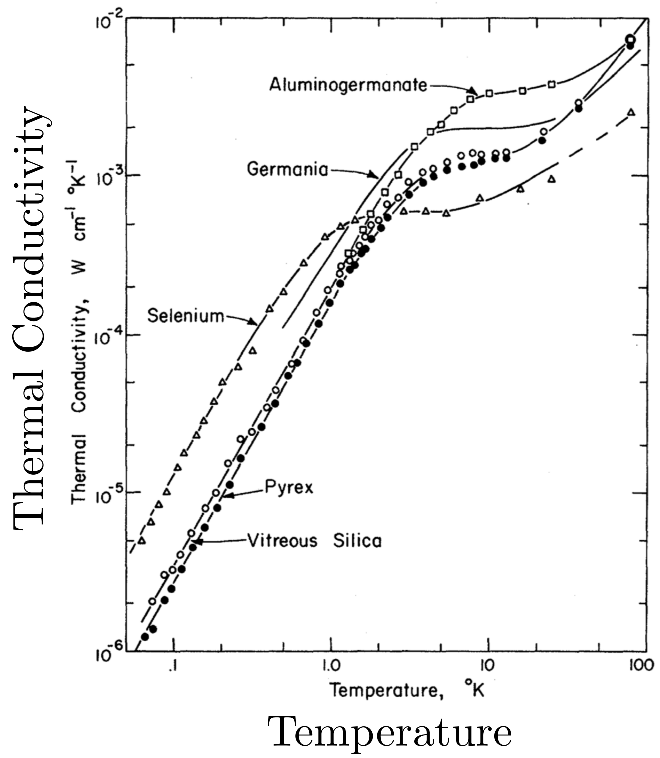
- "Liquids" with extremely high viscosity?

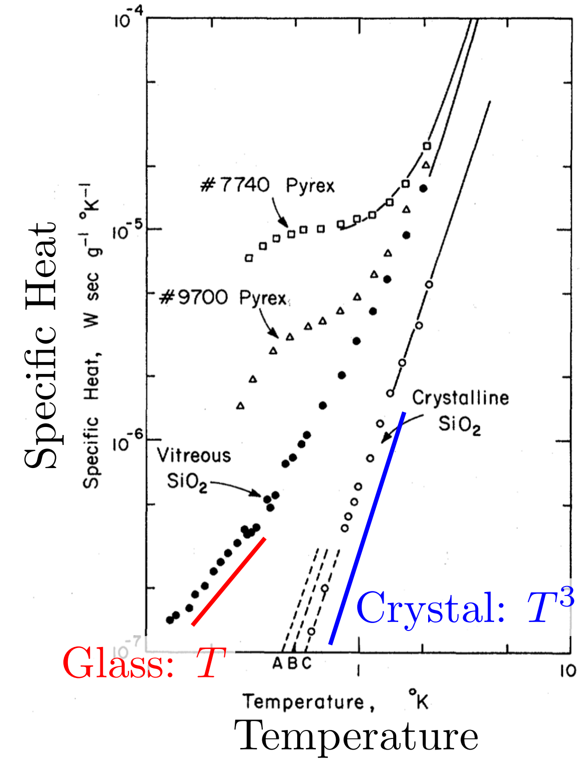
(divergence of viscosity, dynamic heterogeneity etc.)

- "Solids" with random structures?

Low-Temperature Behavior

- Crystal vs. glasses, universal scaling for glasses -





Zeller and Pohl, Phys. Rev. E **4**, 2029 (1971)

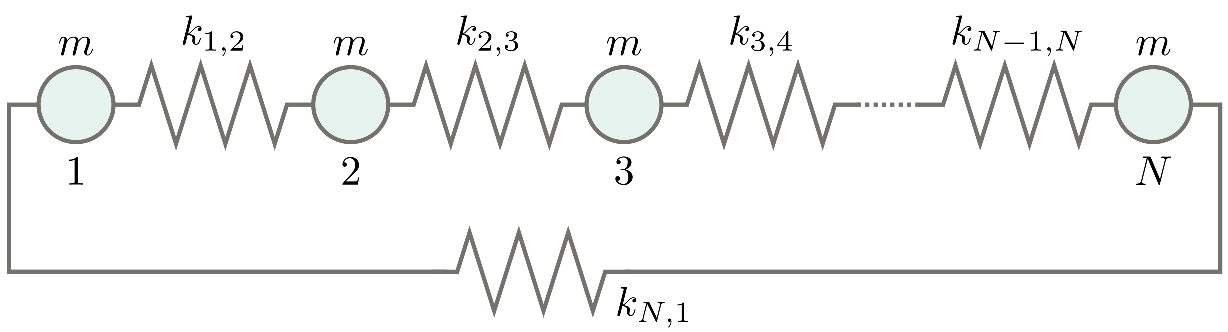
- Different glasses obey the same scaling law
- Amorphous and crystal obey different scaling laws
- Glasses: not dominated by phonon?: Not "solids"?

(Low-temperature: dominated by low-frequency "modes")

統計力学I, II, 田崎 晴明, 培風館

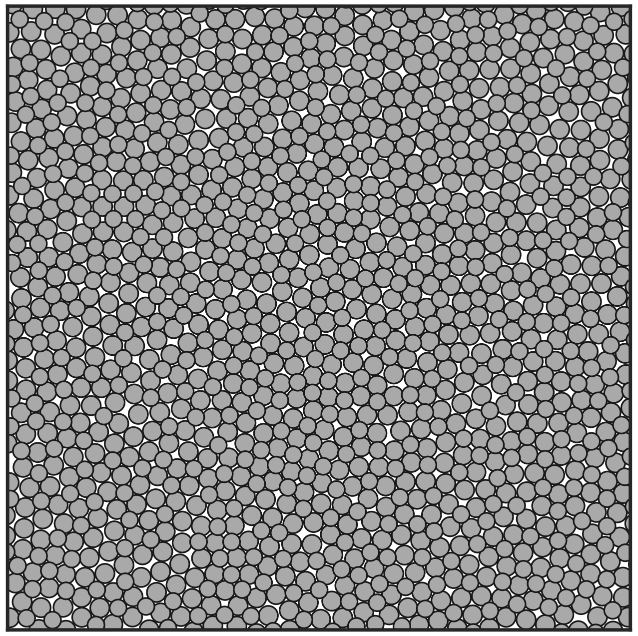
Glasses As Coupled Oscillators

N-body coupled harmonic oscillators



$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_i \\ \vdots \\ \ddot{x}_N \end{bmatrix} = - \begin{bmatrix} (k_{N,1}+k_{1,2}) & -k_{1,2} & 0 & \cdots & & 0 & -k_{N,1} \\ -k_{1,2} & (k_{1,2}+k_{2,3}) & -k_{2,3} & 0 & \cdots & & 0 \\ \vdots & & & & & & \\ \cdots & 0 & -k_{i-1,i} & (k_{i-1,i}+k_{i,i+1}) & k_{i,i+1} & 0 & \cdots \\ \vdots & & & & & & \\ -k_{N,1} & 0 & \cdots & & 0 & -k_{N-1,N} & (k_{N-1,N}, k_{N,1}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ \vdots \\ x_N \end{bmatrix}$$

A glass sample



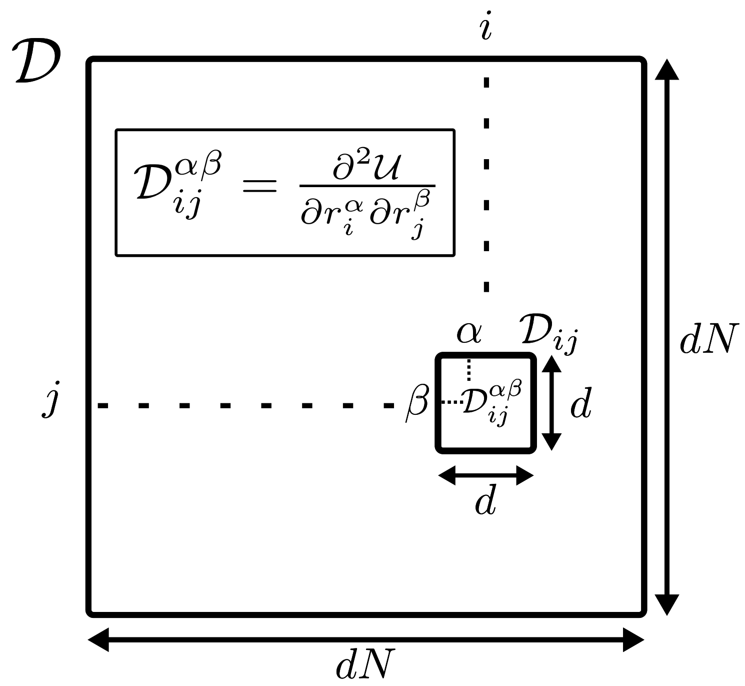
Complexities of glasses

- Complicated "random" structures
- Non-harmonic potentials (e.g., $V_{\mathrm{LJ}}(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$)

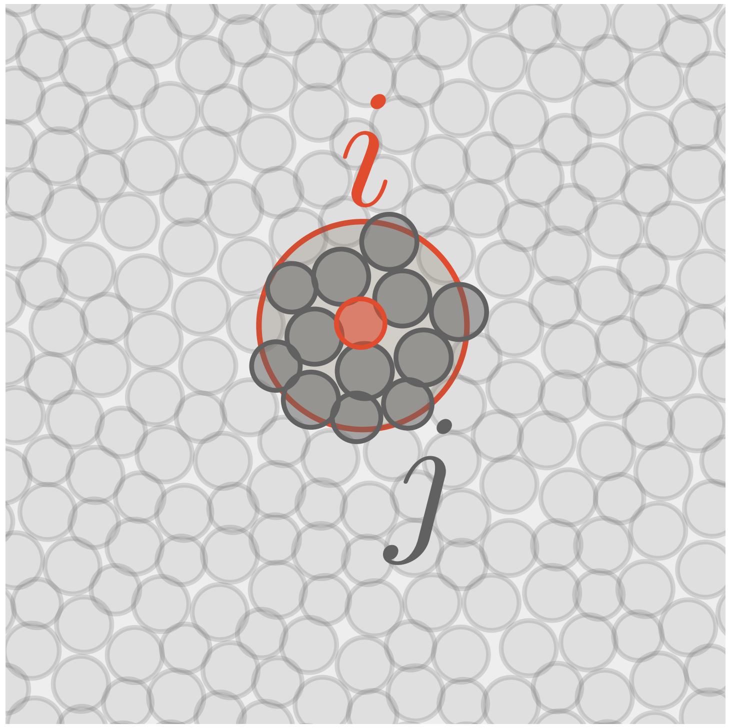
▶ Matrix representation with harmonic approximation for small perturbations (equivalent approximation to ones employed in Debye's theory)

Normal Mode Analysis

Dynamical Matrix



Particles interaction

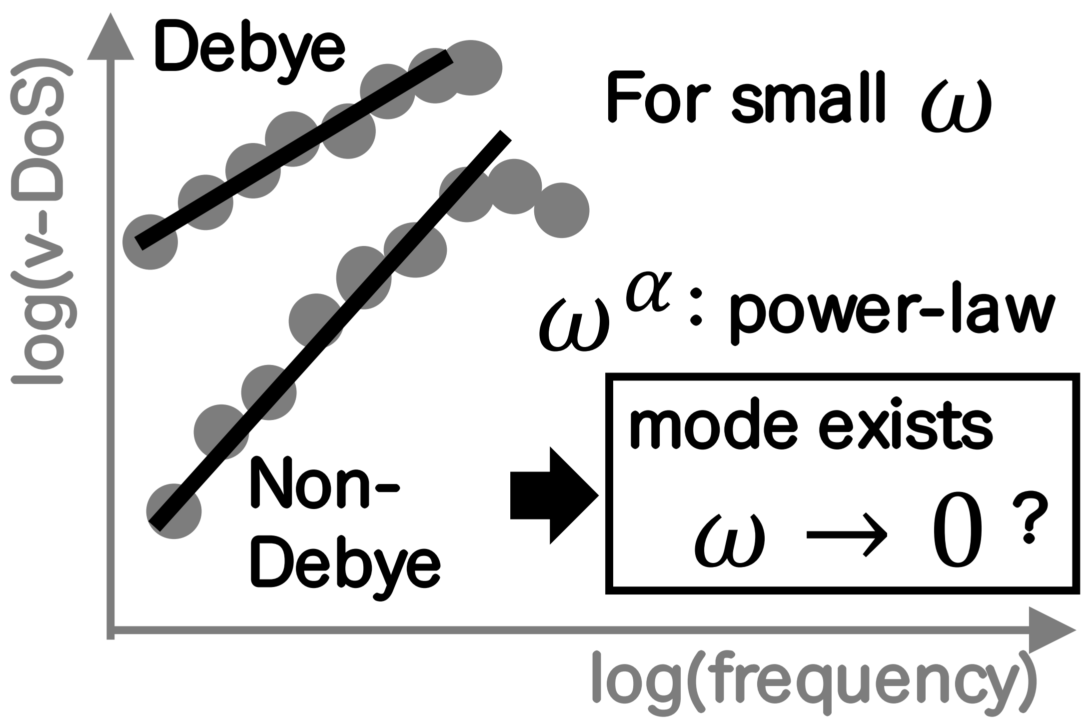


- Mechanical equilibrium state is assumed (zero temperature)
- Total potential $\mathcal{U} = \sum_{ij} V_{ij}(r_{ij})$
- Obtained by the Euler-Lagrange eq. with harmonic approximation
- Eigenvalues λ_k: "Spring consants" to small perturbations
- $\omega_k \equiv \sqrt{\lambda_k}$ gives the eigen-frequency of k-th eigenmode

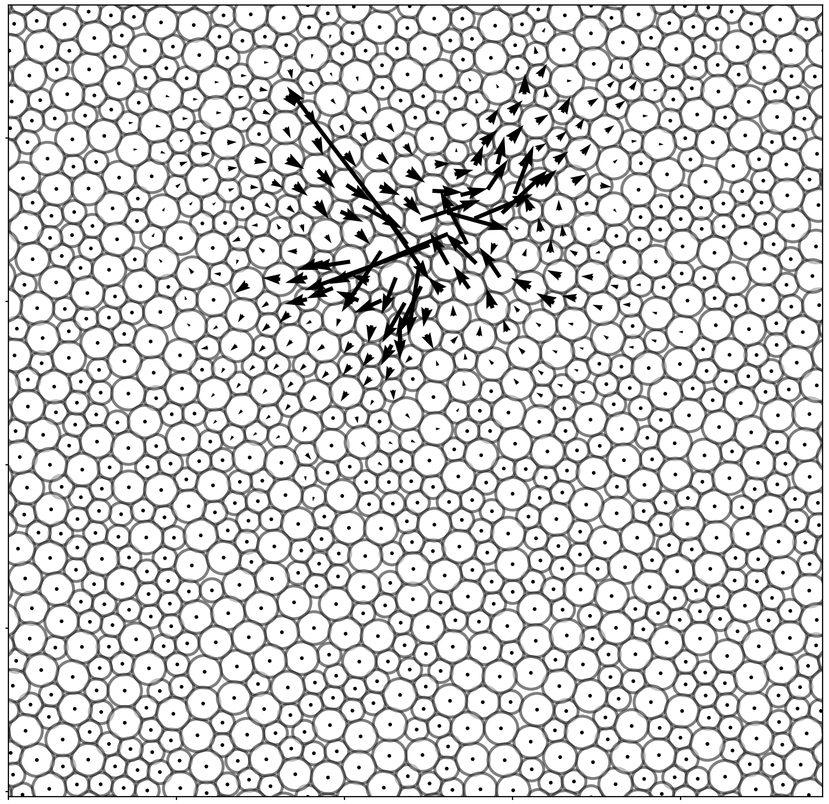
Low-Frequency Modes in Glasses

- Low frequency anomalous properties -

Vibrational density of states (v-DoS)



Low-frequency plastic modes

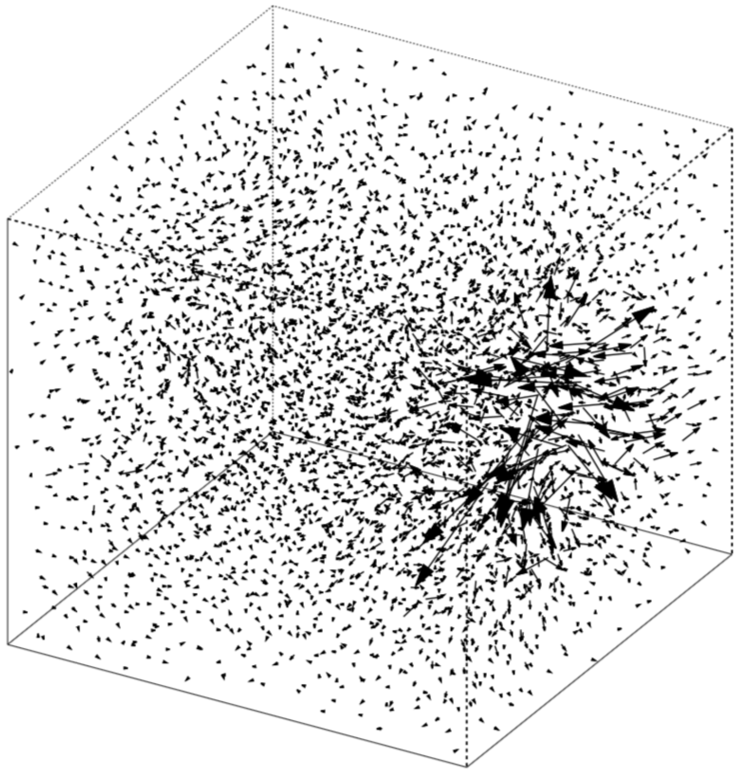


- "Quasi-localized Vibrational Modes (QLVs)" appear
- QLV modes are "plasticity-inducing" pattern
- Infinitesimal perturbation can destabilize the system in $N \to \infty$

QLV vs. Phonon Modes

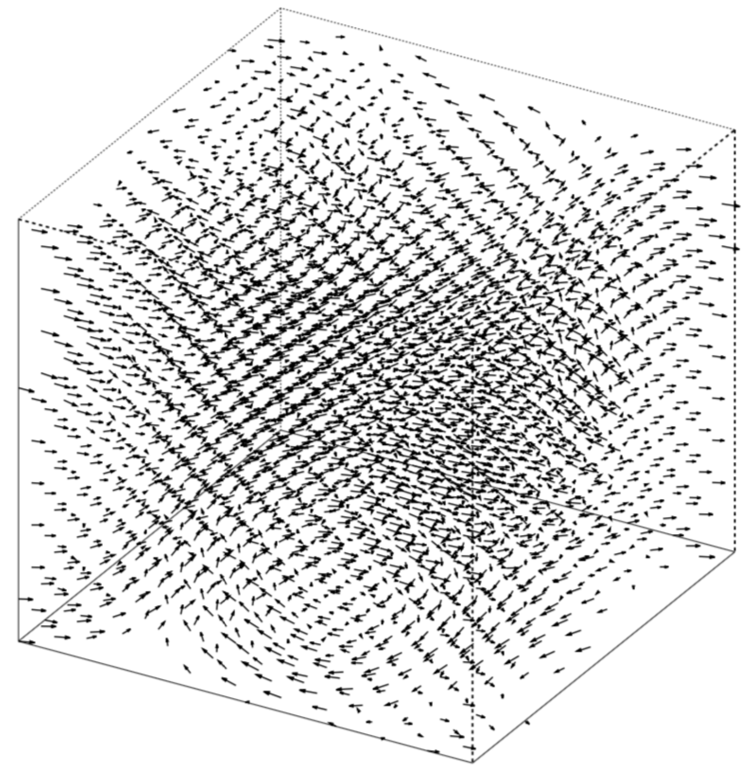
- Difference between low-frequency modes -

Quasi-localized mode



(a) Glass.

