

# Mathematical Characteristics of ROXY Index (I): Distance and Reversed Distance Used as Weighing Factors

Tatsuhiko Kawashima\*

Noriyuki Hiraoka\*

## Contents

- 1 Introduction
  - 2 Spatial-cycle Frameworks: One Original Version and Two Outgrowth Versions
  - 3 Terminological Conventions: Concentration and Centralization
  - 4 ROXY Index: For Spatial Concentration and Deconcentration
  - 5 Two Types of ROXY Indices  $R_d$  and  $R_r$ : For Spatial Centralization and Decentralization
  - 6 Functional Relationship:  $R_d$  (ROXY Index with Distance as Weighing Factor) and  $R_r$  (ROXY Index with Reversed Distance as Weighing Factor)
  - 7 Empirical Results for  $R_d$  and  $R_r$ : Spatial-cycle Paths of Five Railway-line Regions in the Tokyo Metropolitan Area
  - 8 Conclusion
- Notes  
References  
Appendix

## Abstract

There are two types of ROXY indices which have been developed for studies on the phenomena of centralization and decentralization of population and other socio-economic activities in a large metropolitan area. These ROXY indices are  $R_d$  and  $R_r$ . We calculate  $R_d$  by use of a CBD distance as its weighing factor, and  $R_r$  by use of a reversed CBD distance as its weighing factor. This paper, after discussing principal features of  $R_d$  and  $R_r$ , theoretically examines the mathematical relationship between  $R_d$  and  $R_r$ , and concludes that the ratio of  $R_d$  to  $R_r$  is constant. It then empirically compares the value of  $R_d$  with that of  $R_r$  for each of five railway-line regions within the Tokyo metropolitan area to show how the spatial-cycle path of each railway-line region can be represented through  $R_d$  and  $R_r$  respectively. The results of the empirical investigation on the relationship between  $R_d$  and  $R_r$  are found, as expected, to be consistent with the theoretical conclusion drawn from our mathematical examination. Based on the above theoretical and empirical considerations, it is suggested that using  $R_r$  would appear to be a better choice than using  $R_d$  when we want to apply the ROXY-index method to a series of spatial-cycle studies for the investigation of both intra-metropolitan and inter-metropolitan redistribution processes of socio-economic activities.

## Key Words

Centralization, Concentration, Decentralization, Deconcentration, Klaassen,  
Metropolitan area, ROXY index, Spatial-cycle, Tokyo, Urban change

---

\* Kawashima is associated with the Economics Department of Gakushuin University in Tokyo, and Hiraoka with the Social Systems Department of Mitsubishi Research Institute in Tokyo. Kawashima gratefully acknowledges the research support from the Grant-in-Aid for General Scientific Research of the Ministry of Education, Science and Culture. Both authors are indebted to Masumi Morita for her diligent work in typing the original manuscript and to Melanie Mortimer for her editorial suggestions.

## 1 Introduction

Researches in the field of regional and urban economics, as occasion demands, require systematic considerations of changes in spatial distribution patterns of socio-economic activities. Among useful scientific instruments for the investigation of spatial redistribution processes, is the spatial-cycle hypothesis. It argues for the existence of a tendency for spatial redistribution processes to recurrently follow ups-and-downs in four types of major transmuting stages. For the purpose of empirically testing the magnitude of the adequateness of the spatial-cycle hypothesis and of quantitatively analyzing the phenomena of urban changes in a broader sense, the ROXY index was proposed and has been developed since the end of the 1970s.

This paper focuses upon the following five elements concerning the ROXY index method. The first is on three different versions of the spatial-cycle framework: the original version and two outgrowth versions (Section 2). The second element is on the conceptual definitions for spatial concentration and spatial centralization (Section 3). The third element is the discussion on the structural characteristics of three types of ROXY indices; One of them has been developed for inter-metropolitan analyses, while the other two types have been developed for intra-metropolitan analyses (Sections 4 and 5). The fourth element is a mathematical examination of the relationship between the two types of ROXY indices developed for intra-metropolitan analyses; One is the ROXY index whose value we calculate by use of a CBD distance as its weighing factor, and the other is the ROXY index whose value we calculate by use of a reversed CBD distance as its weighing factor (Section 6). The fifth element is an empirical study of spatial-cycle paths for five railway-line regions in the Tokyo metropolitan area, in which we apply the two types of ROXY indices developed for intra-metropolitan analyses to familiarize ourselves with the validity of the theoretical results of our mathematical examination on these two types of ROXY indices (Section 7).

## 2 Spatial-cycle Frameworks: One Original Version and Two Outgrowth Versions

A total of eleven large SMSAs<sup>1)</sup> in the USA experienced a net population loss<sup>2)</sup> in the first half of the 1970s. This phenomenon of net population loss of large metropolitan areas<sup>3)</sup>, which is referred to as disurbanization, presents a striking contrast to the continuous growth of population in large metropolitan areas, with minor exceptions, observed in the US before the 1970s. It should be noted, however, that the “disurbanization did not surge abruptly on the US urban system without any warning signs.”<sup>4)</sup> In fact, the decrease in population of central cities of large metropolitan areas “served as a key omen of its (i.e., disurbanization’s) approach.”<sup>5)</sup>

With this in mind, Klaassen and his collaborative research scholars started to conduct in the middle of the 1970s in Vienna, Austria, extensive empirical studies concerning the dynamic processes of urbanization and suburbanization in a number of relatively large-sized cities in both East and West European countries. A major outcome of their investigation was the conceptualization of the spatial-cycle framework<sup>6)</sup> which is a hypothesis useful for

describing, with reasonably systematic and scientific exactitude, possible tendencies of urban growth and decline. Their hypothetical framework argues that metropolitan areas having relatively large populations tend to follow, as indicated by Table 1, four major recurring transmuting stages with respect to the spatial redistribution pattern of population and other socio-economic activities. The four stages are *urbanization*, *suburbanization*, *counter-urbanization* (or *disurbanization*) and *reurbanization*, each of which is composed of two sub-stages as also shown in Table 1.

Klaassen's original hypothesis relied upon the application of an *absolute level* of increment or decrement of population to identify each of the four cyclical stages. From this original version of the spatial-cycle framework, we have derived two outgrowth versions of spatial-cycle frameworks, both of which follow the cyclical pattern unique to Klaassen's original framework, by applying the *growth ratio*, in place of the absolute level of change in population. One framework has been derived for the intra-metropolitan analyses as illustrated in Table 2, and the other for the inter-metropolitan analyses as illustrated in Table 3<sup>7</sup>. These two outgrowth versions of Klaassen's spatial-cycle framework play key roles in the present paper to discuss cyclical aspects of urban changes.

**Table 1 Spatial-cycle Framework for a Metropolitan Area (Klaassen's Original Version: Spatial-cycles in terms of Absolute Level of Growth and Decline)**

Transmuting stage	Sub-stage	Change in absolute level of population <sup>(1)</sup>			
		Center <sup>(2)</sup> ( $\Delta X$ )	Suburbs <sup>(3)</sup> ( $\Delta Y$ )	Relative size between $\Delta X$ and $\Delta Y$	Metropolitan area as a whole
Urbanization	First half	+	-	$\Delta X > \Delta Y$	+
	Second half	+	+		
Suburbanization	First half	+	+	$\Delta X < \Delta Y$	+
	Second half	-	+		
Counter-urbanization	First half	-	+	$\Delta X < \Delta Y$	-
	Second half	-	-		
Reurbanization	First half	-	-	$\Delta X > \Delta Y$	-
	Second half	+	-		

*Notes*

- (1) Plus and minus signs indicate population increase and decrease respectively.
- (2) The center of a metropolitan area conceptually accords with its core, central city, central part, or inner-ring zone.
- (3) The suburbs of a metropolitan area conceptually accords with ring, outskirts of its center, ring, or outer-ring zone.