Phonon mode



(b) Crystal.

Mizuno and Ikeda, Private Communication

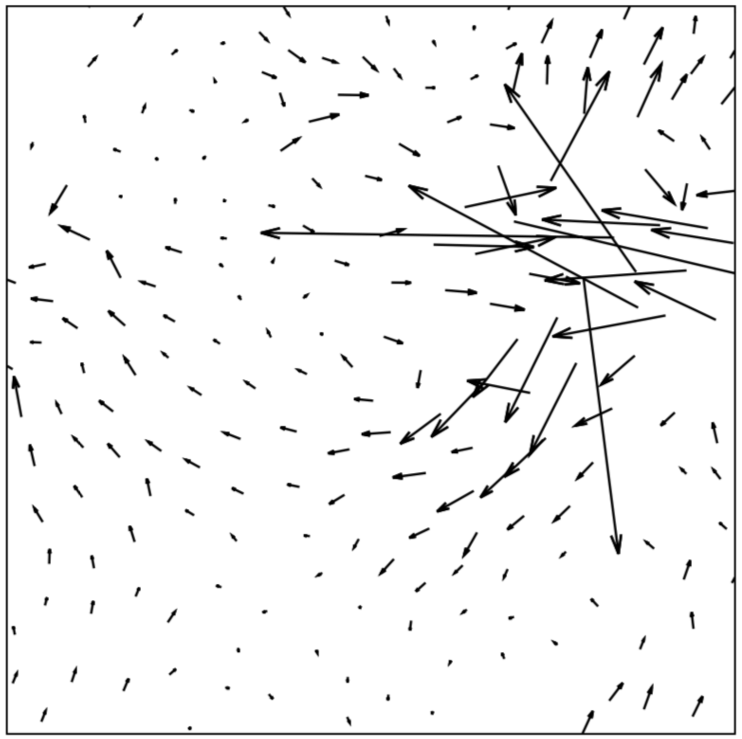
▶ Universally observed in various amorphous systems

Lerner PRL (2016), Mizuno PNAS (2017), Kapteijns PRL (2018), Wang Nat. Commun. (2019), Richard PRL (2020) etc.

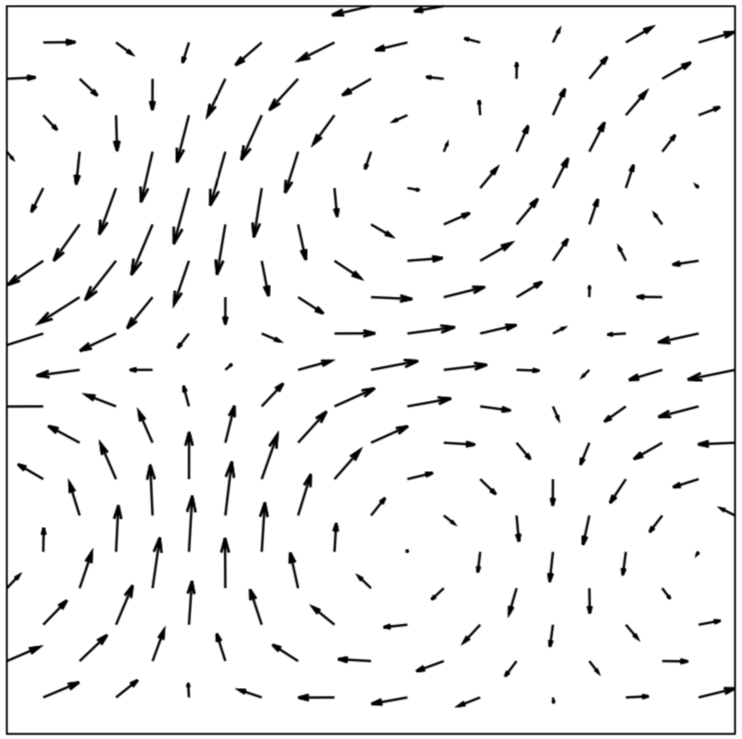
QLV vs. Phonon Modes

- Difference between low-frequency modes -

Quasi-localized mode | **Phonon mode**



(c) Glass.



(d) Crystal.

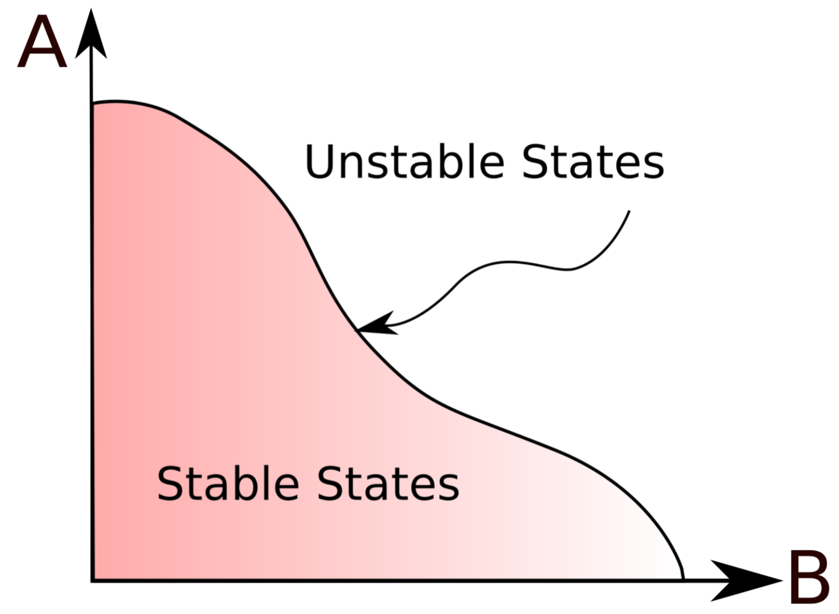
Mizuno and Ikeda, Private Communication

▶ Universally observed in various amorphous systems

Lerner PRL (2016), Mizuno PNAS (2017), Kapteijns PRL (2018), Wang Nat. Commun. (2019), Richard PRL (2020) etc.

Marginal Stability (MS)

- Candidate for the KEY concept -



Müller, and Wyart, Annu. Rev. Condens. Matter Phys. **6**, 177(2015)

- Theoretical predictions are available (e.g. ∞ dimensional mean-field model)

Parisi, Urbani, and Zamponi, "Theory of Simple Glasses", Cambridge University Press (2020)

Franz et al., PNAS **112**, 14539 (2015), Biroli and Urbani, Nat. Phys. **12**, 1130 (2016)

- Glasses: extremely vulnerable solids?

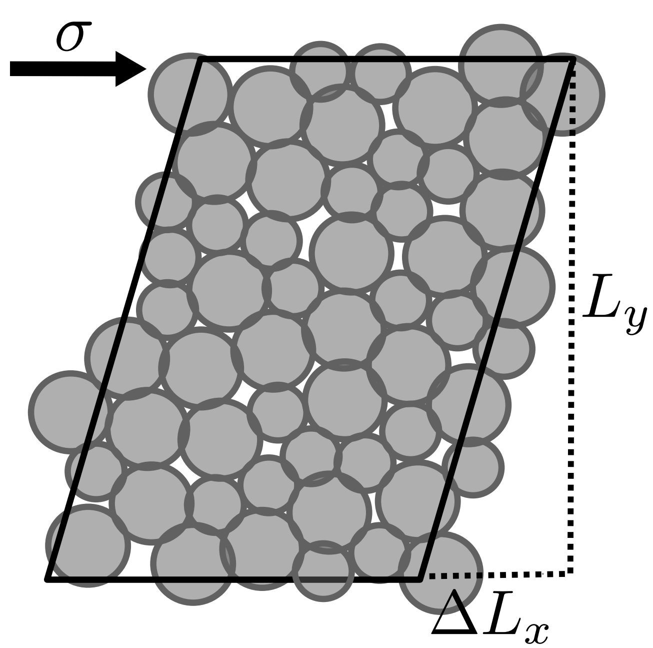
Glasses under External Field

- Non-eq. criticality governing structural failure -

Oyama, Mizuno, and Ikeda, Phys. Rev. E **104**, 015002 (2021), arXiv:2109.08849

Prep: (Shear) Strain and Stress

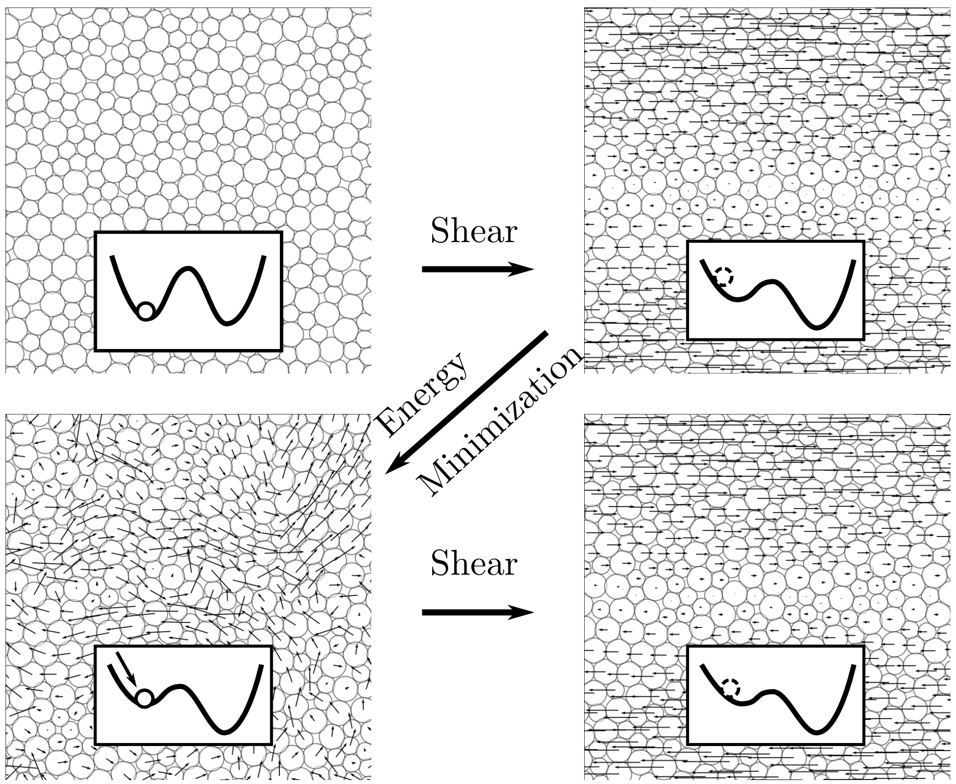
- Fundamental observables for rheology -



- Strain: $\gamma = \Delta L_x / L_y$ (dimensionless)
- Stress: $\sigma = -\frac{1}{V}\sum_{ij}(r_x^{ij} f_y^{ij} + r_y^{ij} f_x^{ij})$
  (dimension of pressure)
- Strain γ (or $\dot{\gamma}$) iscontrolled in this study
  (stress σ can be treated as the response)

MD Simulation under Athermal Quasistatic Drive

- Method to access purely athermal events -

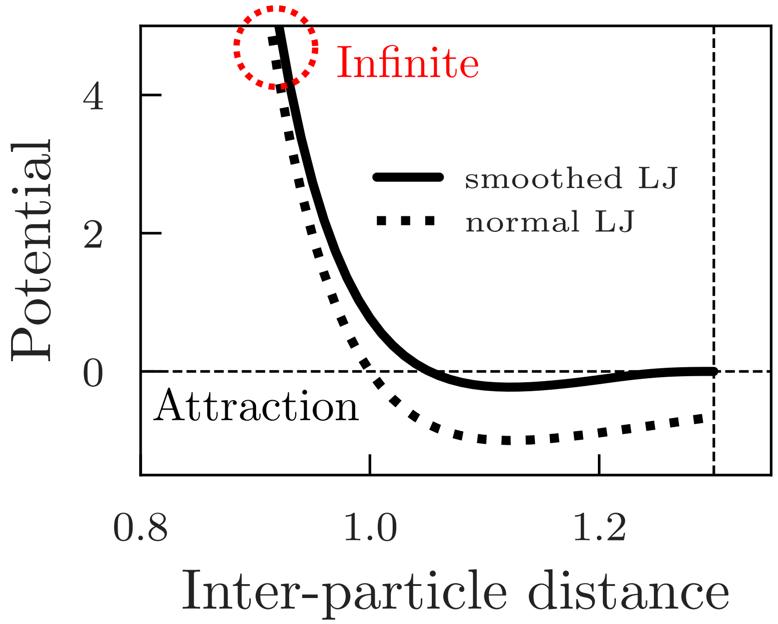


- Corresponding to $\dot{\gamma} = 0$
- $\Delta\gamma = 5 \times 10^{-7} - 5 \times 10^{-6}$ (depending on N)
- Only the steady-state data ($\gamma > 0.25$) is utilized

Inter-particle potentials

- Athermal glass system -

Smoothed Lennard-Jones potential



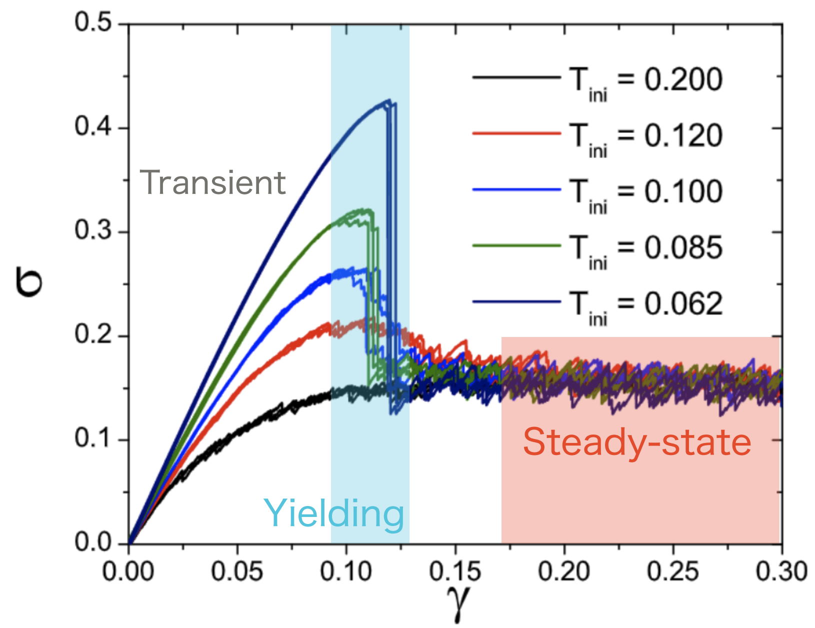
Salerno et al., PRL **109**, 105703 (2012)

- Attraction & excluded volume effect $\rightarrow$ dense: glasses
- Binary mixture (1:1.4 size ratio, 50:50 number ratio)
- 2D, $\rho = 1.09$, $N = 512 - 32768$ particles

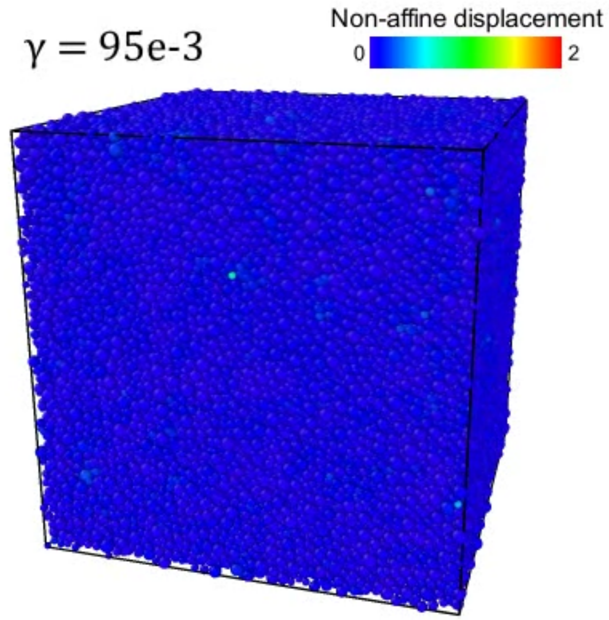
Response under Athermal Quasistatic Shear

- Yielding transition and ... -

Stress-strain cueve



Yielding regime movie



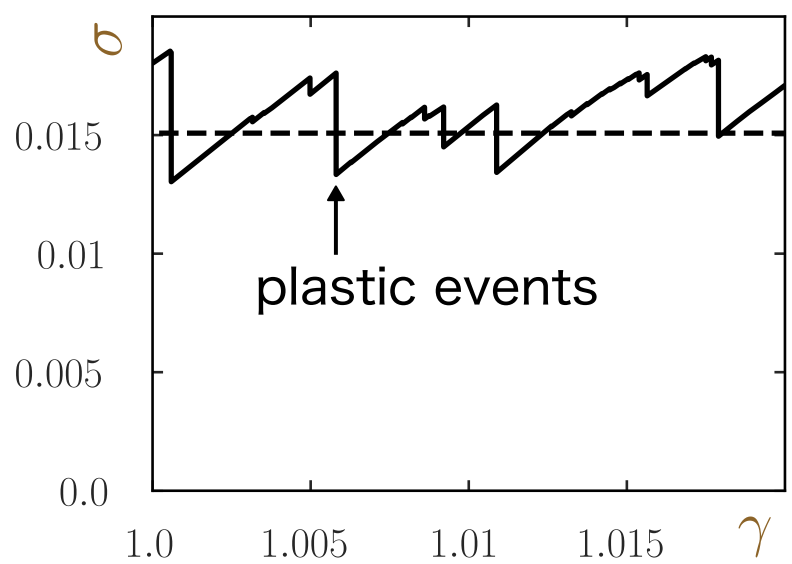
Ozawa et al., PNAS **115**, 6656 (2018)

- Behavior under athermal quasistatic shear
- Yielding singularity at around $\gamma = 0.1$
- Curves self-organize into single one after singularity