*Source:* Constructed from Klaassen *et al.* (pp. 8ff, 1981).

**Table 2 Spatial-cycle Framework for a Metropolitan Area: Spatial-cycles in terms of Growth Ratio**

Transmuting stage	Value relationship of GRCN <sup>(1)</sup> and GRSB <sup>(2)</sup>	Increase or decrease in the value of GRCN/GRSB
Urbanization	GRCN > GRSB	Decreasing
Suburbanization	GRCN < GRSB	Decreasing
Counter-urbanization	GRCN < GRSB	Increasing
Reurbanization	GRCN > GRSB	Increasing

*Notes*

- (1) GRCN : Annual growth *ratio* of population in the center of a metropolitan area. The annual growth *ratio* is defined as  $x^{t+1}/x^t$  where  $x^t$  is population level in year  $t$ .
- (2) GRSB : Annual growth *ratio* of population in the suburbs of a metropolitan area.

**Table 3 Spatial-cycle Framework for a System of Metropolitan Areas: Spatial-cycles in terms of Growth Ratio**

Transmuting stage	Value relationship of GRLM <sup>(1)</sup> and GRSM <sup>(2)</sup>	Increase or decrease in the value of GRLM/GRSM
Urbanization	GRLM > GRSM	Decreasing
Suburbanization	GRLM < GRSM	Decreasing
Counter-urbanization	GRLM < GRSM	Increasing
Reurbanization	GRLM > GRSM	Increasing

*Notes*

- (1) GRLM : Annual growth *ratio* of population of a group of metropolitan areas (which are all in a system of metropolitan areas) with larger population sizes. The annual growth *ratio* is defined as  $x^{t+1}/x^t$  where  $x^t$  is population level in year  $t$ .
- (2) GRSM : Annual growth *ratio* of population of a group of metropolitan areas (which are all in a system of metropolitan areas) with medium and smaller population sizes.

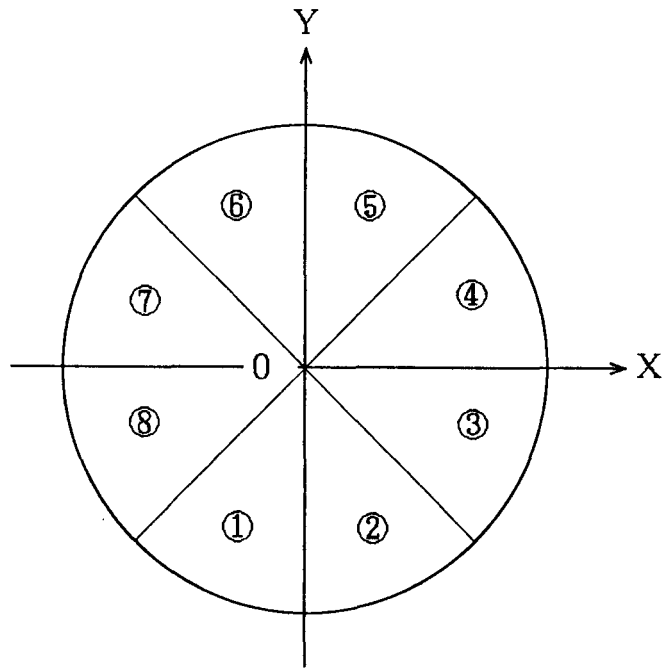
Meanwhile, the intermediate steps which link Klaassen's original hypothesis with our two outgrowth versions of his spatial-cycle framework are as follows.

- (1) From the original version of Klaassen's spatial-cycle framework expressed in Table 1, and through an experimental attempt of the *ideation by analogy*, we may depict a conceptual framework as illustrated by Figure 1 in which the annual growth *rate* of population is applied in place of the absolute level of increment or decrement of population. The spatial-cycle stages for intra-metropolitan analyses and that for inter-metropolitan analyses are both represented in this figure.
- (2) From Figure 1 it would be perhaps reasonable for us to derive, through another experimental attempt of the ideation by analogy, the conceptual framework illustrated by Figure 2. In this figure the annual growth *ratio* of population is applied (instead of growth rate of population) where the growth ratio of population for the period between two years  $t$  and  $t+1$  is defined as the ratio of the population of year  $t+1$  to that of year  $t$ . The spatial-cycle stages for intra-metropolitan analyses and that for inter-metropolitan analyses are both represented in this figure<sup>9)</sup>.
- (3) A logical argument derived from the contents carried by Figure 2 would lead us to the construction of Tables 2 and 3, for intra-metropolitan analyses and inter-metropolitan analyses respectively.

### 3 Terminological Conventions: Concentration and Centralization

The term *urban change*, which is frequently used in the study of urban and regional economics, refers to two aspects of *spatial significance*. These are *urbanization* and *suburbanization*. Urbanization and suburbanization each carry two different conceptual facets. The first facet is associated with the spatial redistribution processes of socio-economic activities (such as population reflecting residential activities) among metropolitan areas<sup>9)</sup> in an urban system. The second facet is associated with the spatial redistribution processes of population within a metropolitan area delineated as a functional urban region<sup>10)</sup>. For either facet, the primary attention of studies on urban changes usually centers around the distinct spatial shifts of population between relatively *densely-populated areas* and *sparsely-populated areas*.

The multifaceted nature of the term urban change is accordingly apt to confuse our discussion when we are involved in urban and regional analyses in general. For the purpose of avoiding unnecessary ambiguity in applying terminologies related to urban change, we will employ four basic terms. These terms are spatial *concentration*, *deconcentration*, *centralization* and *decentralization* of population, as illustrated by Category I of Table 4. The concept of concentration is one type of urbanization, and accords with the notion of inter-metropolitan agglomeration and that of convergence towards larger metropolitan areas<sup>11)</sup>. The concept of deconcentration is one type of suburbanization, and accords with the notion of inter-metropolitan deglomeration and that of divergence (or dispersion) from larger



*Notes*

(1) Notations for intra-metropolitan analyses

X: Annual growth *rate* of population in the center of a metropolitan area

Y: Annual growth *rate* of population in the suburbs of a metropolitan area

(2) Notation for inter-metropolitan analyses

X: Annual growth *rate* of population of a group of larger (in terms of population) metropolitan areas

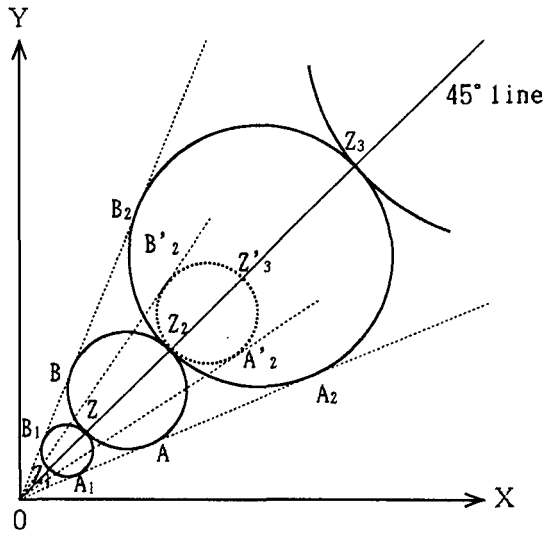
Y: Annual growth *rate* of population of a group of medium and smaller (in terms of population) metropolitan areas