Response under Athermal Quasistatic Shear

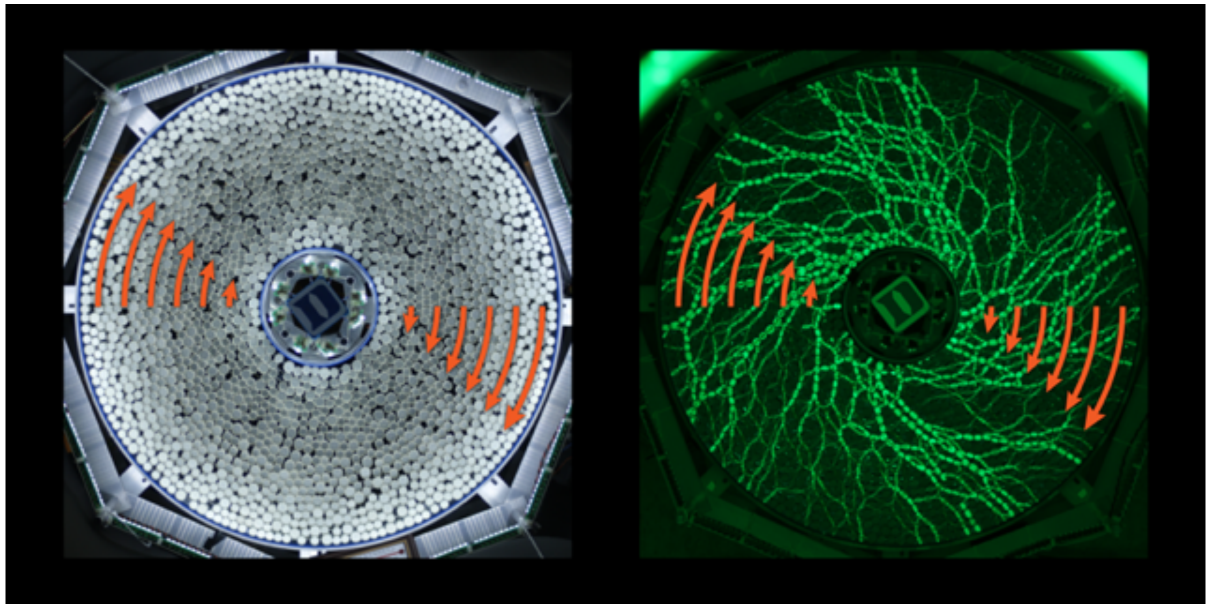
- Steady-state regime -

Stress-strain cueve



Our result

Experiment (Granular flow)



Yiqiu Zhao(Duke University)/ Luding, Physics, **12**, 109 (2019)

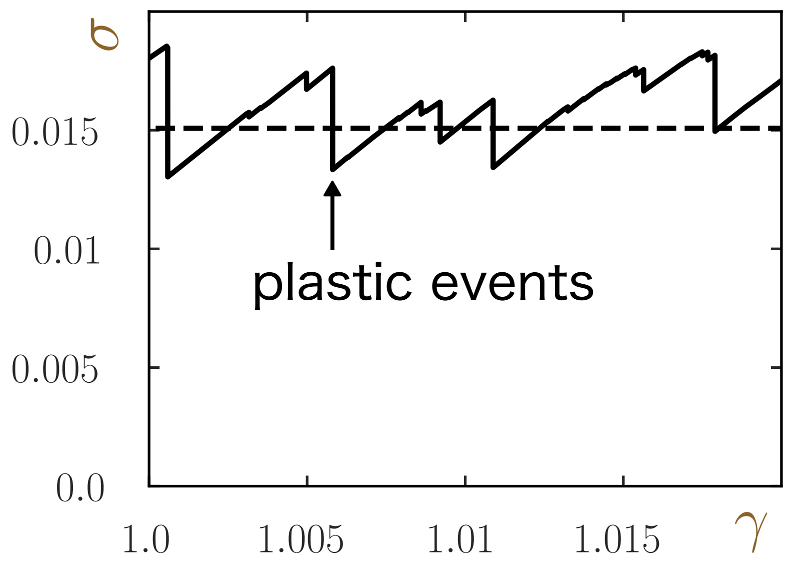
- Focus on steady-state regime hereafter
- Can be viewed as a critical phenomenon?

Dahmen et al., Nat. Phys. **7**, 554 (2011), Lin et al., PNAS **111**, 14382 (2014)

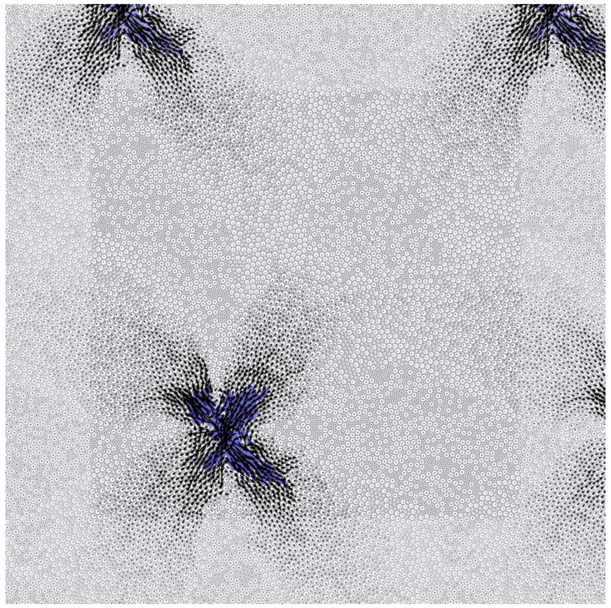
Steady-state Behavior

- Elastic and plastic = elastoplastic response? -

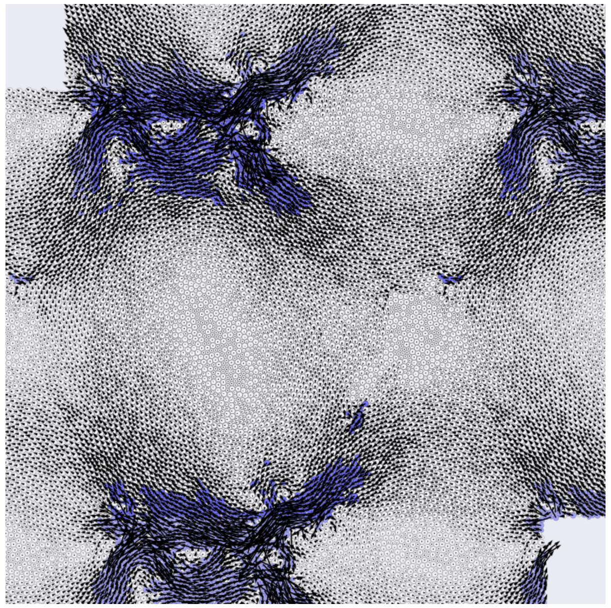
Stress-strain cueve



Single ST



Avalanche of STs

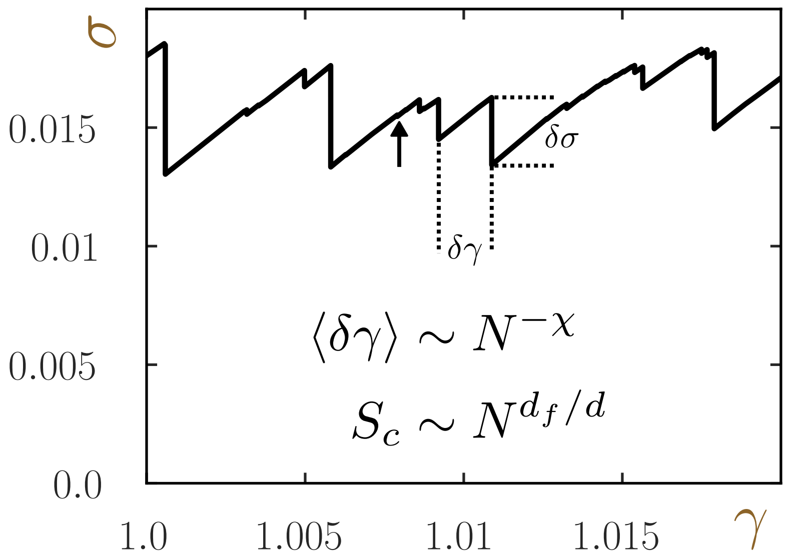


- Elastic behavior is puctuated by intermittent plastic events
- Elementary process of plastisity is shear transformation (ST)
  (characterized by quadrupolar strain field)
- Sometimes organized into avalanches
  (energy released by a ST can trigger other STs)

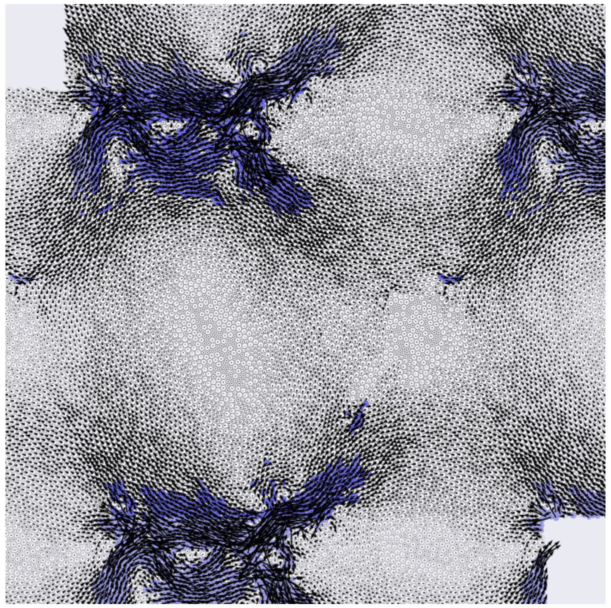
Statistics of ”Avalanches” in Steady State

- One example of avalanche criticalities -

Definitions and scaling laws



Avalanche of STs



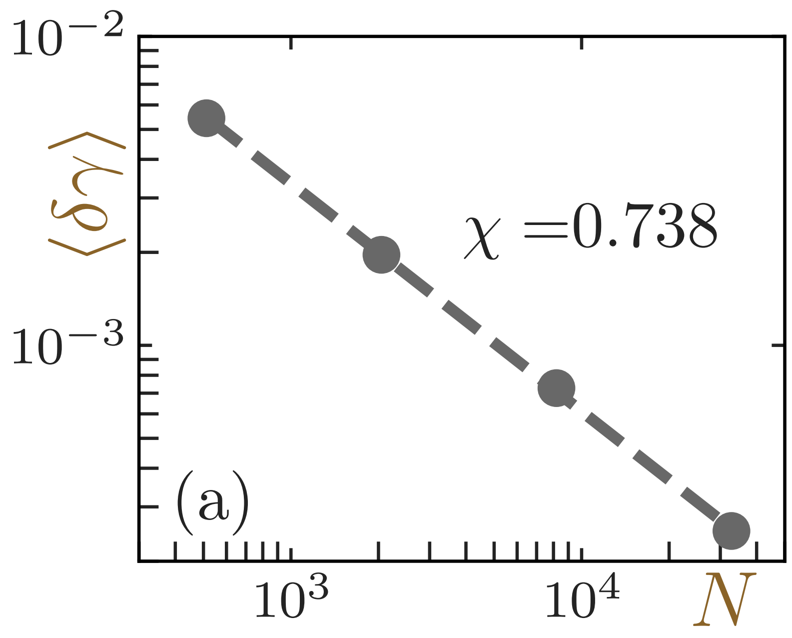
- Avalanche size $S \equiv N\delta\sigma$ (unit of energy $\propto$ volume)
- "Maximum" avalanche size for system with linear dimension L

  $S_c(L) \equiv \langle S^2\rangle/\langle S\rangle$ (assuming $P(S) = S^{-\tau} f(S/S_c)$)

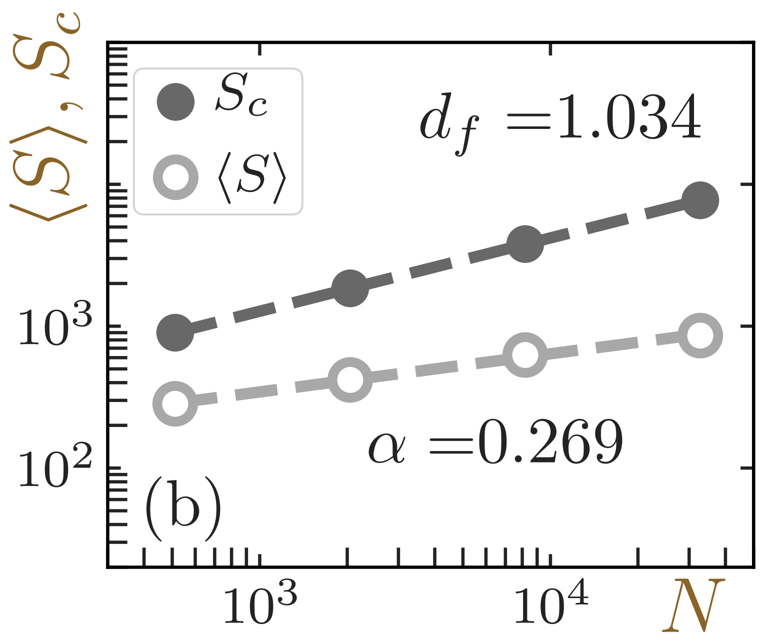
Typical Avalanching Scaling Laws

- System-size-dependence of avg. strain interval and cutoff avalanche size -

Avg. strain interval



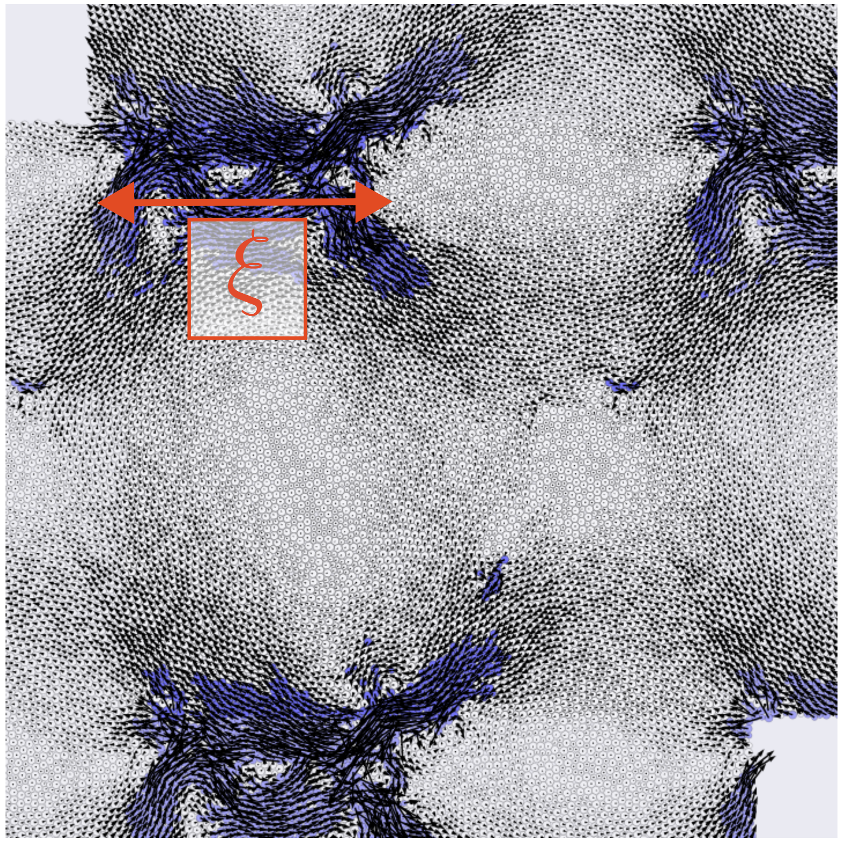
Cutoff avalanche size



- $\langle \Delta\gamma \rangle \sim N^{-\chi} \to 0 (N \to \infty)$: consistent with MS
- $S_c \sim L^{d_f}$ will become important later

Yielding: A Non-eq. Criticality?

Scaling ansatzes



- Correlation length: $\xi \sim |\sigma - \sigma_{\mathrm{Y}}|^{-\nu}$
  - Maximum spanning of avalanches
- Macroscopic strain rate: $\dot{\gamma} \sim |\sigma - \sigma_{\mathrm{Y}}|^{\beta}$
  - Regarded as the order parameter
- Avalanche lifetime: $T \sim \xi^{z}$
  - Inaccessible information
- Cutoff avalanche size: $S_c \sim \xi^{d_f}$
  - Characterized by fractal dimension d_f

Dahmen et al., Nat. Phys. **7**, 554 (2011), Lin et al., PNAS **111**, 14382 (2014)

Effect of Structural Failure: Mechanical Aspect

- Structural origin of a universal rheological law -

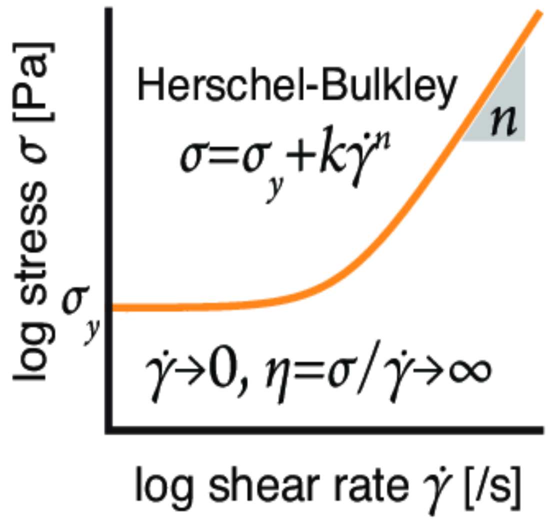
Oyama, Mizuno, and Ikeda, Phys. Rev. Lett. **127**, 108003 (2021)

Herschel-Bulkley Law

- Universal constitutive relation -

Herschel and Bulkley, Kolloid Zeitschrift **39**, 291 (1926)

Schematic Flow Curve



$n<1$ → shear thinning

$n>1$ → shear thickening

$\sigma_y=0,\ n=1$ → Newtonian

de Kort, PhD Thesis (2016)

Vegetables and Fruits

http://www.kuroda-dryer.co.jp,
Diamante and Umemoto, Int. J. Food. Prop. **18**, 1191 (2015)

- Glasses, foams, emulsions, suspensions, blood, etc.
- Yield stress and power-law offset from it
- Structural origin is not yet clarified

MD Simulation under Finite-Rate Shear

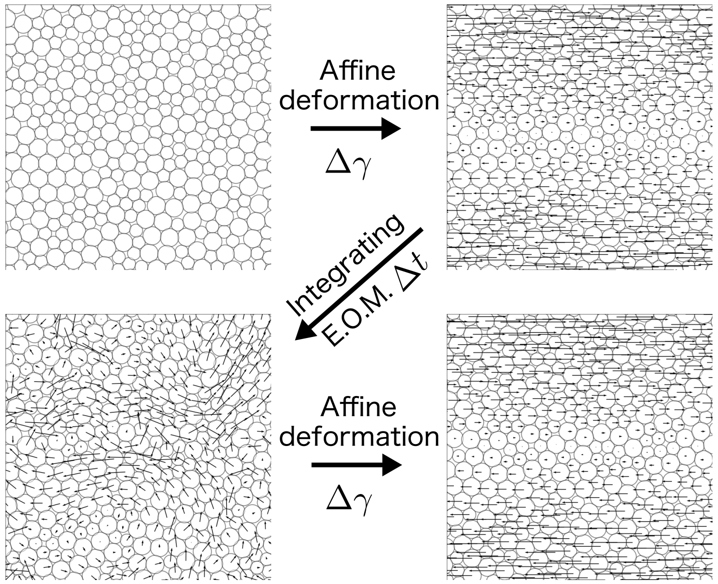
- Athermal system with mechanical noises -

Equation of motion

$$\frac{d\delta \boldsymbol{v}_i}{dt} = -\sum_{j\in\partial i}\frac{\partial V(r_{ij})}{\partial \boldsymbol{r}_i} - \eta\delta\boldsymbol{v}_i$$

$$\frac{d\boldsymbol{r}_i}{dt} = \delta\boldsymbol{v}_i + \dot{\gamma} y_i \boldsymbol{e}_x$$