(3) Spatial-cycle stages

Transmuting stages	Positions in X-Y space
Urbanization	③ and ④
Suburbanization	⑤ and ⑥
Counter-urbanization	⑦ and ⑧
Reurbanization	① and ②

**Figure 1** Spatial-cycle Framework for Intra-metropolitan Analyses and Inter-metropolitan Analyses: Spatial-cycles in terms of Growth Rate

Mathematical Characteristics of ROXY Index (I):  
Distance and Reversed Distance Used as Weighing Factors (Kawashima, Hiraoka)



Notes

(1) Notations for intra-metropolitan analyses

X: Annual growth *ratio* of population in the center of a metropolitan area

Y: Annual growth *ratio* of population in the suburbs of a metropolitan area

(2) Notations for inter-metropolitan analyses

X: Annual growth *ratio* of population of a group of larger (in terms of population) metropolitan areas

Y: Annual growth *ratio* of population of a group of medium and smaller (in terms of population) metropolitan areas

(3) Spatial-cycle stages

Transmuting stages	Position in X-Y space					
	Example-1	Example-2	Example-3		Example-4	
Urbanization	$A^* \rightarrow Z_2$	$A^* \rightarrow Z$	$A_1^* \rightarrow Z$		$A^* \rightarrow Z_2$	$A_2^{\circ} \rightarrow Z_2$
Suburbanization	$Z_2 \rightarrow B$	$Z \rightarrow B$	$Z \rightarrow B$		$Z_2 \rightarrow B_2$	$Z_2 \rightarrow B$
Counter-urbanization	$B \rightarrow Z$	$B \rightarrow Z_2$	$B \rightarrow Z_2^{\#}$		$B_2 \rightarrow Z_3^{\#}$	$B \rightarrow Z^+$
Reurbanization	$Z \rightarrow A^*$	$Z_2 \rightarrow A^*$	$Z_1 \rightarrow A_1^*$	$Z_2^{\#} \rightarrow A_2$	$Z^+ \rightarrow A^*$	$Z_3^{\#} \rightarrow A_2^{\circ}$

(The symbols of \*, #, ©, and + in each column, refer to the continuative direction of the spatial-cycle paths. Among other possible spatial-cycle paths would be  $Z_1 \rightarrow A_1 \rightarrow Z \rightarrow B \rightarrow Z_2 \rightarrow A_2 \rightarrow Z_3 \rightarrow B_2 \rightarrow Z_2 \rightarrow A \rightarrow Z \rightarrow B_1 \rightarrow Z_1$ .)

Figure 2 Spatial-cycle Framework for Intra-metropolitan Analyses and Inter-metropolitan Analyses: Spatial-cycles in terms of Growth Ratio

**Table 4 Terminological Exactitude or Inexactitude?: Five Categories of Technical Terms on Spatial Redistribution Patterns of Population**

Category I		Category II	Category III	Category IV	Category V
Concentration, deconcentration, centralization, and decentralization		Agglomeration and deglomeration	Convergence and divergence	Urbanization and suburbanization	Reurbanization, suburbanization, and counter-urbanization
Spatial concentration	Accelerating concentration	Inter-metropolitan agglomeration	Convergence towards larger metropolitan areas	Urbanization	Reurbanization
	Decelerating concentration				Urbanization
Spatial deconcentration	Accelerating deconcentration	Inter-metropolitan deglomeration	Divergence from larger metropolitan areas	Suburbanization	Suburbanization
	Decelerating deconcentration				Counter-urbanization
Spatial centralization	Accelerating centralization	Intra-metropolitan agglomeration	Convergence within a metropolitan area towards its center	Urbanization	Reurbanization
	Decelerating centralization				Urbanization
Spatial decentralization	Accelerating decentralization	Intra-metropolitan deglomeration	Divergence within a metropolitan area from its center	Suburbanization	Suburbanization
	Decelerating decentralization				Counter-urbanization

*Note* Category V in this table has been arranged based on terminologies coined by L. Klaassen in his original spatial-cycle framework.



metropolitan areas to medium and smaller metropolitan areas. The concept of centralization is another type of urbanization, and accords with the notion of intra-metropolitan agglomeration and that of convergence to the central part of a metropolitan area. The concept of decentralization is another type of suburbanization, and accords with the notion of intra-metropolitan deglomeration and that of divergence (or dispersion) from the center of a metropolitan area to its suburbs.

In this table, we divide each of the four basic terms into two subcomponents. One subcomponent corresponds to the state of acceleration in speed of spatial redistribution processes, and the other to the state of decelerating speed. Consequently, we have eight subcomponents in the column of Category I. They are (i) accelerating concentration, (ii) decelerating concentration, (iii) accelerating deconcentration, (iv) decelerating deconcentration, (v) accelerating centralization, (vi) decelerating centralization, (vii) accelerating decentralization, and (viii) decelerating decentralization.

In Category V of Table 4 which represents Klaassen's original version of the spatial-cycle framework, the subcomponents (ii) and (vi) of Category I correspond to the urbanization stage, subcomponents (iii) and (vii) to the suburbanization stage, subcomponents (iv) and (viii) to the counter-urbanization<sup>12)</sup> stage, and subcomponents (i) and (v) to the reurbanization stage.

By integrating these definitions into our discussion below, we can make more accurate applications of the technical terms specified under Category I in Table 4.

#### 4 ROXY Index: For Spatial Concentration and Deconcentration

The ROXY index is a comprehensive measure that would be useful for quantitative analyses of the spatial-cycle phenomena in general. This index has been proposed and developed by Kawashima<sup>13)</sup> as an analytical instrument to identify spatial-cycle stages for (i) a system of metropolitan areas and (ii) a metropolitan area, whereby the basic characteristics of the ROXY index can be consistent with the urban change concepts described by Tables 2, 3, and 4, and Figure 2.<sup>14)</sup>

Table 5 furnishes the definition of the ROXY index. This definition is one for a ROXY index which would be useful for inter-metropolitan analyses, namely, for studying the phenomena of spatial concentration and deconcentration<sup>15)</sup>. In defining this type of ROXY index, the population of each metropolitan area is employed as the weighing factor necessary for the calculation of the value of the ROXY index<sup>16)</sup>.

Implications of the values of this type of ROXY index are summarized in Table 6. It can be seen from this table, that the value of the ROXY index is positive if the population is spatially concentrating, negative if the population is spatially deconcentrating, and zero if the spatial redistribution pattern of population is neutral from the movements of both concentration and deconcentration. Among major causes of this neutrality are the balanced, bell-shaped and cup-shaped growths or declines in population, as described in the notes to Table 6.