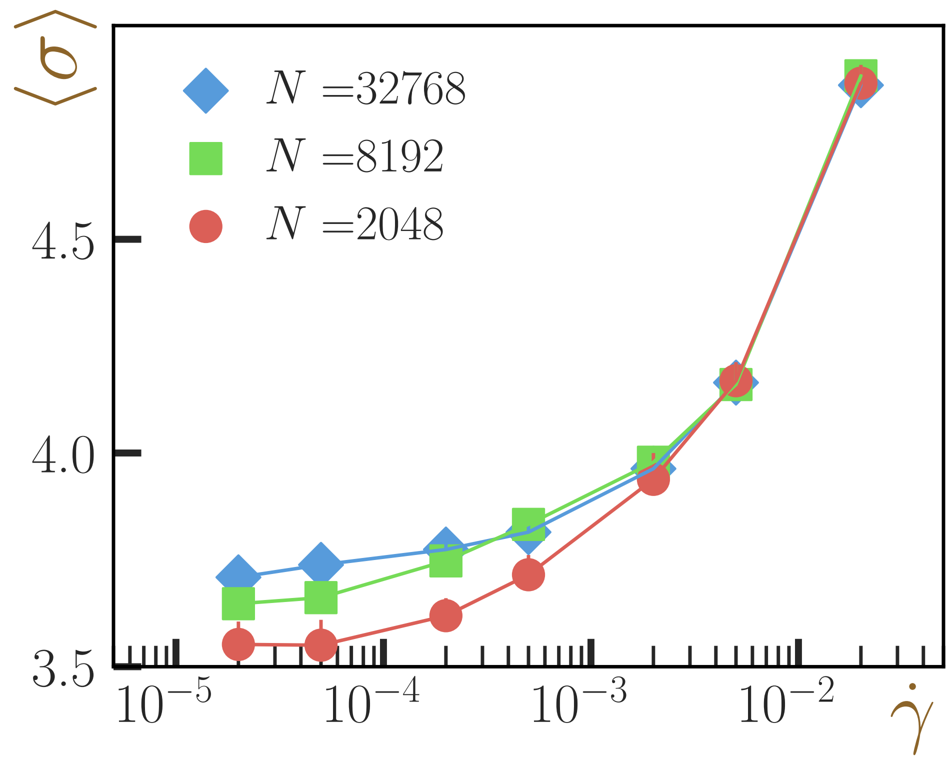
Sketch of algorithm



- $2\times 10^{-5} < \dot{\gamma} \equiv \Delta\gamma/\Delta t < 2\times 10^{-2}$
- Focus on purely structurally-induced rheology: zero temperature (still treated as a classical system)
- Dissipation: Stokes drag

Flow Curves

- Shear-rate dependent mechanical response -

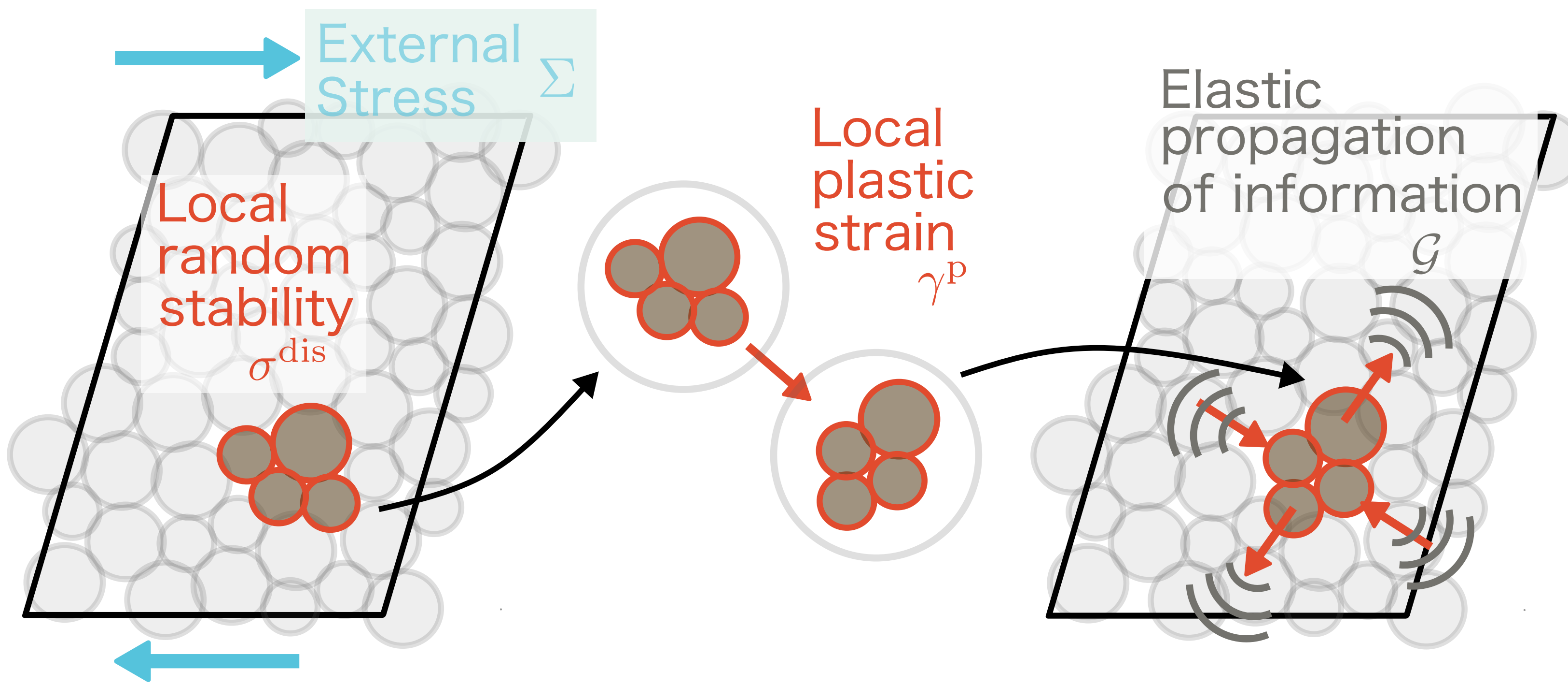


- Herschel-Bulkley law: $\langle\sigma\rangle = \sigma_{\mathrm{Y}} + k\dot{\gamma}^{n}$
- Strong finite size effect (FSE)
- HB parameters cannot be obtained by simple fitting

Plasticity in Sheared Glasses

- Phenomenological description -

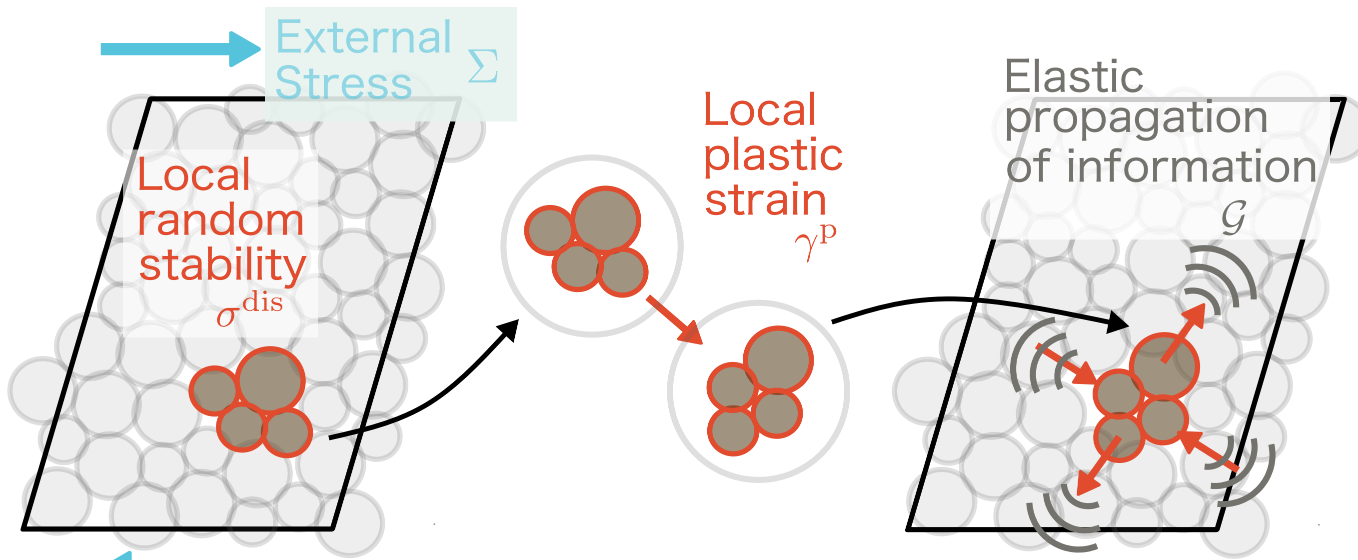
Schematic picture of cause and effect of plastic events



- Driven by external shear stress
- Local stability is randomly distributed
- Local instability triggers plastic events
- Local plastic strain propagate through elastic kernel

Theoretical Formulation

- Continuum description -



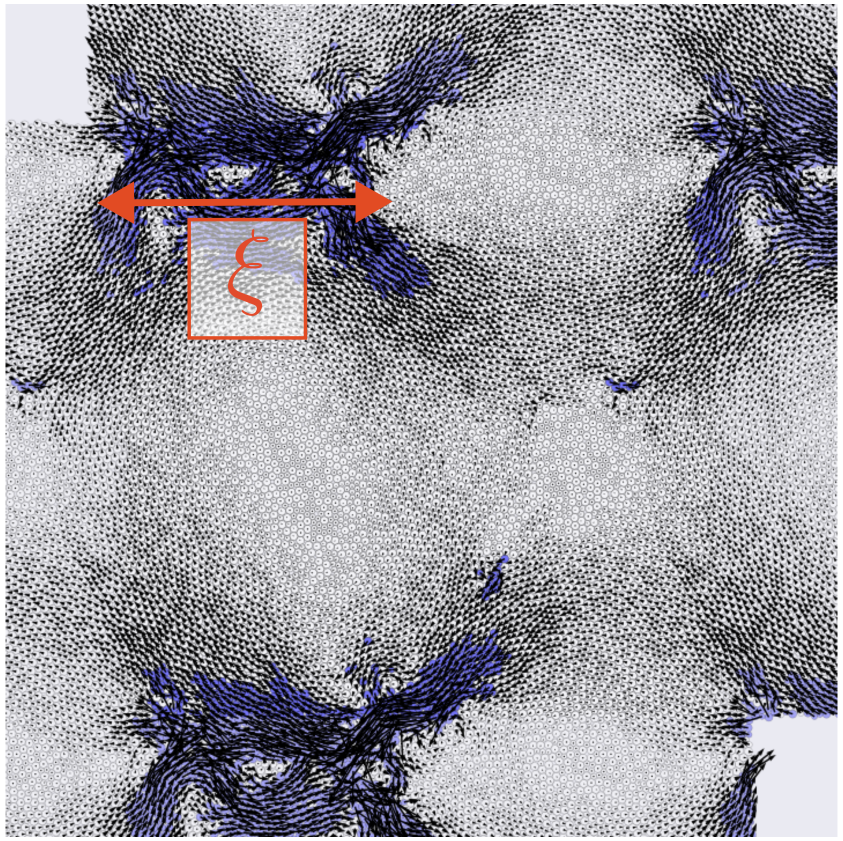
$$\eta\partial_t\gamma(\boldsymbol{r},t) = \Sigma + \int_{\boldsymbol{r}'} \mathcal{G}(\boldsymbol{r}-\boldsymbol{r}')\gamma(\boldsymbol{r}',t) - \sigma^{\mathrm{dis}}[\gamma(\boldsymbol{r},t),\boldsymbol{r}]$$

Lin et al., PNAS **111**, 14382 (2014)

- External stress Σ: Spatially uniform and time independent
- Local yield stress $\sigma^{\mathrm{dis}}[\gamma, x]$: Not explicitly depend on time (controlled solely by the applied total shear γ -> so is γ^{P})
- Interaction kernel $\mathcal{G}$: Eshelby-type quadrupolar propagator (Nonmonotonicity is crucial to reproduce MS: $\langle\Delta\gamma\rangle \sim N^{-\chi}$)

Yielding: A Non-eq. Criticality?

Scaling ansatzes



- Interaction kernel: $\mathcal{G} \sim \xi^0$ (non-monotonic)
  - Eshelby-like quadrupolar pattern ($\mathcal{G}(\boldsymbol{k}) = 4k_x^2 k_y^2 / k^4$ in $\boldsymbol{k}$-space)
- Correlation length: $\xi \sim |\sigma - \sigma_{\mathrm{Y}}|^{-\nu}$
  - Maximum spanning of avalanches
- Macroscopic strain rate: $\dot{\gamma} \sim |\sigma - \sigma_{\mathrm{Y}}|^{\beta}$
  - Regarded as the order parameter
- Avalanche lifetime: $T \sim \xi^z$
- Cutoff avalanche size: $S_c \sim \xi^{d_f}$
  - Characterized by fractal dimension d_f

▶ Useful hyperscaling relations among them?

Statistical Tilt Symmetry (STS)

- Special symmetry of the system -

Introduction of "tilt"

$$\eta\partial_t\gamma(\boldsymbol{r},t) = \Sigma + \sigma^{\text{tilt}} + \int_{\boldsymbol{r}'} \mathcal{G}(\boldsymbol{r}-\boldsymbol{r}')\gamma(\boldsymbol{r}',t) - \sigma^{\text{dis}}[\gamma(\boldsymbol{r},t),\boldsymbol{r}]$$

$$\Leftrightarrow \eta\partial_t\tilde{\gamma}(\boldsymbol{r},t) = \Sigma \qquad + \int_{\boldsymbol{r}'} \mathcal{G}(\boldsymbol{r}-\boldsymbol{r}')\tilde{\gamma}(\boldsymbol{r}',t) - \sigma^{\text{dis}}[\tilde{\gamma} - \int \mathcal{G}^{-1}\sigma^{\text{tilt}},\boldsymbol{r}]$$

Narayan and Fisher, Phys. Rev. B **48**, 7030 (1993); Lin et al., PNAS **111**, 14382 (2014)

- Tilt σ^{tilt}: random stress field with vanishing spatial average

  (tilted strain field: $\tilde{\gamma}(\boldsymbol{r}) = \gamma(\boldsymbol{r}) + \int_{\boldsymbol{r}'} \mathcal{G}^{-1}(\boldsymbol{r}-\boldsymbol{r}')\sigma^{\text{tilt}}(\boldsymbol{r}')$)

- Absorbed by the random local yield stress distribution σ^{dis}
- On average (over randomness), tilt does not matter: $\overline{\delta\tilde{\gamma}/\delta\sigma^{\text{tilt}}} = 0$

Consequences of STS

- Scaling relations from susceptibility -

Susceptibility estimated by tilt

$$\chi_1 \equiv \frac{\overline{\partial \gamma}}{\partial \sigma^{\mathrm{tilt}}} = \frac{\overline{\partial(\tilde{\gamma} + \int \mathcal{G}^{-1} \sigma^{\mathrm{tilt}})}}{\partial \sigma^{\mathrm{tilt}}} \sim \mathcal{G}^{-1} \sim \xi^0$$

- *Tilt does not play any role:* $\overline{\partial \tilde{\gamma}} / \partial \sigma^{\mathrm{tilt}} = 0$
- *Interactino kernel is nonmonotonic:* $\mathcal{G} \sim \xi^0$

Susceptibility estimated by global stress increment

$$\chi_2 \equiv \frac{\Delta \gamma}{\Delta |\sigma - \sigma^{\mathrm{Y}}|} \sim \frac{\xi^{d_f} / \xi^d}{\xi^{-1/\nu}} \sim \xi^{1/\nu + d_f - d}$$

- *Perturbing* σ *induces avalanhce with size* $\xi \sim |\sigma - \sigma_{\mathrm{Y}}|^{-\nu}$
- *Strain: (avalanche volume* ξ^{d_f} *) / (total volume* ξ^d *))*

Narayan and Fisher, Phys. Rev. B **48**, 7030 (1993); Lin et al., PNAS **111**, 14382 (2014)

▶ Hyperscaling relation: $\chi_1 \sim \chi_2 \Rightarrow \nu = 1/(d - d_f)$

Simple Scaling Argument

- Other useful relations -

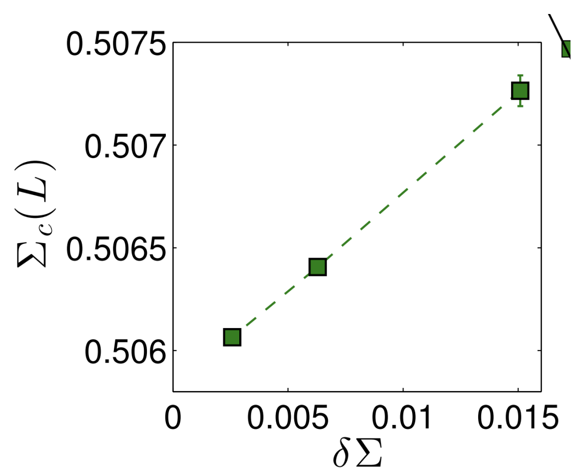
Scaling relation for strain rate

$$\dot{\gamma} = \frac{\gamma}{T} \sim \frac{\xi^{d_f}/\xi^d}{\xi^z} \sim \xi^{d_f-d-z}$$

$$\dot{\gamma} \sim |\sigma - \sigma_{\rm Y}|^{\beta} \sim \xi^{-\beta/\nu}$$

$$\therefore \beta = \nu(z + d - d_f)$$

Determination of the intrinsic yield stress



Lin et al., PNAS **111**, 14382 (2014)

$$\xi \sim |\sigma - \sigma_{\rm Y}|^{-\nu} \Leftrightarrow \langle\sigma\rangle(\xi) = \sigma_{\rm Y} + k_1\xi^{-1/\nu}$$

$$\Delta_\sigma(\xi) \sim \langle\sigma\rangle(\xi) - \sigma_{rmY} \sim k_2\xi^{-1/\nu}$$

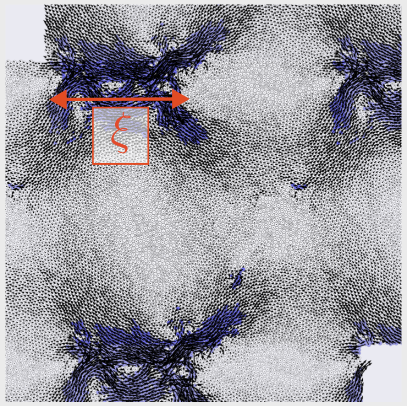
$$\therefore \langle F\rangle(\xi) = F_c^* + k_3\sigma_F(\xi)$$

$$(\Delta_\sigma(\xi)\text{: Standard deviation of } \sigma(\xi))$$

Vandembroucq et al., Phys. Rev. E **70**, 051101 (2004)

Yielding: A Non-eq. Criticality?