In the stage of concentration, the value of the ROXY index increases if the speed of

concentration is accelerating, remains the same for a constant speed of concentration, and decreases for a deceleration in the speed of concentration. In the stage of deconcentration, the value of the ROXY index decreases, remains the same, or increases, when the speed of deconcentration accelerates, stays constant, or decelerates respectively. For the neutral situation, the value of the ROXY index increases from zero at the onset of accelerating concentration, remains zero for the continuation of the neutral situation, and decreases from zero at the onset of accelerating deconcentration.

As to the above implications of the ROXY index, it should be noted that conditions appearing in column (i) of Table 6 are *necessary* conditions for their corresponding phenomena listed in column (ii), and that conditions appearing in column (iii) are also *necessary* conditions for their corresponding phenomena listed in column (iv). A clear understanding of these relations should be born in mind for the discussion in the next section.

**Table 5 Definition of ROXY Index: With Metropolitan Population Used as Weighing Factor**

$$\begin{aligned} \text{ROXY Index} &= \left( \frac{WAGR_{t,t+1}}{SAGR_{t,t+1}} - 1.0 \right) \times 10^4 \\ &= \left\{ \frac{\sum_{i=1}^n (x_i^t \times r_i^{t,t+1})}{\sum_{i=1}^n x_i^t} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \right\} \times 10^4 \end{aligned}$$

where

- $x_i^t$  : population of metropolitan area  $i$  in year  $t$
- $r_i^{t,t+1}$  : annual growth *ratio* of population in metropolitan area  $i$  for the period between years  $t$  and  $t+1$ , which is defined as the  $k$ -th root of  $x_i^{t+k}/x_i^t$
- $n$  : number of metropolitan areas
- $WAGR_{t,t+1}$  : weighted average of annual growth *ratios* of population in  $n$  metropolitan areas for the period between years  $t$  and  $t+1$ , which is equal, in case population level of each metropolitan area is used as the weighing factor, to

$$\frac{\sum_{i=1}^n (x_i^t \times r_i^{t,t+1})}{\sum_{i=1}^n x_i^t}$$

- $SAGR_{t,t+1}$  : simple average of annual growth *ratios* of population in  $n$  metropolitan areas for the period between years  $t$  and  $t+1$ , which is equal to

$$\frac{\sum_{i=1}^n r_i^{t,t+1}}{n}$$

**Table 6 ROXY Index for Inter-metropolitan Analyses: With Metropolitan Population Used as Weighing Factor**

(i)	(ii)	(iii)	(iv)
Value of ROXY index	Pattern of spatial redistribution of population	State of changes in value of ROXY index	Speed of spatial redistribution process of population
Positive	Concentration	Increasing	Accelerating
		Leveling-off	Constant
		Decreasing	Decelerating
Zero	Neutrality from both concentration and deconcentration ( <i>viz.</i> symmetric growth or decline <sup>(1)</sup> )	Increasing	Start of ACon <sup>(2)</sup>
		Leveling-off	Continuation of neutrality
		Decreasing	Start of ADcon <sup>(3)</sup>
Negative	Deconcentration	Increasing	Decelerating
		Leveling-off	Constant
		Decreasing	Accelerating

*Notes*

- (1) The spatial redistribution pattern of the 'symmetric growth or decline' includes the following three sub-patterns.
  - (i) Balanced growth or decline (BGD): The growth-rate curve is nearly flat, reflecting a fixed share of population by metropolitan areas.
  - (ii) Bell-shaped growth or decline (BSGD): The growth-rate curve is bell-shaped, reflecting the 'medianization' of population over metropolitan areas of different sizes in population. 'Medianization' refers to the increases in population share by metropolitan areas of medium sizes in population, accompanied by decreases in population share by metropolitan areas of smaller and larger sizes in population.
  - (iii) Cup-shaped growth or decline (CSGD): The growth-rate curve is cup-shaped, reflecting the 'bipolarization' of population over metropolitan areas of different sizes in population. 'Bipolarization' means increases in population share of smaller and larger metropolitan areas, along with decreases in population share of medium-sized metropolitan areas.
- (2) 'ACon' stands for accelerating concentration.
- (3) 'ADcon' stands for accelerating deconcentration.

## 5 Two Types of ROXY Indices $R_s$ and $R_d$ : For Spatial Centralization and Decentralization

We have learned that the ROXY index approach could assist our studies of spatial concentration and deconcentration. Consequently, we have also become interested in searching for a ROXY index which can be applied to the studies on the phenomena of spatial centralization and decentralization. Two types of ROXY indices have been developed by Kawashima to meet this demand. The first type is the ROXY index whose value we calculate by use of a CBD distance<sup>17)</sup> of each locality as the weighing factor. The formula of this type of ROXY index is delineated in Table 7, with the implications of its values as summarized in Table 8. The second type is the ROXY index whose value we calculate by use of a reversed CBD distance<sup>18)</sup> of each locality as the weighing factor. The formula of this type of ROXY index is delineated in Table 9, with the implications of its values as summarized in Table 10.

**Table 7 Definition of ROXY Index: With CBD Distance Used as Weighing Factor**

$$\begin{aligned} \text{ROXY Index} &= \left( \frac{WAGR_{t,t+1}}{SAGR_{t,t+1}} - 1.0 \right) \times 10^4 \\ &= \left\{ \frac{\sum_{i=1}^n (d_i \times r_i^{t,t+1})}{\sum_{i=1}^n d_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \right\} \times 10^4 \end{aligned}$$

where

$r_i^{t,t+1}$  : annual growth *ratio* of the population in locality  $i$  for the period between years  $t$  and  $t+1$ , which is defined as the  $k$ -th root of  $x_i^{t+k}/x_i^t$  where  $x_i^t$  is the population of locality  $i$  in year  $t$

$d_i$  : CBD distance of locality  $i$

$n$  : number of localities

$WAGR_{t,t+1}$  : weighted average of annual growth *ratios* of population in  $n$  localities for the period between years  $t$  and  $t+1$ , which is equal, in case the CBD distance of each locality is used as the weighing factor, to

$$\frac{\sum_{i=1}^n (d_i \times r_i^{t,t+1})}{\sum_{i=1}^n d_i}$$

$SAGR_{t,t+1}$  : simple average of annual growth *ratios* of population in  $n$  localities for the period between years  $t$  and  $t+1$ , which is equal to

$$\frac{\sum_{i=1}^n r_i^{t,t+1}}{n}$$

**Table 8 ROXY Index for Intra-metropolitan Analyses: With CBD Distance Used as Weighing Factor**

(i)	(ii)	(iii)	(iv)
Value of ROXY index	Pattern of spatial redistribution of population	State of changes in value of ROXY index	Speed of spatial redistribution process of population
Positive	Decentralization	Increasing	Accelerating
		Leveling-off	Constant
		Decreasing	Decelerating
Zero	Neutrality from both centralization and decentralization ( <i>viz.</i> symmetric growth or decline <sup>(1)</sup> )	Increasing	Start of ADcen <sup>(2)</sup>
		Leveling-off	Continuation of neutrality
		Decreasing	Start of ACen <sup>(3)</sup>
Negative	Centralization	Increasing	Decelerating
		Leveling-off	Constant
		Decreasing	Accelerating

*Notes*

- (1) The spatial redistribution pattern of 'symmetric growth or decline' includes the following three sub-patterns.
  - (i) Balanced growth or decline (BGD): The growth-rate curve is nearly flat, reflecting a fixed share of population by localities.
  - (ii) Bell-shaped growth or decline (BSGD): The growth-rate curve is bell-shaped, reflecting the 'medianization' of population over localities with different CBD distances. 'Medianization' refers to the increases in population share by localities with medium distances, accompanied by decreases in population share by localities with near and far distances.
  - (iii) Cup-shaped growth or decline (CSGD): The growth-rate curve is cup-shaped, reflecting the 'bipolarization' of population over localities with different CBD distances. 'Bipolarization' means increases in population share of localities with near and far distances, along with decreases in population share of localities with medium distances.
- (2) 'ADcen' stands for accelerlating decentralization.
- (3) 'ACen' stands for accelerating centralization.

Looking back upon the development processes of the ROXY index for the intra-metropolitan analyses, it is pointed out that the ROXY index with a weighing factor of CBD distance<sup>19)</sup> was conceptualized and empirically applied before the ROXY index with a weighing factor of a reversed CBD distance<sup>20)</sup>. Following the chronological order of their development, let us examine the fundamental characteristics of the two types of ROXY indices, by starting with the ROXY index having a weighing factor of a CBD distance. Table 8 indicates that the value of the ROXY index is positive, zero, or negative, when the metropolitan area under investigation is decentralizing, neutral, or centralizing respectively along its spatial-cycle path. In the stage of decentralization, the value of the ROXY index increases, remains the same, or decreases, when the speed of decentralization accelerates,

**Table 9 Definition of ROXY Index: With Reversed CBD Distance Used as Weighing Factor**

$$\begin{aligned} \text{ROXY Index} &= \left( \frac{WAGR_{t,t+1}}{SAGR_{t,t+1}} - 1.0 \right) \times 10^4 \\ &= \left\{ \frac{\sum_{i=1}^n (s_i \times r_i^{t,t+1})}{\sum_{i=1}^n s_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \right\} \times 10^4 \end{aligned}$$

where

- $r_i^{t,t+1}$  : annual growth *ratio* of the population in locality  $i$  for the period between years  $t$  and  $t+1$ , which is defined as the  $k$ -th root of  $x_i^{t+k}/x_i^t$  where  $x_i^t$  is the population of locality  $i$  in year  $t$
- $s_i$  : reversed CBD distance of locality  $i$  which is defined as  $d_{min} + d_{max} - d_i$ , where  $d_i$  is the CBD distance of locality  $i$ , and  $d_{min}$  and  $d_{max}$  respectively indicate the minimum and maximum values of  $d_i$  (for  $i=1, 2, \dots, n$ )
- $n$  : number of localities
- $WAGR_{t,t+1}$  : weighted average of annual growth *ratios* of population in  $n$  localities for the period between years  $t$  and  $t+1$ , which is equal, in case the reversed CBD distance of each locality is used as the weighing factor, to

$$\frac{\sum_{i=1}^n (s_i \times r_i^{t,t+1})}{\sum_{i=1}^n s_i}$$

- $SAGR_{t,t+1}$  : simple average of annual growth *ratios* of population in  $n$  localities for the period between years  $t$  and  $t+1$ , which is equal to

$$\frac{\sum_{i=1}^n r_i^{t,t+1}}{n}$$

**Table 10 ROXY Index for Intra-metropolitan Analyses: With Reversed CBD Distance Used as Weighing Factor**

(i)	(ii)	(iii)	(iv)
Value of ROXY index	Pattern of spatial redistribution of population	State of changes in value of ROXY index	Speed of spatial redistribution process of population
Positive	Centralization	Increasing	Accelerating
		Leveling-off	Constant
		Decreasing	Decelerating
Zero	Neutrality from both centralization and decentralization ( <i>viz.</i> symmetric growth or decline <sup>(1)</sup> )	Increasing	Start of ACen <sup>(2)</sup>
		Leveling-off	Continuation of neutrality
		Decreasing	Start of ADcen <sup>(3)</sup>
Negative	Decentralization	Increasing	Decelerating
		Leveling-off	Constant
		Decreasing	Accelerating

*Notes*

- (1) See note (1) to Table 8 for the meanings of 'symmetric growth or decline.'
- (2) 'ACen' stands for accelerating centralization.
- (3) 'ADcen' stands for accelerating decentralization.

stays constant, or decelerates respectively. In the stage of centralization, the value of the ROXY index decreases, remains the same, or increases, when the speed of centralization accelerates, stays constant, or decelerates respectively. For the neutral situation, the value of the ROXY index increases from zero at the onset of accelerating decentralization, remains zero for the continuation of the neutral situation, and decreases from zero at the onset accelerating centralization.

From the above discussion, we might conclude that the ROXY index with the weighing factor of a CBD distance would assist our studies on the phenomena of spatial centralization and decentralization. It is, however, important to notice that the sign of the value of this type of ROXY index is opposite to that of the ROXY index with the weighing factor of metropolitan population which we previously discussed as to Table 6 for the study of spatial concentration and deconcentration. Taking this fact into consideration, the ROXY index with the weighing factor of a reversed CBD distance, has been structuralized as defined in Table 9 in such a way that its value can show the same sign as the ROXY index with the weighing factor of metropolitan population. As for the ROXY index with the weighing factor of a reversed CBD distance, Table 10 indicates that its value is positive, zero, or

negative, when the metropolitan area is centralizing, neutral, or decentralizing respectively. In the stage of centralization, the value of the ROXY index increases, remains the same, or decreases, when the speed of centralization accelerates, stays constant, or decelerates respectively. In the stage of decentralization, the value of the ROXY index decreases, remains the same, or increases, when the speed of decentralization accelerates, stays constant, or decelerates respectively. For the neutral situation, the value of the ROXY index increases from zero at the onset of accelerating centralization, remains zero for the continuation of the neutral situation, and decreases for the start of accelerating decentralization.

Accordingly, it would appear that the two types of ROXY indices developed for intra-metropolitan analyses practically provide us with almost similar information on the spatial-cycle stages of a metropolitan area. The only exception would be that the signs of their values for a given stage of the spatial-cycle path are different from each other. In the next section, we talk in more detail about the relationship between the ROXY index using a CBD distance and the ROXY index using a reversed CBD distance as their respective weighing factors.

## 6 Functional Relationship: $R_d$ (ROXY Index with Distance as Weighing Factor) and $R_s$ (ROXY Index with Reversed Distance as Weighing Factor)

In this section we use the following notational conventions for our examination of the relationship between the two ROXY indices developed for intra-metropolitan analyses;

- $r_i^{t,t+1}$  : annual growth *ratio* of the population in locality  $i$  for the period between years  $t$  and  $t+1$
- $d_i$  : CBD distance of locality  $i$
- $\bar{d}$  : average of CBD distance
- $d_{min}$  : minimum CBD distance, *viz.* the minimum value of  $d_i$  (for  $i=1, 2, \dots, n$ )
- $d_{max}$  : maximum CBD distance, *viz.* the maximum value of  $d_i$  (for  $i=1, 2, \dots, n$ )
- $s_i$  : reversed CBD distance of locality  $i$  which is defined as  
 $d_{min} + d_{max} - d_i$
- $\bar{s}$  : average of reversed CBD distance
- $n$  : number of localities
- $R_d$  : value of ROXY index which we calculate by use of a CBD distance as its weighing factor
- $R_s$  : value of ROXY index which we calculate by use of a reversed CBD distance as its weighing factor