Scaling ansatzes



- *Interaction kernel:* $\mathcal{G} \sim \xi^0$ *(non-monotonic)*
  ($\mathcal{G}(\boldsymbol{k}) = 4k_x^2 k_y^2 / k^4$ *in* k*-space)*
- *Correlation length:* $\xi \sim |\sigma - \sigma_\mathrm{Y}|^{-\nu}$
- *Macroscopic strain rate:* $\dot{\gamma} \sim |\sigma - \sigma_\mathrm{Y}|^{\beta} \sim \xi^{-\beta/\nu}$
- *Avalanche lifetime:* $T \sim \xi^z$
- *Cutoff avalanche size:* $S_c \sim \xi^{d_f}$

Hyperscaling relation

- Correlation length : $\nu = 1/(d - d_f)$
- Strain rate : $\beta = \nu(z + d - d_f)$
- Critical stress estimation : $\langle \sigma \rangle(\xi) = \sigma_\mathrm{Y} + k\Delta_\sigma(\xi)$

▶ These relations were verified using continuum model

Lin et al., PNAS **111**, 14382 (2014)

▶ No verification for paritcle-based data -> WHY?

Yielding: A Non-eq. Criticality?

Hyperscaling relation

- Correlation length : $\nu = 1/(d - d_f)$
- Strain rate : $\beta = \nu(z + d - d_f)$
- Critical stress estimation : $\langle\sigma\rangle(\xi) = \sigma_{\mathrm{Y}} + k\Delta_\sigma(\xi)$

▶ Red values can be measured by quasistatic simulation

Oyama, Mizuno, and Ikeda, PRE **104**, 015002 (2021)

▶ Critical stress σ_{Y} is determined!

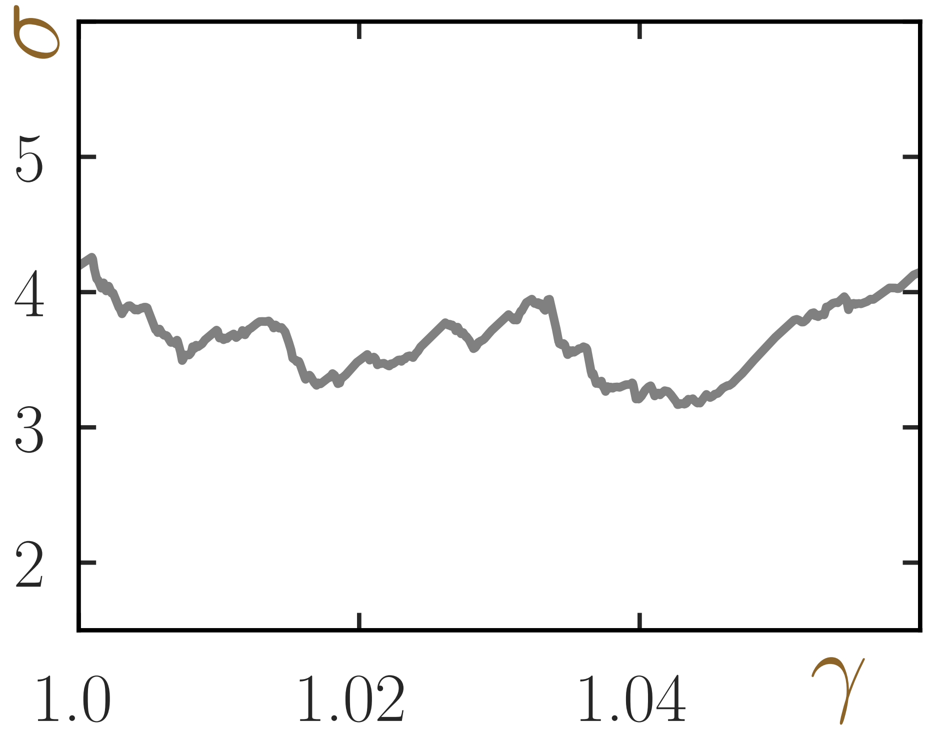
(HB law: $\dot{\gamma} \sim |\sigma - \sigma_{\mathrm{Y}}|^{\beta}$)

▶ HB exponent β can be obtained if z is measured, but...

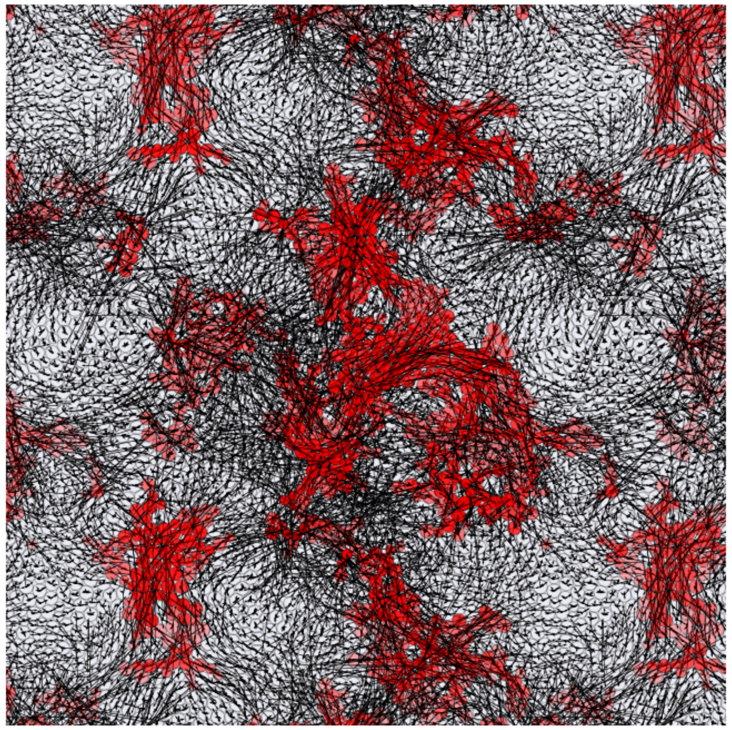
Dynamical information?

- Inaccessible information -

Stress-strain curve



"Displacement" field



- Avalanche lifetime: $T \sim \xi^{z}$
- Both T and ξ cannot be extracted unambiguously
- Cannot utilize this relation for particle data...

Our Strategy and Aim

- To by-pass the problem above... -

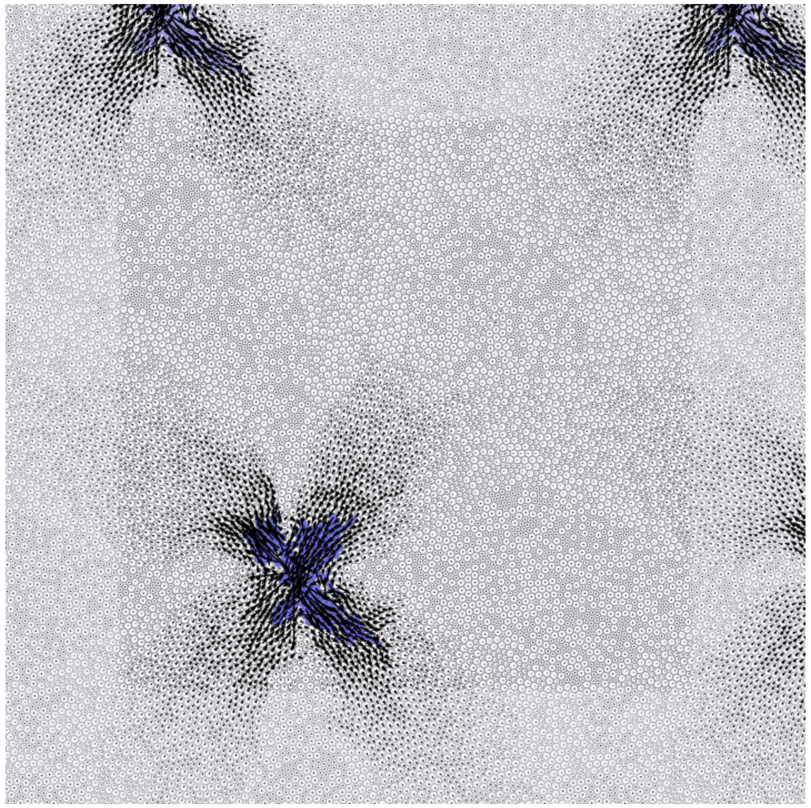
Utilize the microscopic structures
that are available only in MD simulations

▶ (Instantaneous) Normal Modes

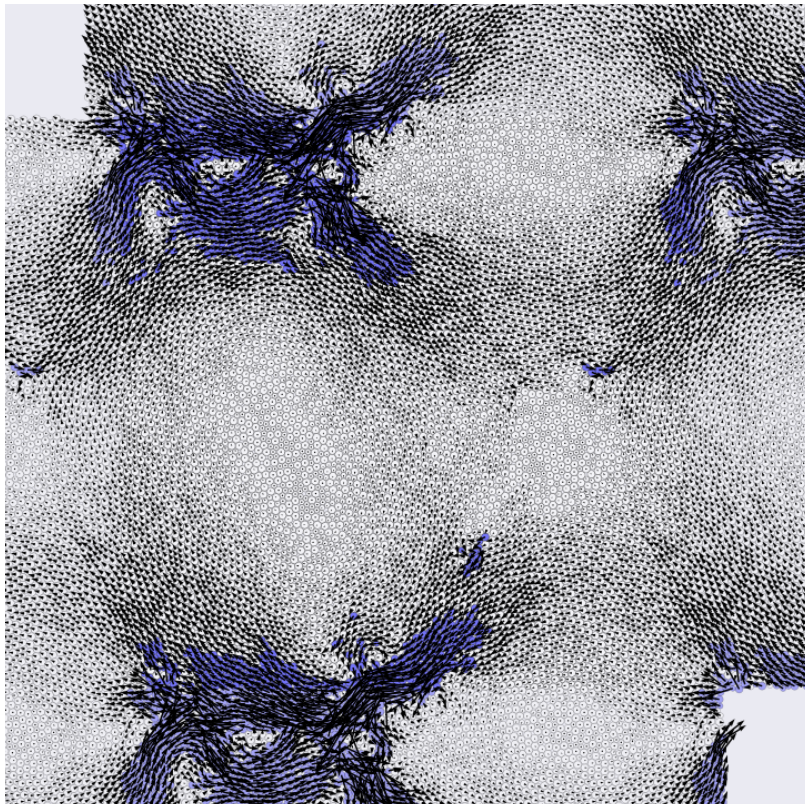
Shear Transformations (STs)

- Elementary processes of plasticity -

Single ST



Avalanche of STs

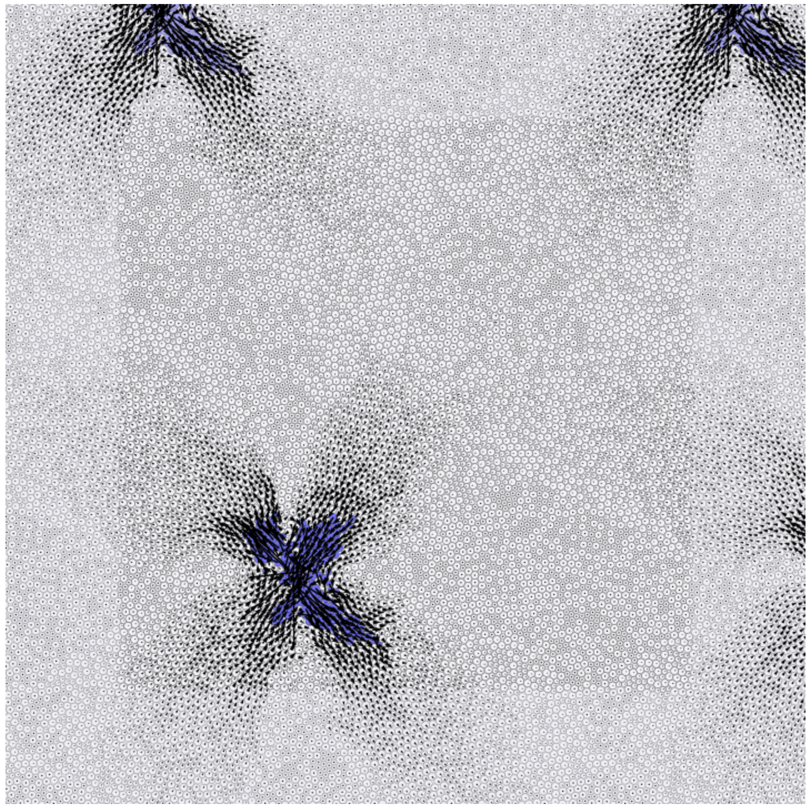


- Considered to be the elementary process
- Sometimes form "avalanches"

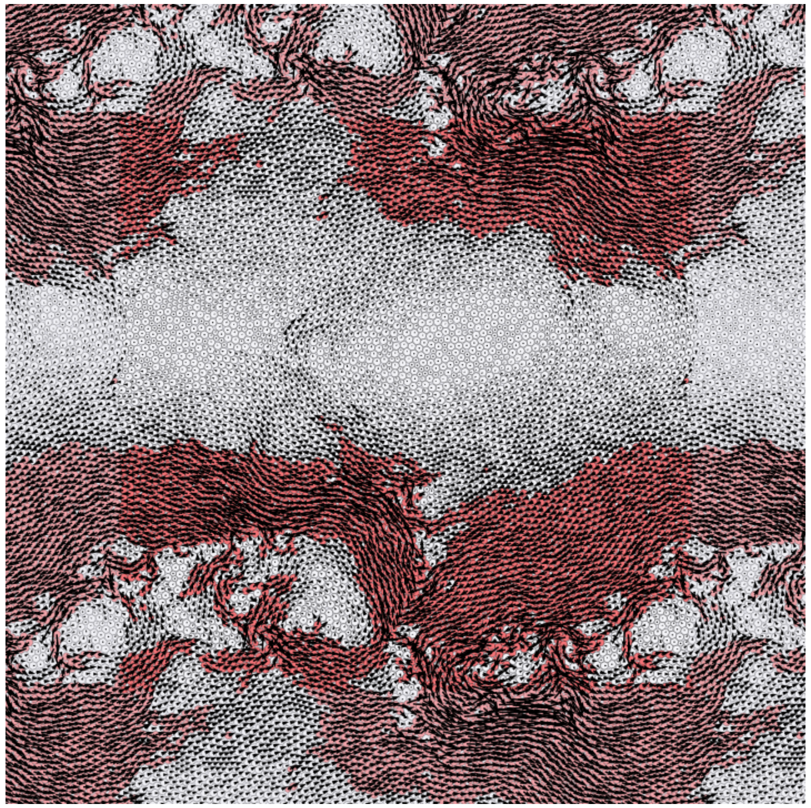
Shear Transformations (STs)

- Elementary processes of plasticity -

Single ST



System-spanning Avalanche



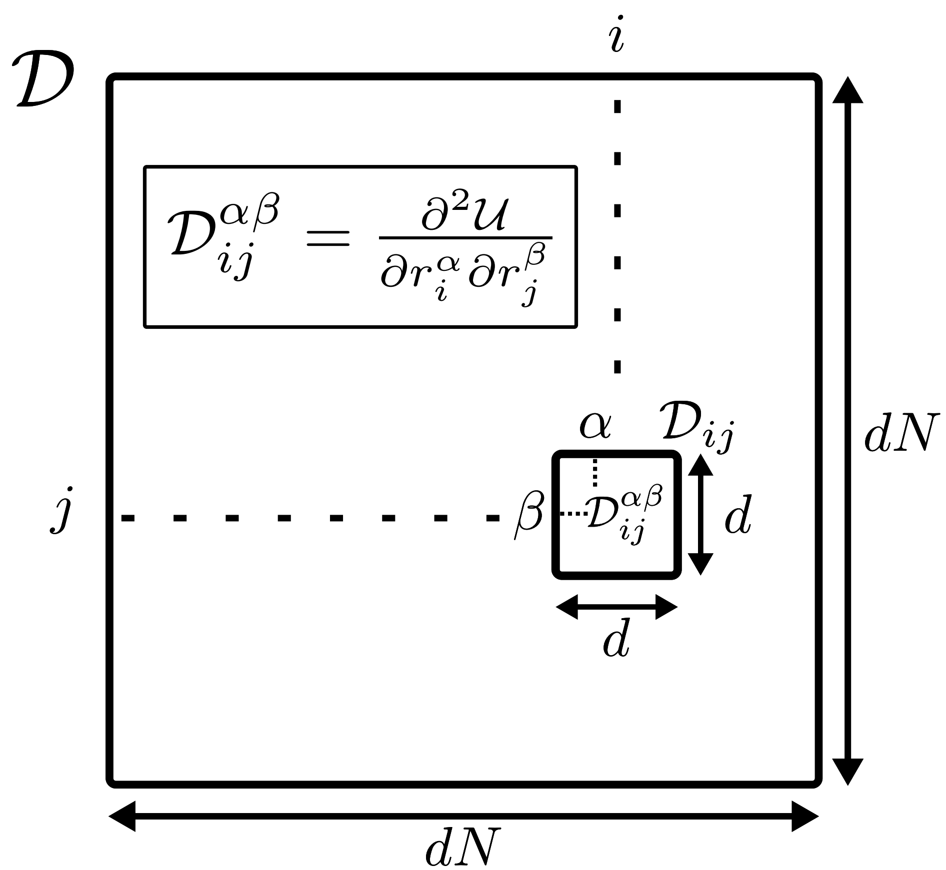
- Considered to be the elementary process
- Sometimes form "avalanches"

Knowledge Under Quasistatic Shear

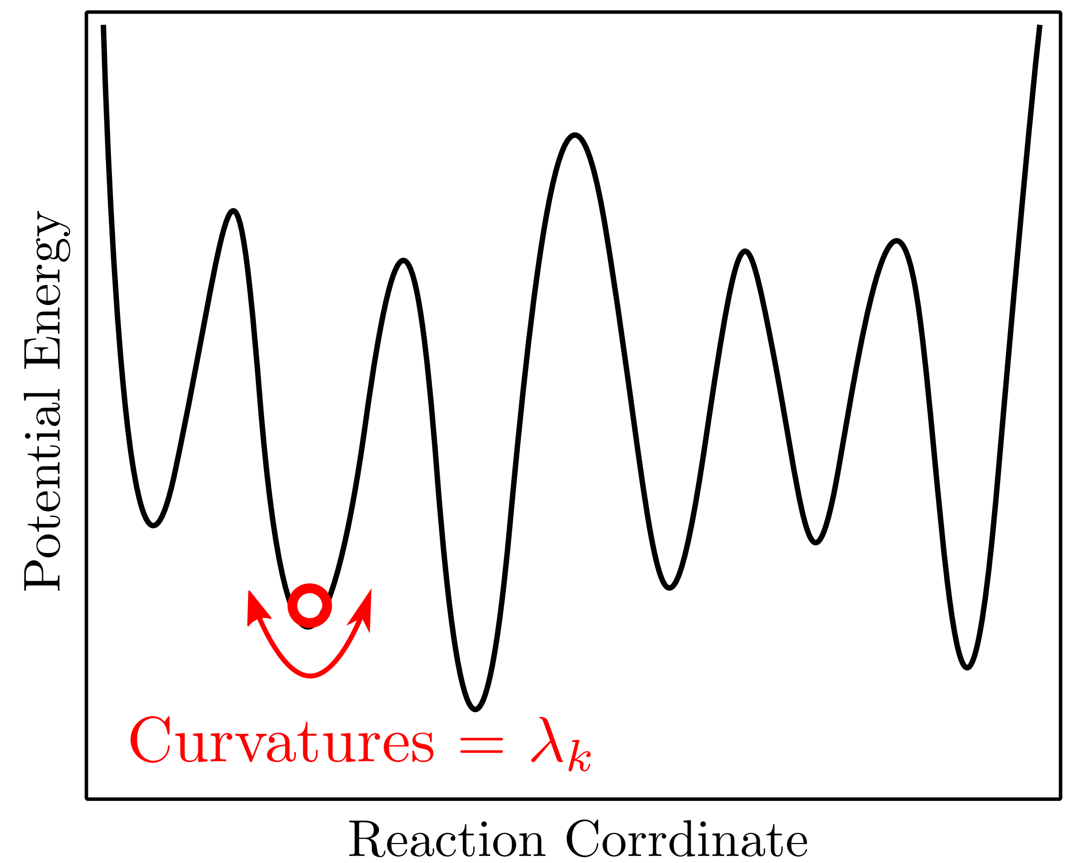
- How are plastic events evoked? -

Normal Mode Analysis

Dynamical Matrix



Potential energy landscape (PEL)

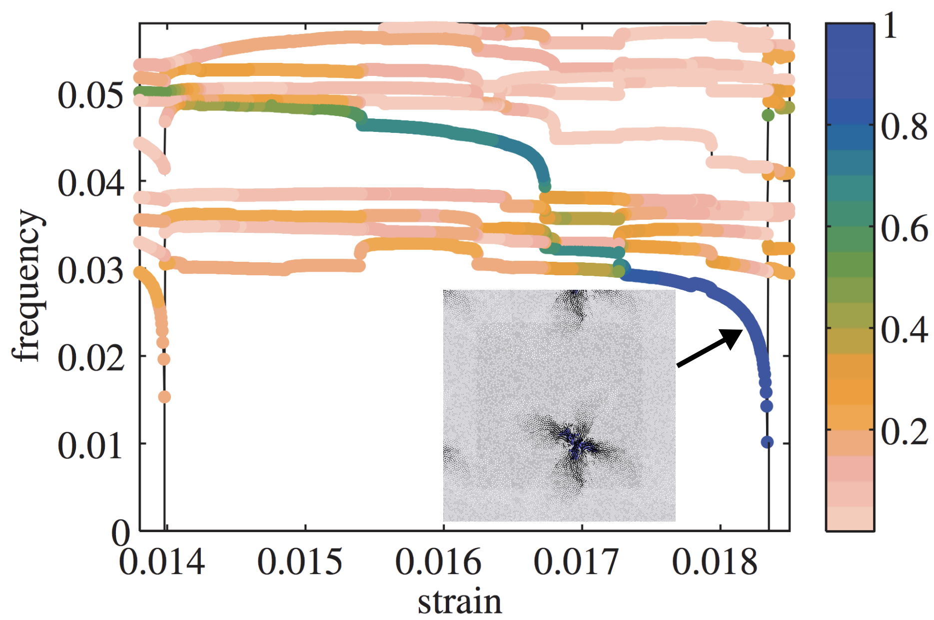


- Measured with no noise sources (basin bottom of PEL)
- All eigenvalues λ_k are positive in this case
- Eigenvalues λ_k give the curvatures of the PEL
- $\omega_k \equiv \sqrt{\lambda_k}$: eigen-frequencies along the curvature

”Cause” of Plastic Events

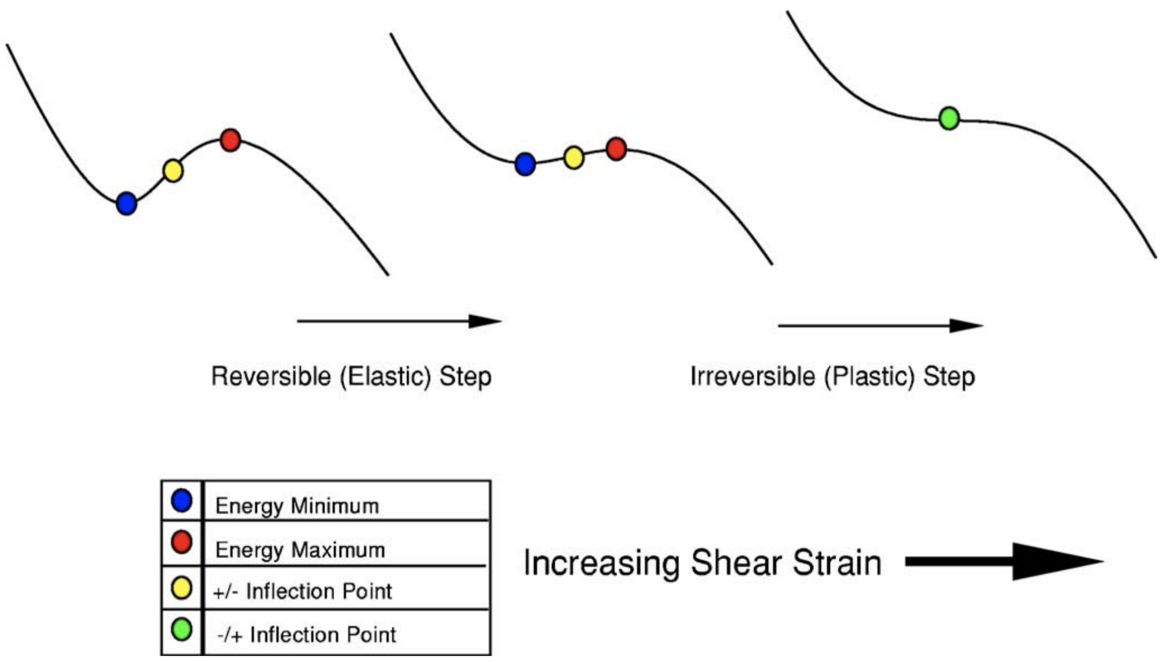
- Under athermal quasistatic shear -

Evolution of normal modes



Manning and Liu, Phys. Rev. Lett. **107**,108302 (2011)

Evolution of potential energy landscape



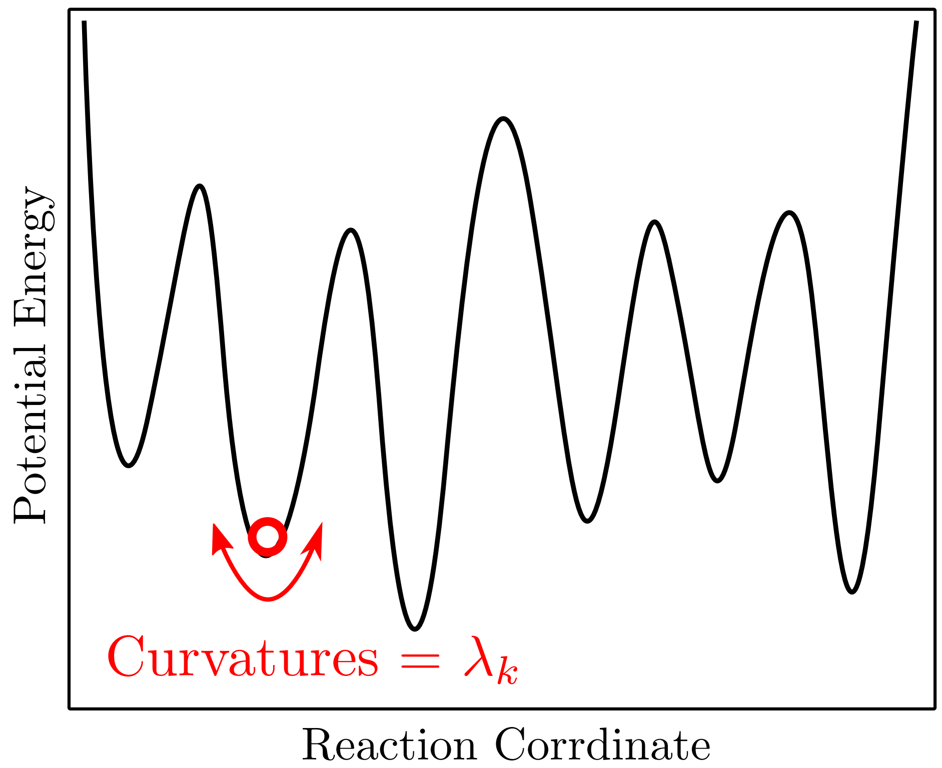
Maloney and Lemaître, Phys. Rev. E **74**,016118 (2006)

- Lowest eigenvalue goes to zero just before plastic event
- The decaying mode has a quadrupolar pattern
- Disappearance of energy basin: saddle-node bifurcation
- What if under finite-rate shear?

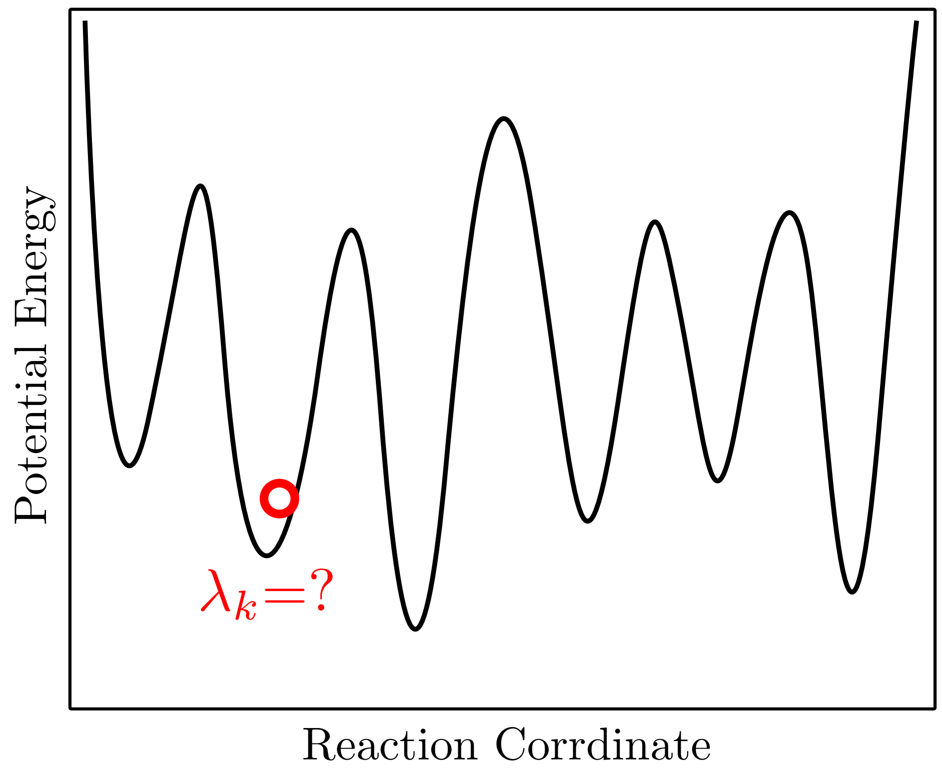
Normal Modes Under Finite-rate Shear

- Generalization of normal mode analysis -

Standard NM analysis



NM analysis under finite-rate shear

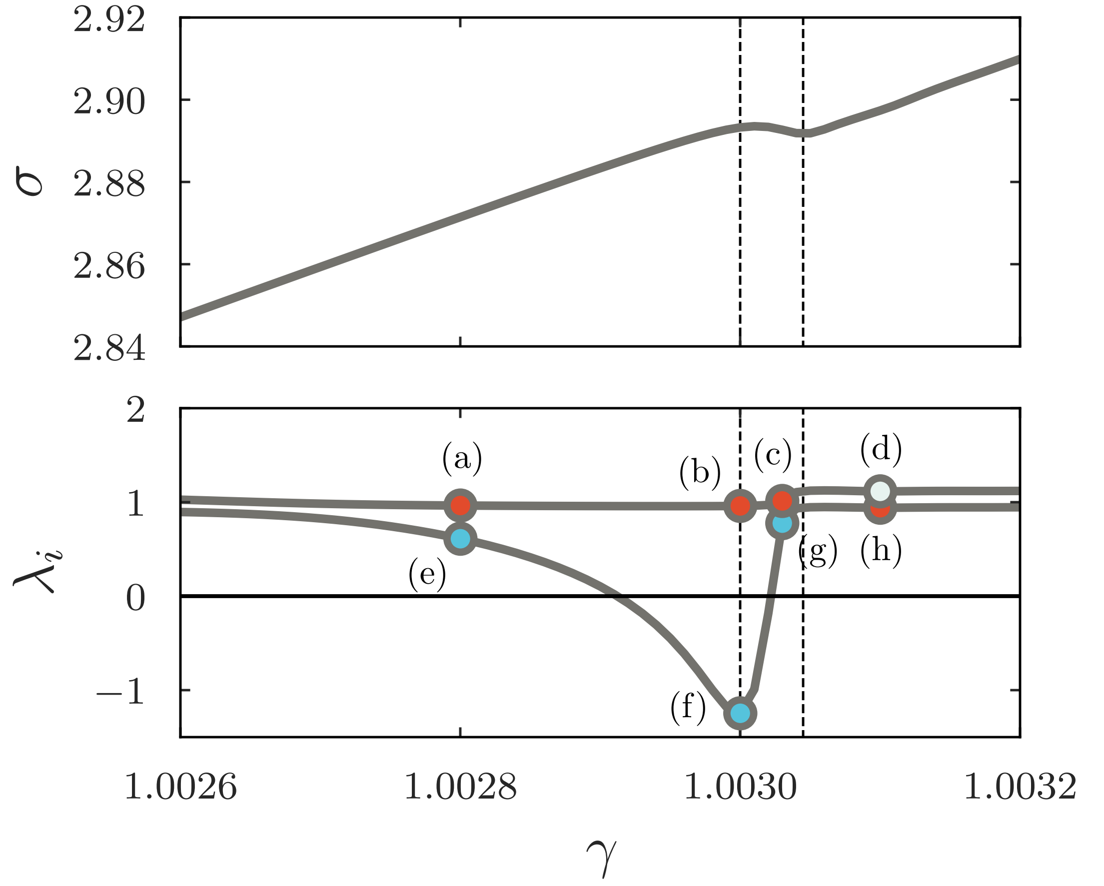


- What do eigenmodes stand for in such a situation?

Emergence of Negative Mode?

- Cause of plasticity under finite-rate shear -

At very slow rate: $\dot{\gamma} = 2 \times 10^{-5}$

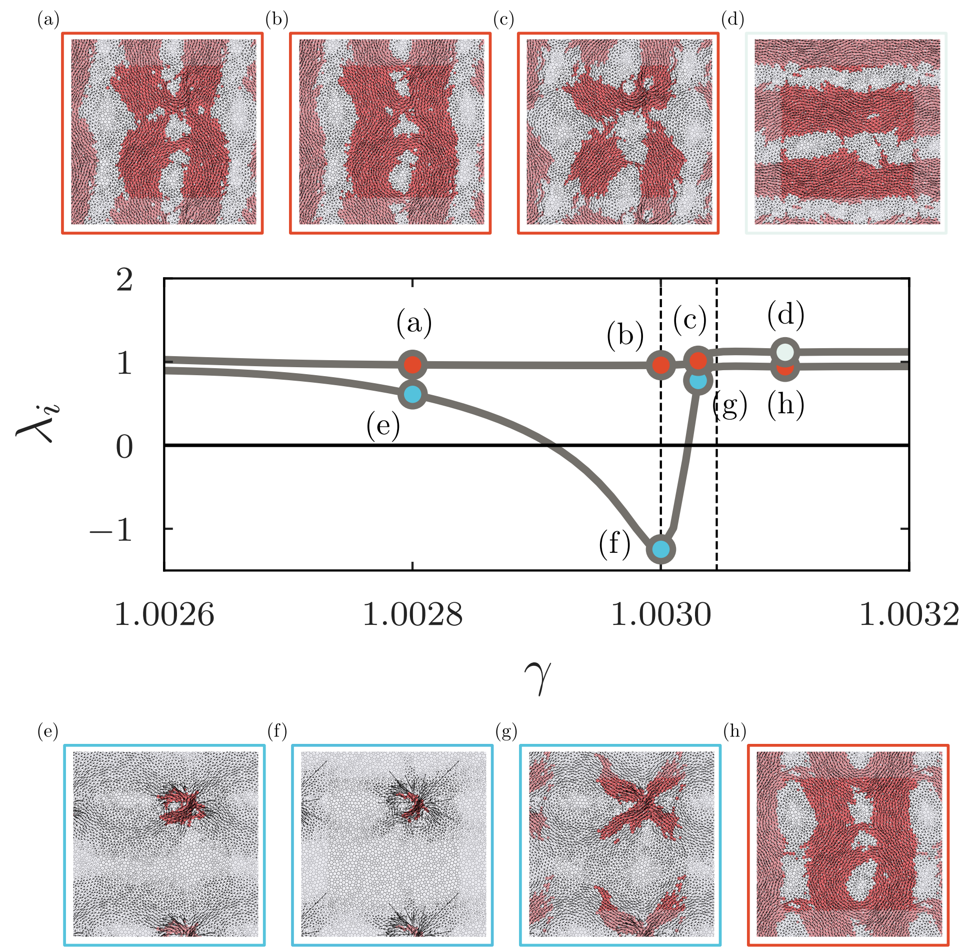


- Lowest eigenvalue goes down similarly to quasistatic case
- No stress drop (plasticity) at the onset of $\lambda_1 = 0$

Emergence of Negative Mode?