By definition, we have



$$R_d \equiv \left\{ \frac{\sum_{i=1}^n (d_i \times r_i^{t,t+1})}{\sum_{i=1}^n d_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \right\} \times 10^4$$

$$R_s \equiv \left\{ \frac{\sum_{i=1}^n (s_i \times r_i^{t,t+1})}{\sum_{i=1}^n s_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \right\} \times 10^4$$

It then follows that

$$\begin{aligned} R_s \times 10^4 &= \frac{\sum_{i=1}^n \{(d_{\min} + d_{\max} - d_i) \times r_i^{t,t+1}\}}{\sum_{i=1}^n s_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \\ &= \frac{\sum_{i=1}^n \{(d_{\min} + d_{\max}) \times r_i^{t,t+1}\} - \sum_{i=1}^n (d_i \times r_i^{t,t+1})}{\sum_{i=1}^n s_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \\ &= \frac{(d_{\min} + d_{\max}) \times \sum_{i=1}^n r_i^{t,t+1}}{\sum_{i=1}^n s_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - \frac{\sum_{i=1}^n d_i}{n \times \bar{s}} \times \frac{\sum_{i=1}^n (d_i \times r_i^{t,t+1})}{\sum_{i=1}^n d_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \\ &= \frac{d_{\min} + d_{\max}}{\bar{s}} - \frac{\bar{d}}{\bar{s}} \times \frac{\sum_{i=1}^n (d_i \times r_i^{t,t+1})}{\sum_{i=1}^n d_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \\ &= \frac{\bar{s} + \bar{d}}{\bar{s}} - \frac{\bar{d}}{\bar{s}} \times \frac{\sum_{i=1}^n (d_i \times r_i^{t,t+1})}{\sum_{i=1}^n d_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \\ &= -\frac{\bar{d}}{\bar{s}} \times \left\{ \frac{\sum_{i=1}^n (d_i \times r_i^{t,t+1})}{\sum_{i=1}^n d_i} \times \frac{n}{\sum_{i=1}^n r_i^{t,t+1}} - 1.0 \right\} \\ &= -\frac{\bar{d}}{\bar{s}} \times R_d \times 10^4 \end{aligned}$$

where  $\bar{d}$  : average of distance, viz.  $\sum_{i=1}^n d_i / n$

$\bar{s}$  : average of reversed distance, viz.  $\sum_{i=1}^n s_i / n$

Hence

$$R_s = - \frac{\bar{d}}{\bar{s}} \times R_d \dots\dots\dots (1)^{21}$$

Since we can rewrite the average of reversed CBD distance as

$$\bar{s} = d_{min} + d_{max} - \bar{d},$$

we obtain from Equation-1

$$R_s = - \frac{\bar{d}}{d_{min} + d_{max} - \bar{d}} \times R_d$$

Hence

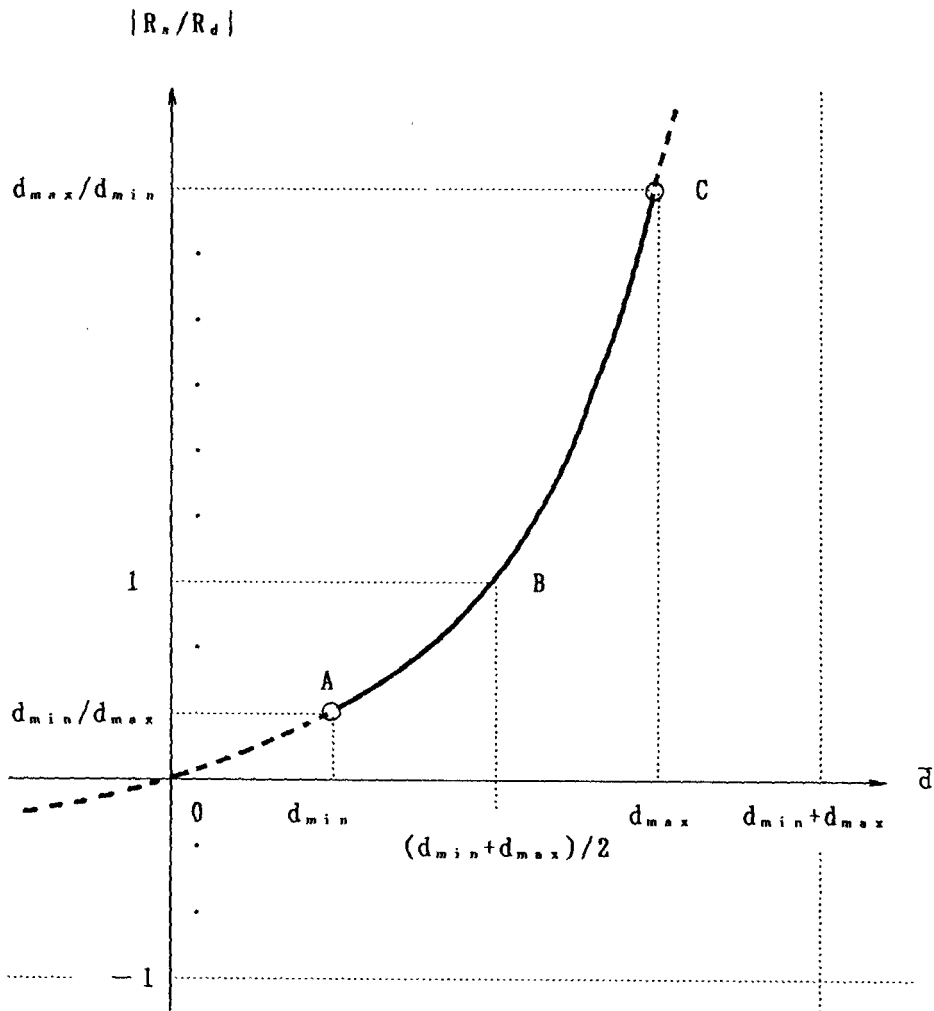
$$R_s = - \left( \frac{d_{min} + d_{max}}{d_{min} + d_{max} - \bar{d}} - 1 \right) \times R_d \dots\dots\dots (2)$$

From Equation-2, we develop Figure 3, which diagrammatically summarizes the relationship between  $\bar{d}$  and  $|R_s/R_d|$ . Meanwhile, since  $d_{min} < \bar{d} < d_{max}$ , the following can be deduced from Equation-2:

- (i) If  $\bar{d}$  is infinitesimally close to  $d_{min}$ , then  
 $|R_s/R_d| \cong d_{min}/d_{max} (< 1)$
- (ii) If  $d_{min} < \bar{d} < (d_{min} + d_{max})/2$ , then  
 $d_{min}/d_{max} < |R_s/R_d| < 1$
- (iii) If  $\bar{d}$  is equal to  $(d_{min} + d_{max})/2$ , then  
 $|R_s/R_d| = 1$
- (iv) If  $(d_{min} + d_{max})/2 < \bar{d} < d_{max}$ , then  
 $1 < |R_s/R_d| < d_{max}/d_{min}$
- (v) If  $\bar{d}$  is infinitesimally close to  $d_{max}$ , then  
 $|R_s/R_d| \cong d_{max}/d_{min} (> 1)$

Considering the above, we can summarize the following about the relationship between  $R_s$  and  $R_d$ :