- Cause of plasticity under finite-rate shear -

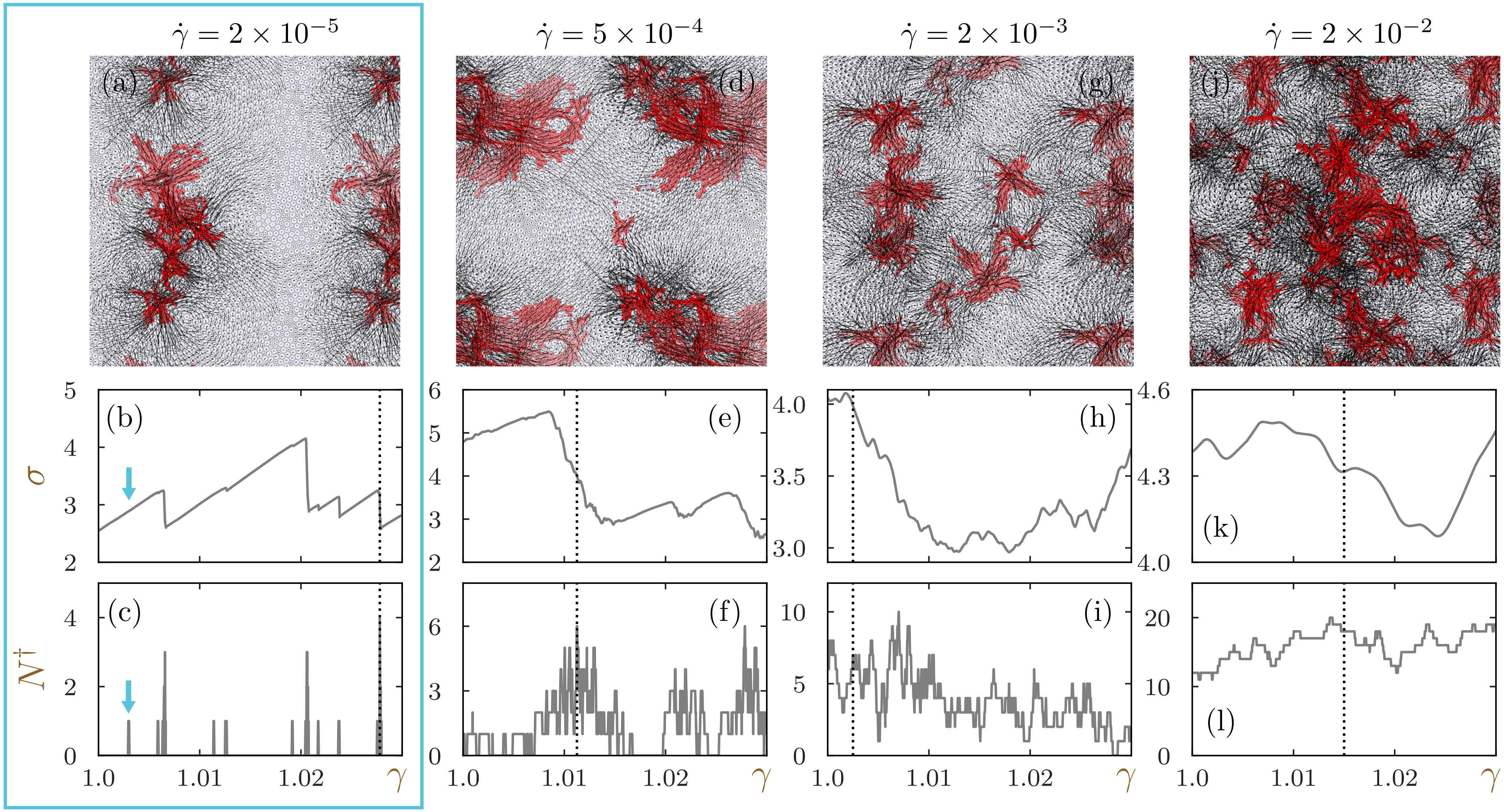
At very slow rate: $\dot{\gamma} = 2 \times 10^{-5}$



- Mode crossing zero: ST-inducing eigenvector -> phonon
- Negative normal mode corresponds to "active" ST

Number of Negative Modes $N^\dagger$

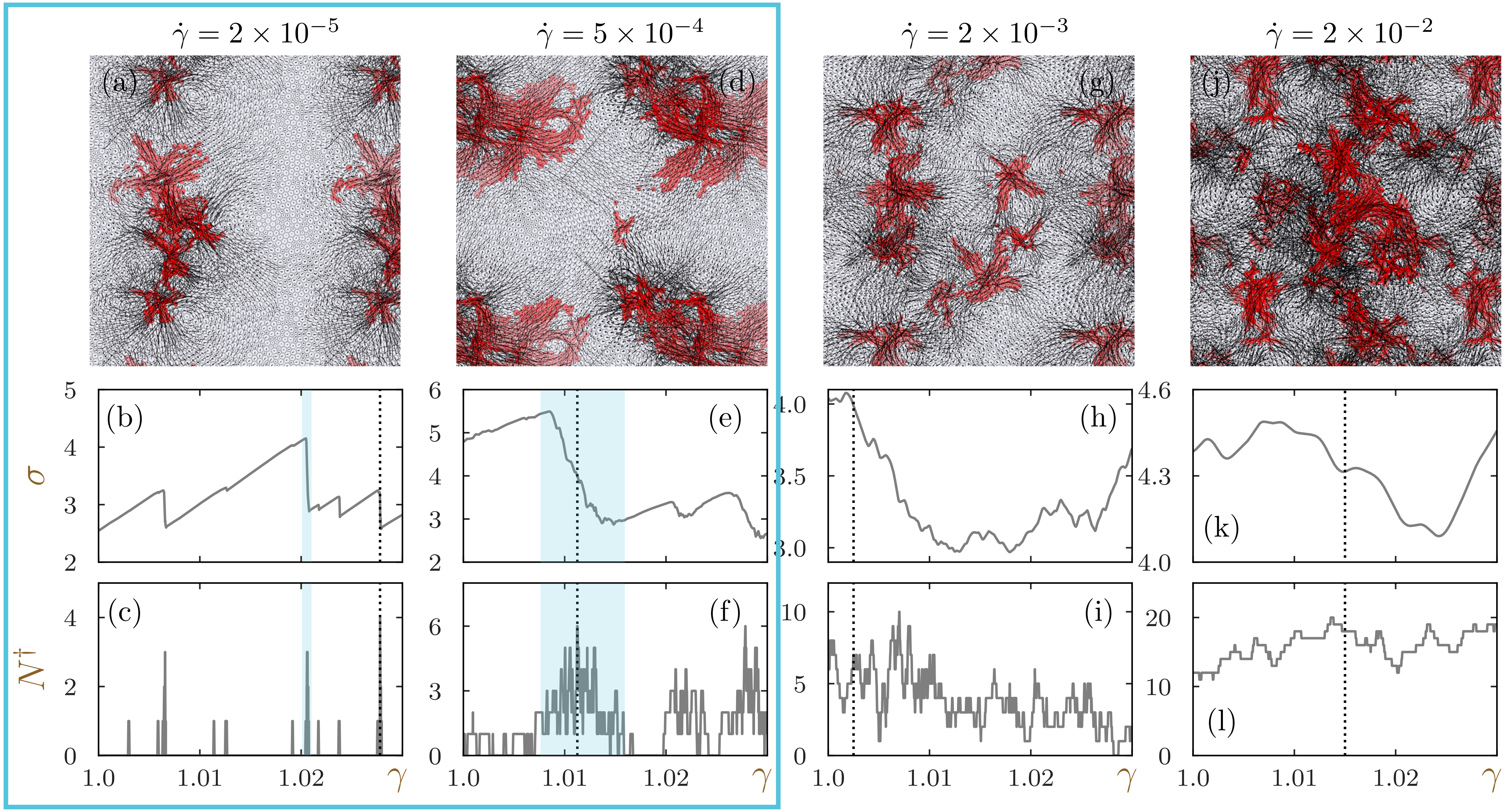
- Shear rate dependent change in morphology of avalanches-



- Sometimes form avalanches: large stress drop and $N^\dagger > 1$

Number of Negative Modes $N^\dagger$

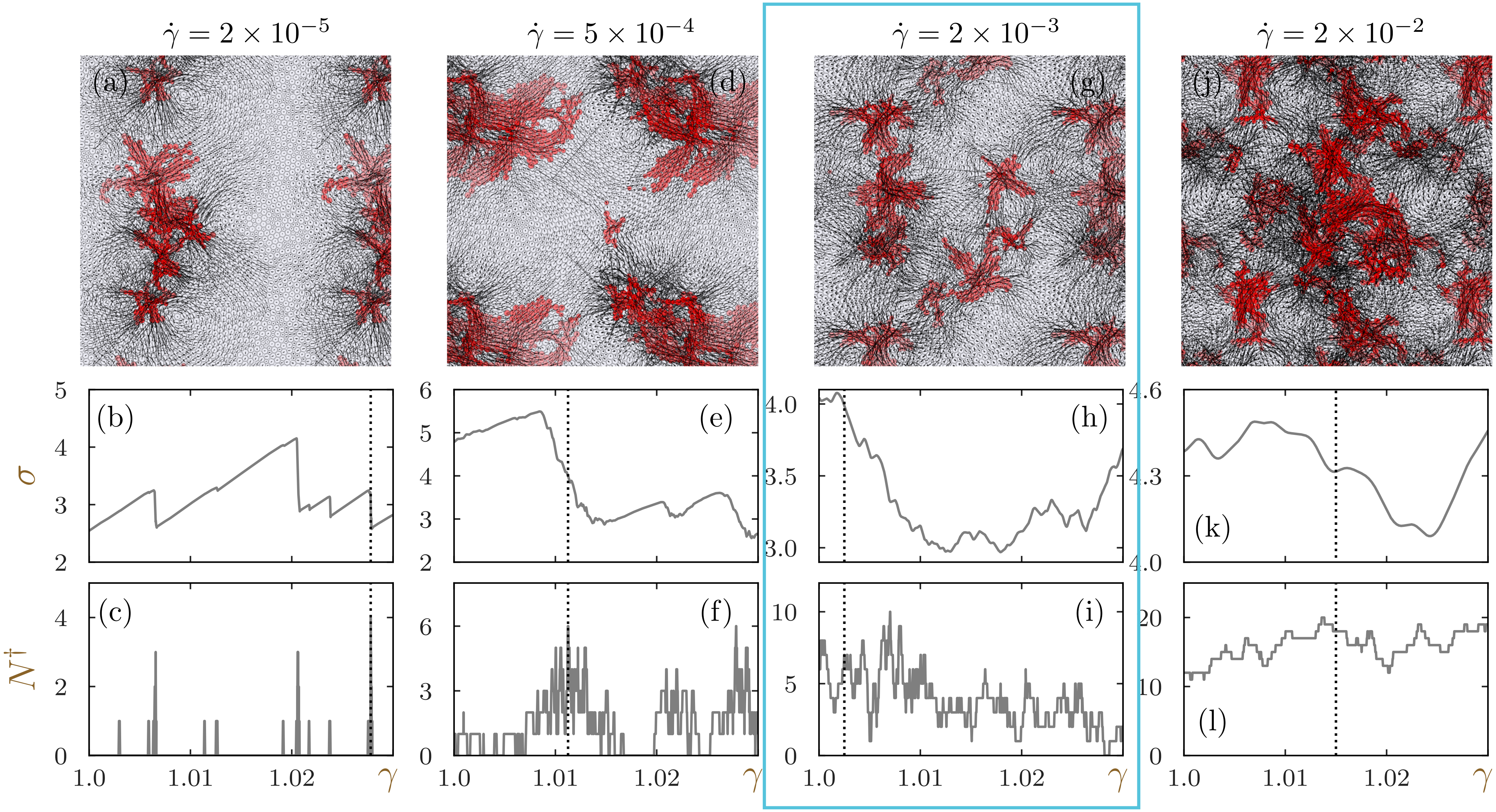
- Shear rate dependent change in morphology of avalanches-



▶ Stress drop events span broad (abscissa : γ, not t)

Number of Negative Modes $N^\dagger$

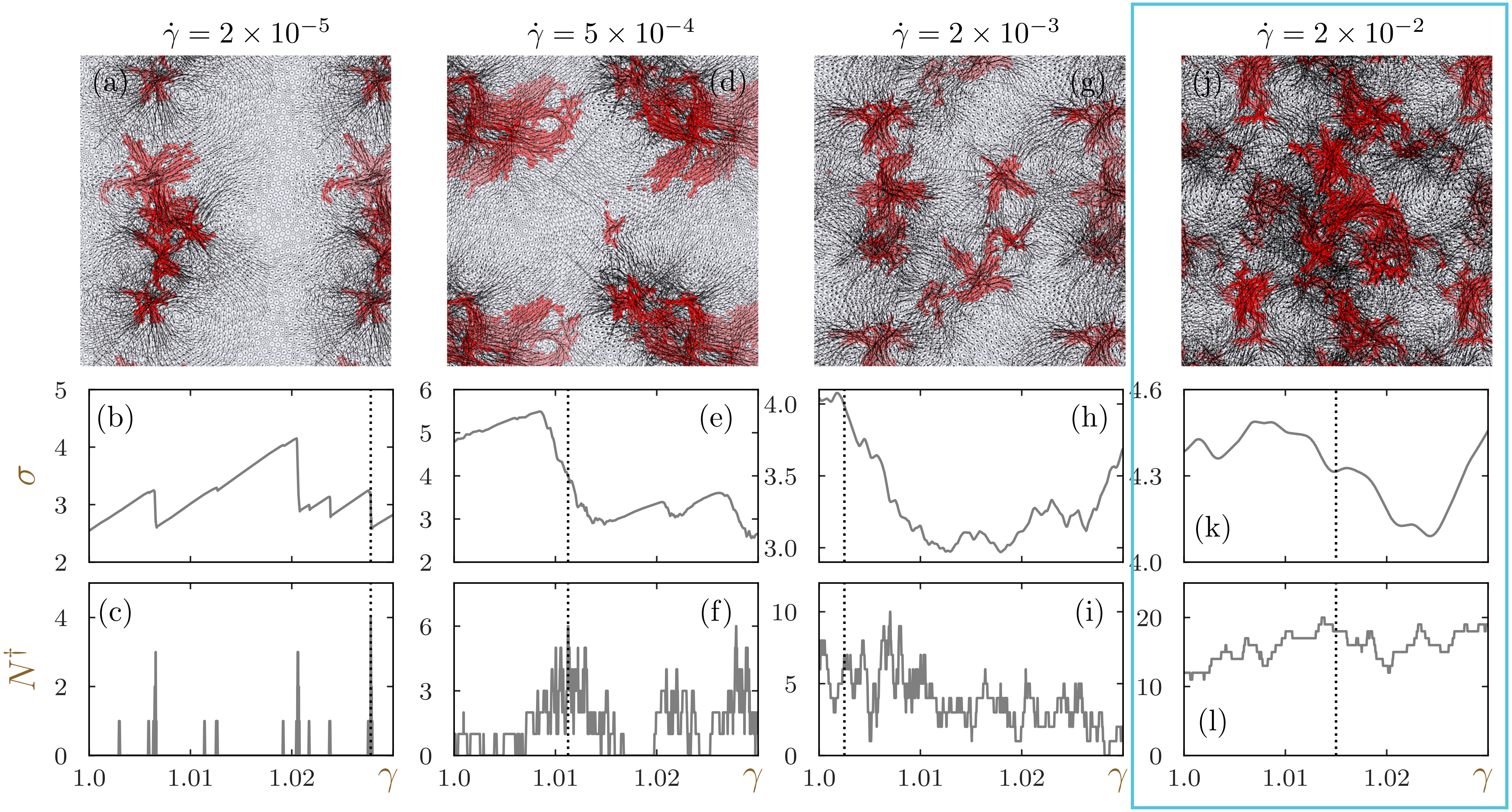
- Shear rate dependent change in morphology of avalanches-



▶ Multiple avalanches (cause of decrease in ξ)

Number of Negative Modes $N^\dagger$

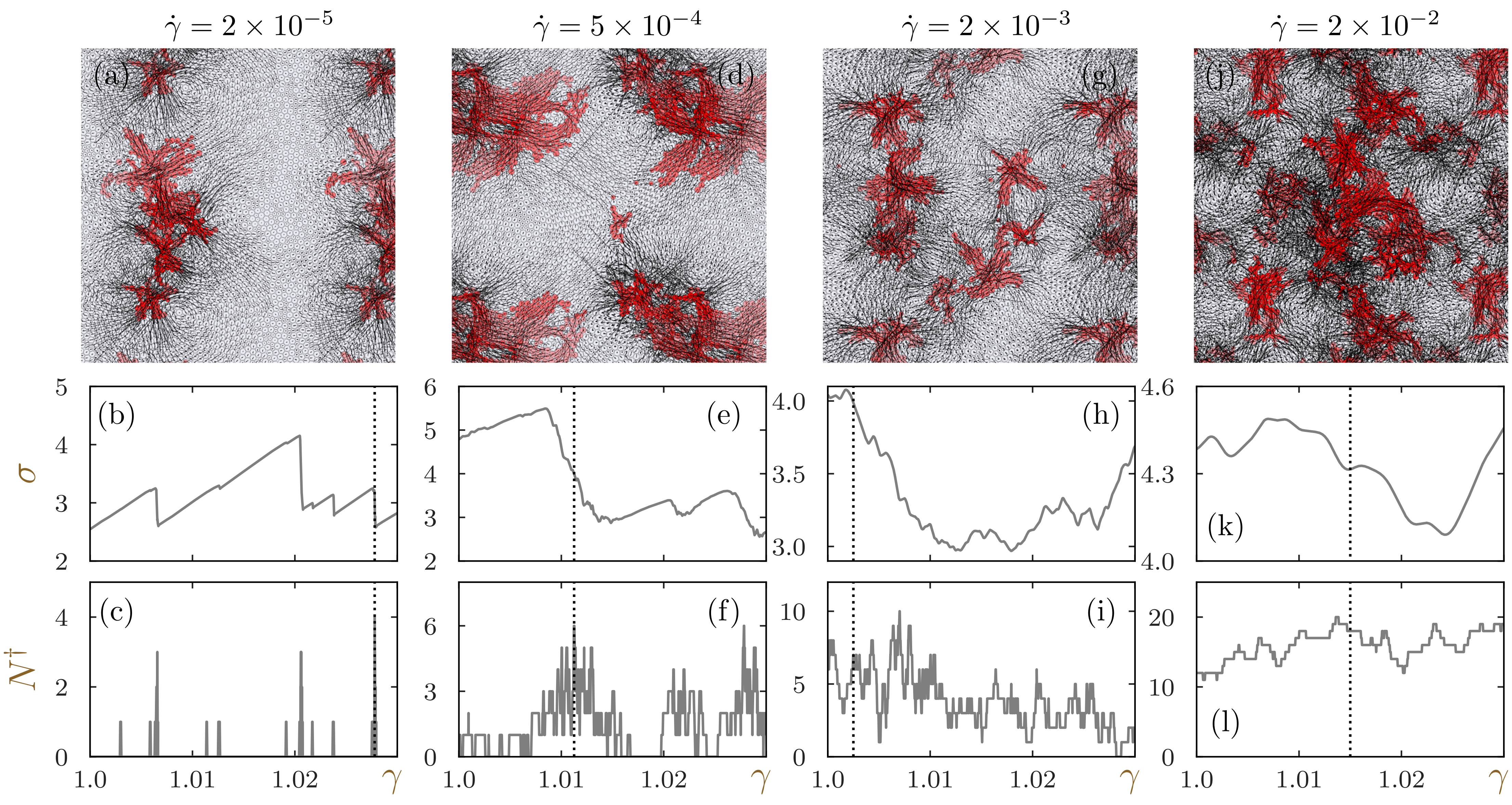
- Shear rate dependent change in morphology of avalanches-



▶ Compicated spatial pattern (avalanches cannot be identified)

Number of Negative Modes $N^\dagger$

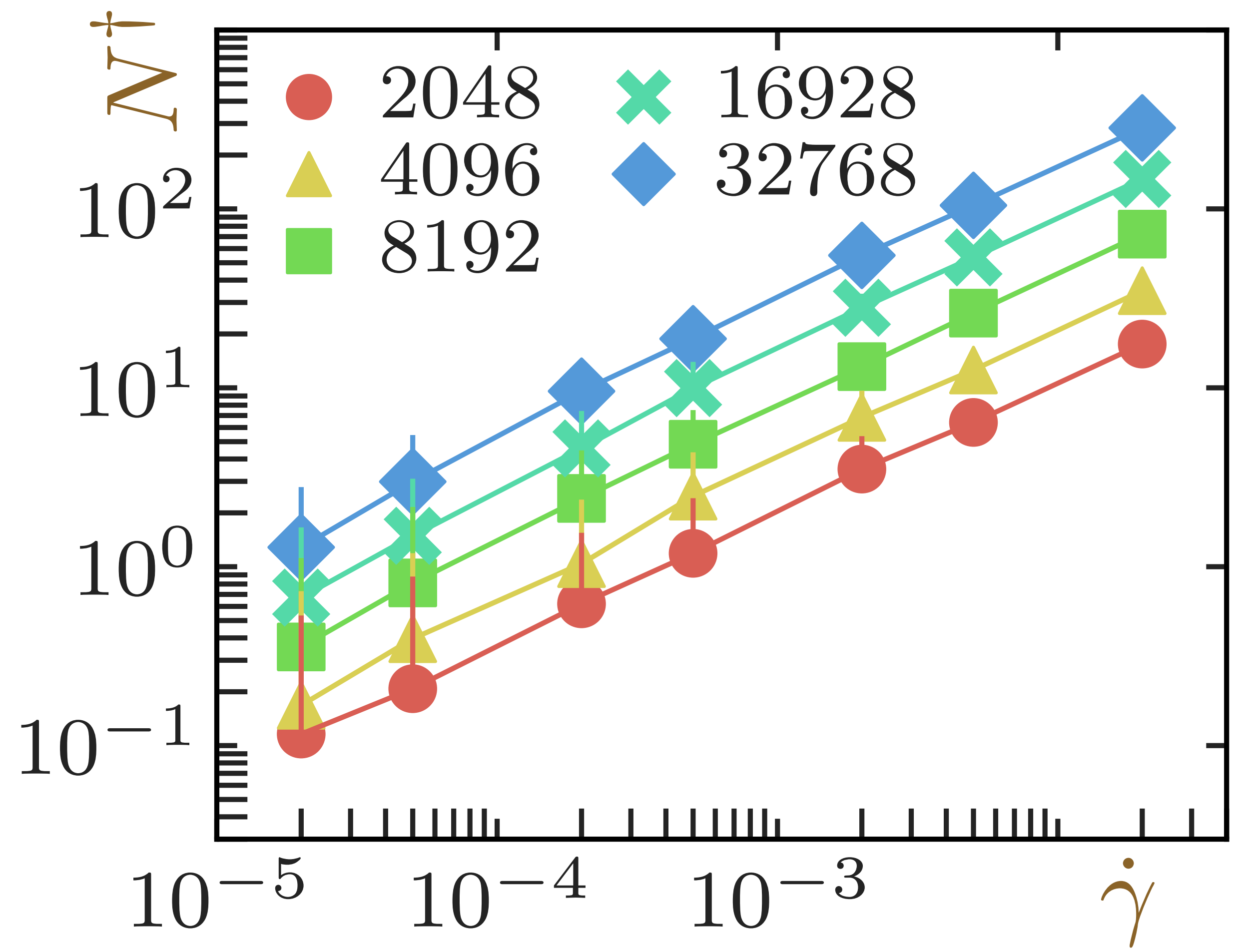
- Shear rate dependent change in morphology of avalanches-



▶ Information of shape and number of avalanches is encoded in $N^\dagger$?