- (1) The absolute value of the ratio of  $R_s$  to  $R_d$  is equal to the ratio of  $\bar{d}$  to  $\bar{s}$  which is constant;  $R_s/R_d = - \bar{d}/\bar{s}$ .
- (2) The sign of  $R_s$  is opposite to that of  $R_d$



$$|R_s/R_d| = (d_{\min} + d_{\max}) / (d_{\min} + d_{\max} - \bar{d}) - 1$$

Figure 3 Relationship of  $|R_s/R_d|$  to  $\bar{d}$

- (3) The ratio of  $|R_s|$  to  $|R_d|$  ranges from  $d_{min}/d_{max}$  through  $d_{max}/d_{min}$ .
- (4)  $|R_s| = |R_d|$  if  $\bar{d} = (d_{min} + d_{max}) / 2$ , (i.e., if  $\bar{d} = \bar{s}$ ).
- (5) If the distribution pattern of the CBD distance of localities is skewed toward  $d_{min}$ , then the value of  $|R_s|$  can be reasonably smaller than that of  $|R_d|$ .
- (6) If the distribution pattern of the CBD distance of localities is skewed toward  $d_{max}$ , then the value of  $|R_s|$  can be reasonably greater than that of  $|R_d|$ .

Now let  $R_p$  denote the value of the ROXY index with the weighing factor of metropolitan population as defined by Table 5. Using this notation, the following observations can be pointed out about the interface among  $R_s$ ,  $R_d$  and  $R_p$ :

- (1) Since  $R_s$  and  $R_d$  are in proportion, the information which they each provide is basically identical, with respect to the movements of the spatial-cycle path.
- (2)  $R_s$  and  $R_p$  share the same sign for the same given stage of the spatial-cycle path. Namely, they show the plus sign for spatial agglomeration and minus sign for spatial deglomeration.
- (3) The sign of  $R_d$  and that of  $R_p$  are opposite for the same given stage of the spatial-cycle path.
- (4) Let us define the reversed population for metropolitan area  $i$  by the formulation of  $x_{min} + x_{max} - x_i$  where  $x_i$  is denoted as population of metropolitan area  $i$ ,  $x_{min}$  and  $x_{max}$  as minimum and maximum values of  $x_i$  (for  $i = 1, 2, \dots, n$ ) respectively, and  $n$  as the number of metropolitan areas. In addition, let  $R_q$  be denoted as the value of the ROXY index with the weighing factor of reversed population. For this setting, we have the relationship  $R_q/R_p = -\bar{p}/\bar{q}$ , where  $\bar{p}$  and  $\bar{q}$  are the average of population and the average of reversed population respectively. It should be noted here that both  $\bar{p}$  and  $\bar{q}$  are not fixed, but variable when time changes. Therefore the ratio of  $R_q$  to  $R_p$  would not remain the same with a time change.

From the aforementioned, it follows (i) that  $R_s$  is more compatible with  $R_p$  than  $R_d$  is, and (ii) that the functional relationship between  $R_s$  and  $R_d$  is firmer than that between  $R_q$  and  $R_p$ .

Consequently, when we want to utilize *both*  $R_p$  *and* either  $R_d$  or  $R_s$  in a series of spatial-cycle studies on both inter-metropolitan and intra-metropolitan areas, using  $R_s$  would appear to be a better choice than using  $R_d$ . The next section will empirically investigate how the spatial-cycle path of each major railway-line regions in the Tokyo metropolitan area can be represented through  $R_d$  *and*  $R_s$  respectively in order to familiarize ourselves to the validity of the outcomes of our theoretical considerations on the mathematical relation between  $R_d$  and  $R_s$ .

## 7 Empirical Results for $R_d$ and $R_r$ : Spatial-cycle Paths of Five Railway-line Regions in the Tokyo Metropolitan Area

In this section, we investigate the values of  $R_d$  and  $R_r$  for five major railway-line regions in the Tokyo metropolitan area<sup>22</sup>. They are the Chuo-line, Takasaki-line, Joban-line, Tokaido-line and Sobu-line regions. The member localities and their local codes for the five railway-line regions are listed in Table 11. For each railway-line regions, we set two cases. One is the 'aggregated case' in which each of Tokyo city (*i.e.*, *Tokyo-tokubetsu-ku*), Kawasaki-city (*i.e.*, *Kawasaki-shi*) and Yokohama-city (*i.e.*, *Yokohama-shi*) is considered as one spatial unit. Another one is the 'disaggregated case' in which each of the above three cities is spatially disaggregated into wards (*ku*). In the disaggregated case, individual wards can be considered as separate spatial units, and those wards that are *on* or *close to* each of the railway-line regions, are picked up to be member localities of that railway-line region.

The number of localities, and minimum and maximum CBD distances for the five railway-line regions are furnished by Table 12 (for aggregated case) and Table 13 (for disaggregated case). The CBD distance, reversed CBD distance, and population (for every fifth year from 1960 through 1990) of member localities are given for each of the five railway-line regions by Table A-1 in Appendix. From this table, we obtain Table A-2 which shows five-year growth ratios of population for each member locality of the five railway-line regions. Based on Table A-2, we can construct Figure 4. In this figure are illustrated 'five-year growth-rate curves' for the six five-year periods, by railway-line region for both aggregated and disaggregated cases. The growth-rate curves in Figure 4 would tell us the following three *general* characteristics for both aggregated and disaggregated cases, about their dynamic movements which we can observe as time goes by<sup>23</sup>.

- (1) The peak point of the growth-rate curve almost successively shifts from localities with a shorter CBD distance to localities with a longer CBD distance (*i.e.*, the existence of a tendency for the peak point to move outwards).
- (2) The height of the peak point of the growth-rate curve gradually becomes lower (*i.e.*, the existence of a reductive tendency in maximum growth-rate value).
- (3) The growth-rate curve levels off (*i.e.*, the existence of a flattening tendency of the shape of the growth-rate curve).

Figure 4 can also help us notice *individual* characteristics of growth-rate curves for each railway-line regions<sup>24</sup>. Taking into consideration the nature of the above-mentioned *general* and *individual* characteristics of growth-rate curves, the following three points may possibly be suggested concerning the processes of centralization and decentralization of population for the five railway-line regions of the Tokyo metropolitan area during the 1960-90 period<sup>25</sup>.

- (1) A movement from the stage of centralization to that of decentralization seems to have taken place for all the railway-line regions.