Statistics of Number of Negative Modes $N^\dagger$

- Structural information about plasticity -

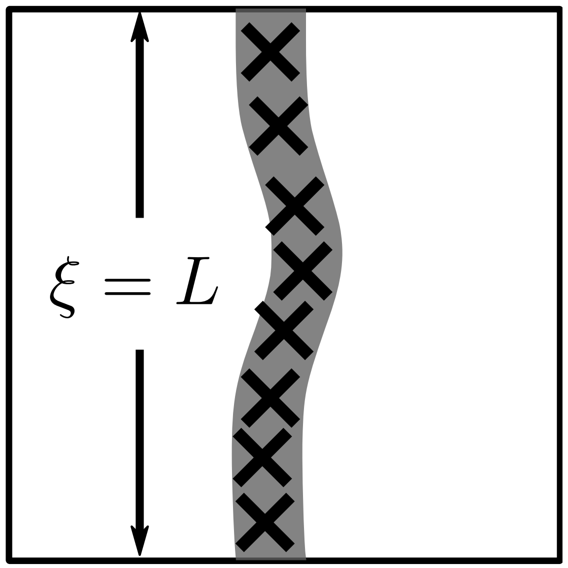


▶ Power-law function of $\dot{\gamma}$?

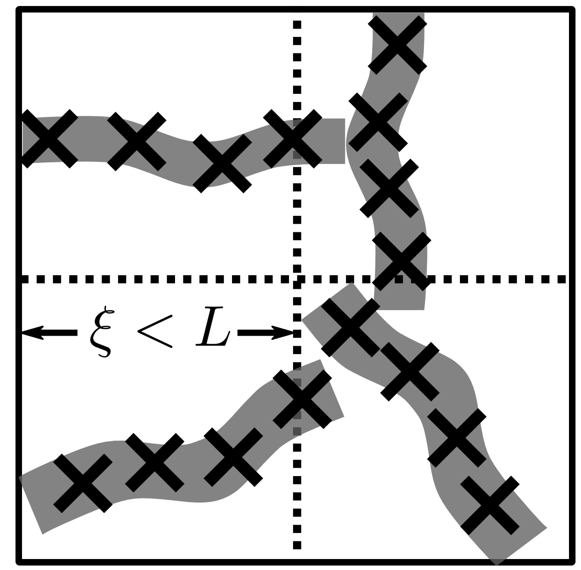
Scaling Estimation of $N^\dagger$

- Plastic "structures" indicate mechanics -

$\xi = L$



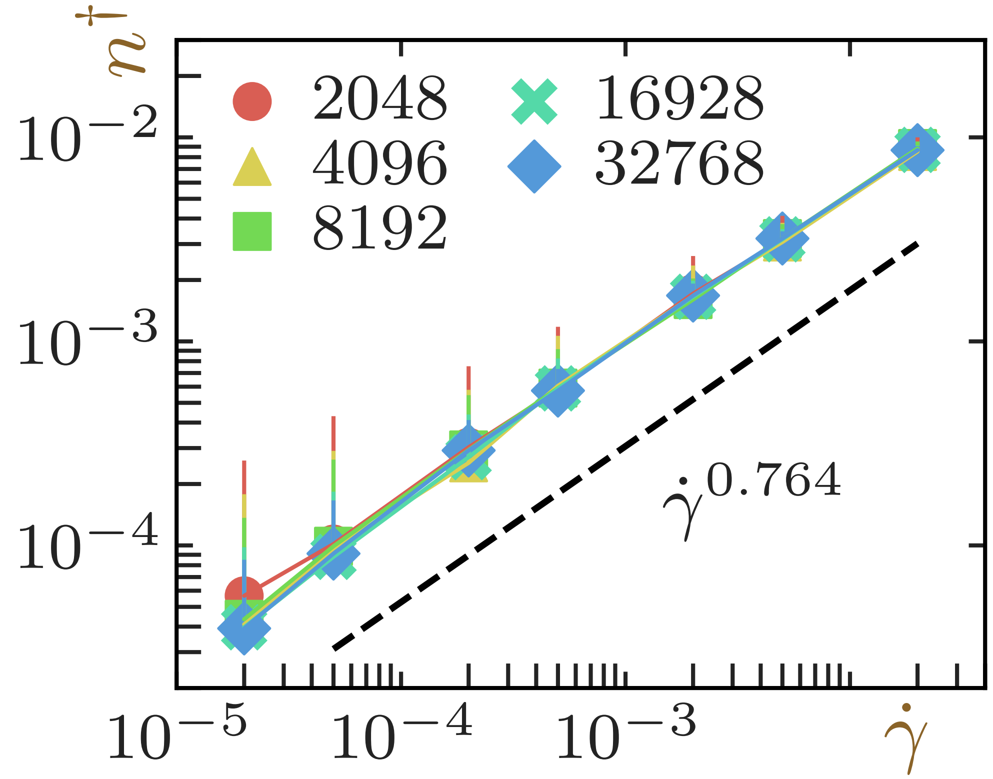
$\xi < L$



- # of avalanches : $N_{\mathrm{ava}} \sim L^d/\xi^d$
  White and Dahmen, PRL **91**, 085702 (2003), Lin et al., PNAS **111**, 14382 (2014)
- # of STs per avalanche : $N_{\mathrm{ST/ava}} \sim \xi^{d_f}$
- $N^\dagger = N_{\mathrm{ava}} \times N_{\mathrm{ST/ava}} \sim L^d \xi^{d_f - d} \sim N\dot{\gamma}^{1/\beta}$
- Number density: $n^\dagger \equiv N^\dagger/N \sim N^0 \dot{\gamma}^{1/\beta}$

Number Density of Negative Modes $n^\dagger$

- Validation of scaling law 1 -



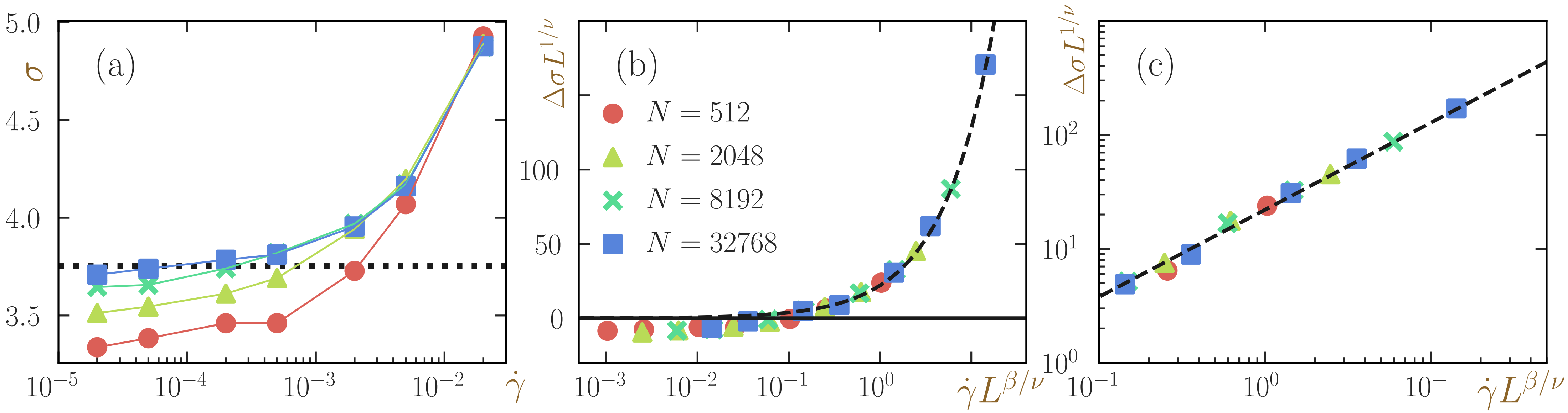
$$n^\dagger \sim N^0 \dot\gamma^{1/\beta}$$

- $n^\dagger$ for different N are collapsed as predicted!
- β is determined to be $\beta \approx 1/0.764 \approx 1.31$

Scaling Collapse of Flow Curves

- Validation of scaling law 2 -

Unscaled plot | Scaled semi-log plot | Scaled log-log plot



$$\dot\gamma \sim \Delta\sigma^{\beta} f(\Delta\sigma/\Delta\sigma_0(L))$$

$$\sim L^{-\beta/\nu}(\Delta\sigma \cdot L^{1/\nu})^{\beta} f(\Delta\sigma L^{1/\nu})$$

$$\Leftrightarrow \dot\gamma L^{\beta/\nu} \sim (\Delta\sigma \cdot L^{1/\nu})^{\beta} f(\Delta\sigma L^{1/\nu})$$

- Beautiful collapse with β obtained by negative NM analysis
- Negative modes serve as the structural signature of HB law

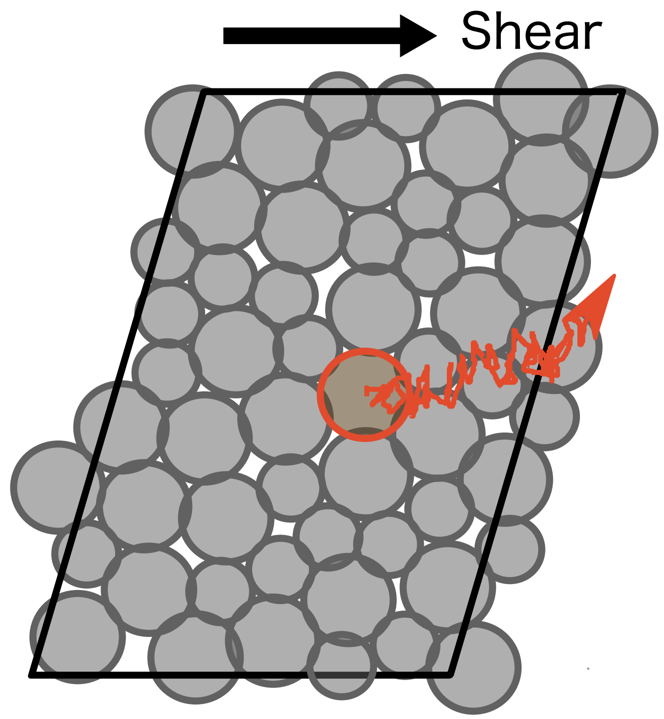
Effect of Structural Failure: Dynamical Aspect

- A distinct correlation length emerges? -

Oyama, Kawasaki, Kim, and Mizuno, in preparation

Effective Diffusion under Shear

- Purely mechanically induced diffusivity -



- Diffusive motion in perpendicular direction to shear
- Quantified by mean squared displacements (MSD) $\Delta^{\perp}$:

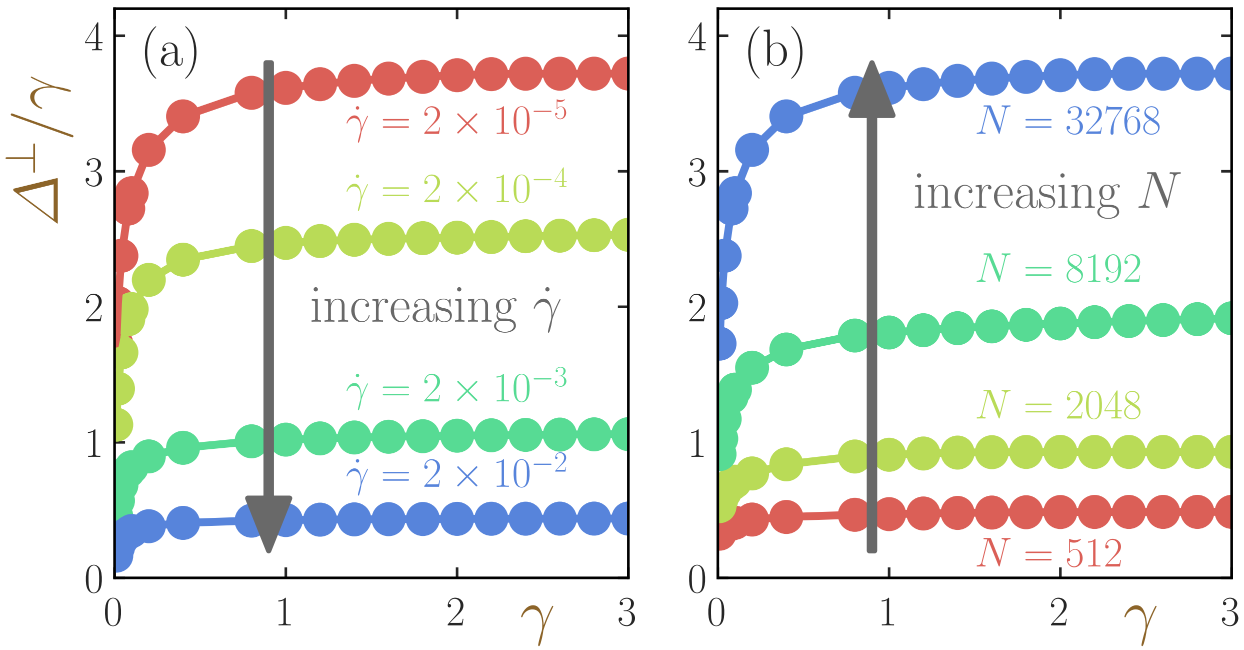
$$\Delta^{\perp}(t) \equiv \frac{1}{N} \sum_i \langle y_i(t_0+t) y_i(t_o) \rangle_{t_0}$$

Mean Squared Displacements

- Shear-rate dependent dynamical response -

Constant N ($N = 32768$)

Constant $\dot{\gamma}$ ($\dot{\gamma} = 2 \times 10^{-5}$)

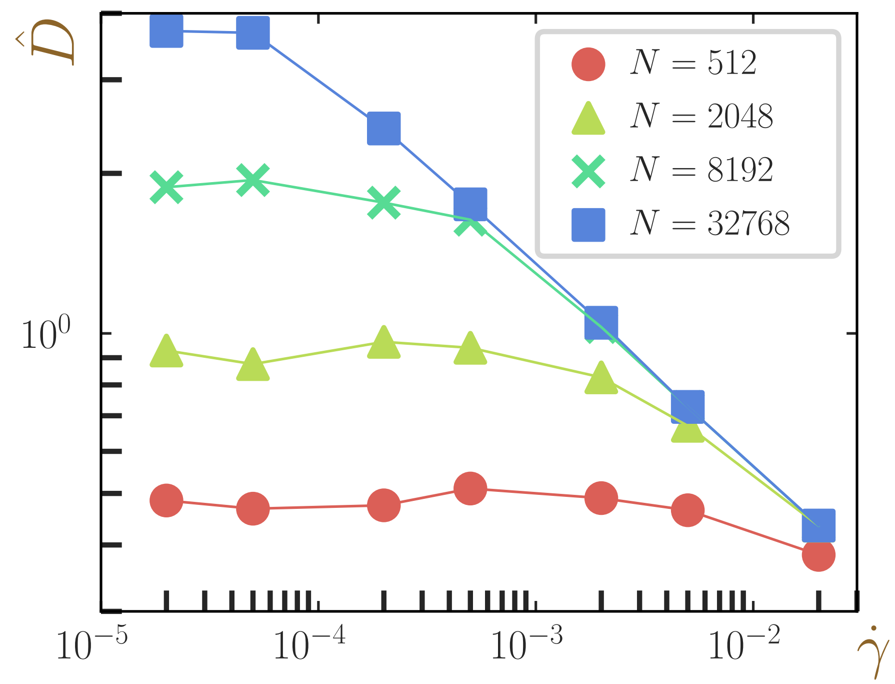


- $\Delta^{\perp}$ decreases with shear rate $\dot{\gamma}$
- $\Delta^{\perp}$ increases with system size N
- Quantified by per-strain diffusion constant

$$\hat{D} \equiv D^{\perp}/\dot{\gamma} = \lim_{\gamma\to\infty} \Delta^{\perp}/\gamma$$

Per-strain Diffusion Constant

- Total parameter dependence -



- $\hat{D}$ decreases with shear rate as: $\hat{D}(\dot{\gamma} \gg 0) \sim \dot{\gamma}^{-\upsilon}$?
- $\hat{D}$ increases with system size as: $\hat{D}_0 \equiv \hat{D}(\dot{\gamma} \to 0) \sim L$
- Consistent with a previous work

Lemaître and Caroli, Phys. Rev. Lett. **103**, 065501 (2009)

Summary

- Take-home messages -

☑ Structural failures of glasses: yielding criticality

- ▶ Closely related to marginal stability

☑ Universal rheological law: Herschel-Bulkley law

- ▶ Governed by the correlation length ξ

☑ Shear-induced self-diffusion dynamics active STs

- ▶ Can be governed by another length

  (depending on dissipation mechanism)

Overview

- What should be done next? -

- Checking the "universality" of the finidng
  - Potentials, dissipation sources, ...
- Taking into account thermal fluctuations
  - Phase diagram gets complicated
- Thermal relaxation without shear?
  - Is there any connection?