**Table 11 Localities for Five Railway-line Regions**

(a) Chuo-line region

Code	Locality
13100	Tokyo-tokubetsu-kubu
13102	Chuo-ku
13101	Chiyoda-ku
13104	Shinjuku-ku
13113	Shibuya-ku
13114	Nakano-ku
13115	Suginami-ku
13203	Musashino-shi
13204	Mitaka-shi
13210	Koganei-shi
13206	Fuchuh-shi
13214	Kokubunji-shi
13215	Kunitachi-shi
13202	Tachikawa-shi
13212	Hino-shi
13201	Hachioji-shi
14424	Fujino-machi

(b) Takasaki-line region

Code	Locality
13100	Tokyo-tokubetsu-kubu
13106	Taito-ku
13118	Arakawa-ku
13117	Kita-ku
11203	Kawaguchi-shi
11223	Warabi-shi
11204	Urawa-shi
11220	Yono-shi
11205	Ohmiya-shi
11219	Ageo-shi
11231	Okegawa-shi
11233	Kitamoto-shi
11217	Kohnosu-shi
11304	Fukiage-shi
11206	Gyohda-shi

(c) Joban-line region

Code	Locality
13100	Tokyo-tokubetsu-kubu
13106	Taito-ku
13118	Arakawa-ku
13121	Adachi-ku
13122	Katsushika-ku
12207	Matsudo-shi
12220	Nagareyama-shi
12217	Kashiwa-shi
12222	Abiko-shi
8217	Toride-shi
8563	Fujishiro-shi
8208	Ryuhgasaki-shi
8219	Ushiku-shi

(d) Tokaido-line region

Code	Locality
13100	Tokyo-tokubetsu-kubu
13101	Chiyoda-ku
13103	Minato-ku
13109	Shinagawa-ku
13111	Ohta-ku
14130	Kawasaki-shi
14132	Saiwai-ku
14131	Kawasaki-ku
14100	Yokohama-shi
14101	Tsurumi-ku
14102	Kanagawa-ku
14103	Nishi-ku
14106	Hodogaya-ku (including 14112 Asahi-ku)
14110	Totsuka-ku (including 14115 Sakae-ku)
14204	Kamakura-shi
14205	Fujisawa-shi
14207	Chigasaki-shi

(e) Sobu-line region

Code	Locality
13100	Tokyo-tokubetsu-kubu
13101	Chiyoda-ku
13107	Sumida-ku
13106	Taito-ku
13108	Kohtoh-ku
13123	Edogawa-ku
13122	Katsushika-ku
12203	Ichikawa-shi
12204	Funabashi-shi
12216	Narashino-shi
12201	Chiba-shi
12228	Yotsukaido-shi
12212	Sakura-shi
12322	Shisui-shi
12323	Yachimata-shi

**Table 12 Number of Localities, and Minimum and Maximum CBD Distances of Five Railway-line Regions (For Aggregated Case)**

(unit of distance: km)

Railway-line region	Number of localities	Minimum distance	Maximum distance
Chuo-line region	11	7.4	55.5
Takasaki-line region	12	7.4	58.0
Joban-line region	9	7.4	48.0
Tokaido-line region	6	7.4	50.1
Sobu-line region	9	7.4	49.8

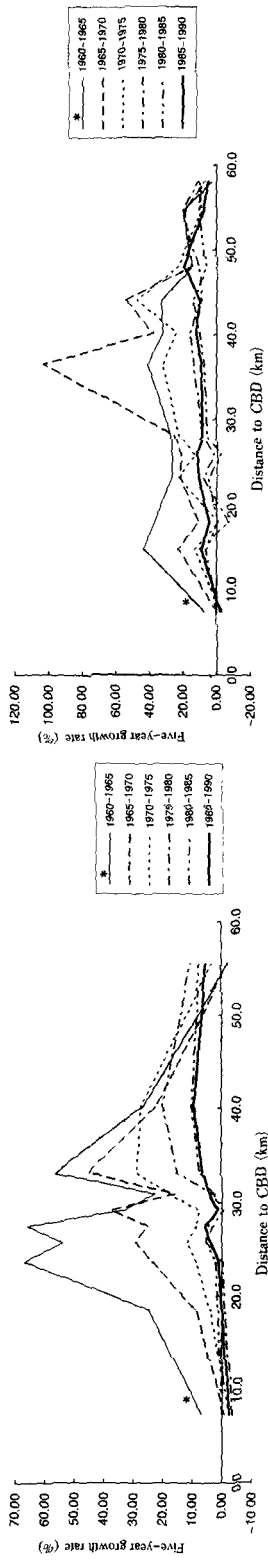
**Table 13 Number of Localities, and Minimum and Maximum CBD Distances of Five Railway-line Regions (For Disaggregated Case)**

(unit of distance: km)

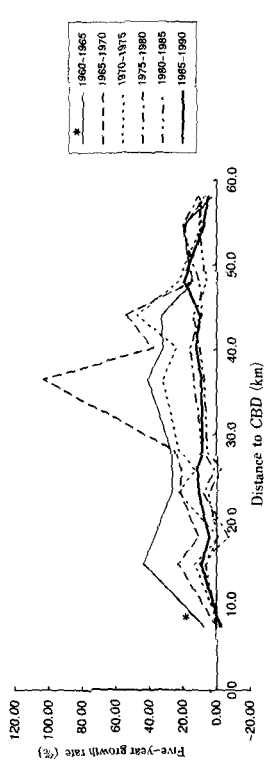
Railway-line region	Number of localities	Minimum distance	Maximum distance
Chuo-line region	16	1.1	55.5
Takasaki-line region	14	4.2	58.0
Joban-line region	12	4.2	48.0
Tokaido-line region	14	2.1	50.1
Sobu-line region	14	2.1	49.8

- (2) A movement from the stage of accelerating decentralization to that of decelerating decentralization seems to have taken place for most of the railway-line regions.
- (3) A movement from the stage of decelerating decentralization to that of accelerating re-decentralization may have taken place towards the end of the 1980s for the Chuo-line, Tokaido-line and Sobu-line regions.

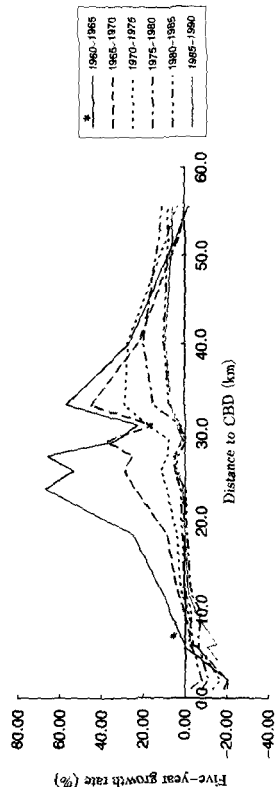
From Table A-2, meanwhile, we can prepare Table 14 showing (i) the value of the ROXY index with a CBD distance as its weighing factor and (ii) the marginal change in the value of the ROXY index, for five railway-line regions (for aggregated case). In addition, we can similarly prepare Table 15 for the ROXY index with a reversed CBD distance (for aggregated case), Table 16 for the ROXY index with a CBD distance (for disaggregated case), and Table 17 for the ROXY index with a reversed CBD distance (for disaggregated case).



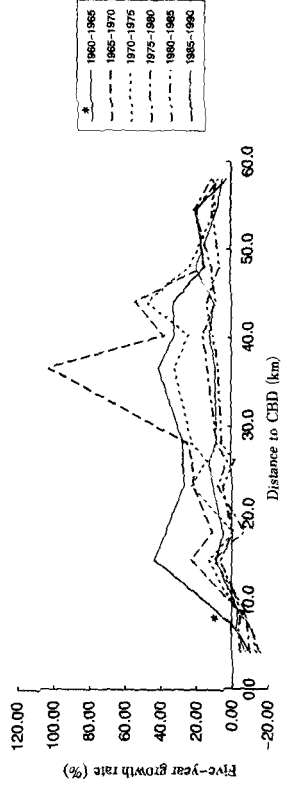
(AG-a) Chuo-line region



(AG-b) Takasaki-line region



(DAG-a) Chuo-line region



(DAG-b) Takasaki-line region

Figure 4 Five-year Growth-rate Curves for Five Railway-line Regions in the Tokyo Metropolitan Area: Aggregated Case (AG) and Disaggregated Case (DAG)



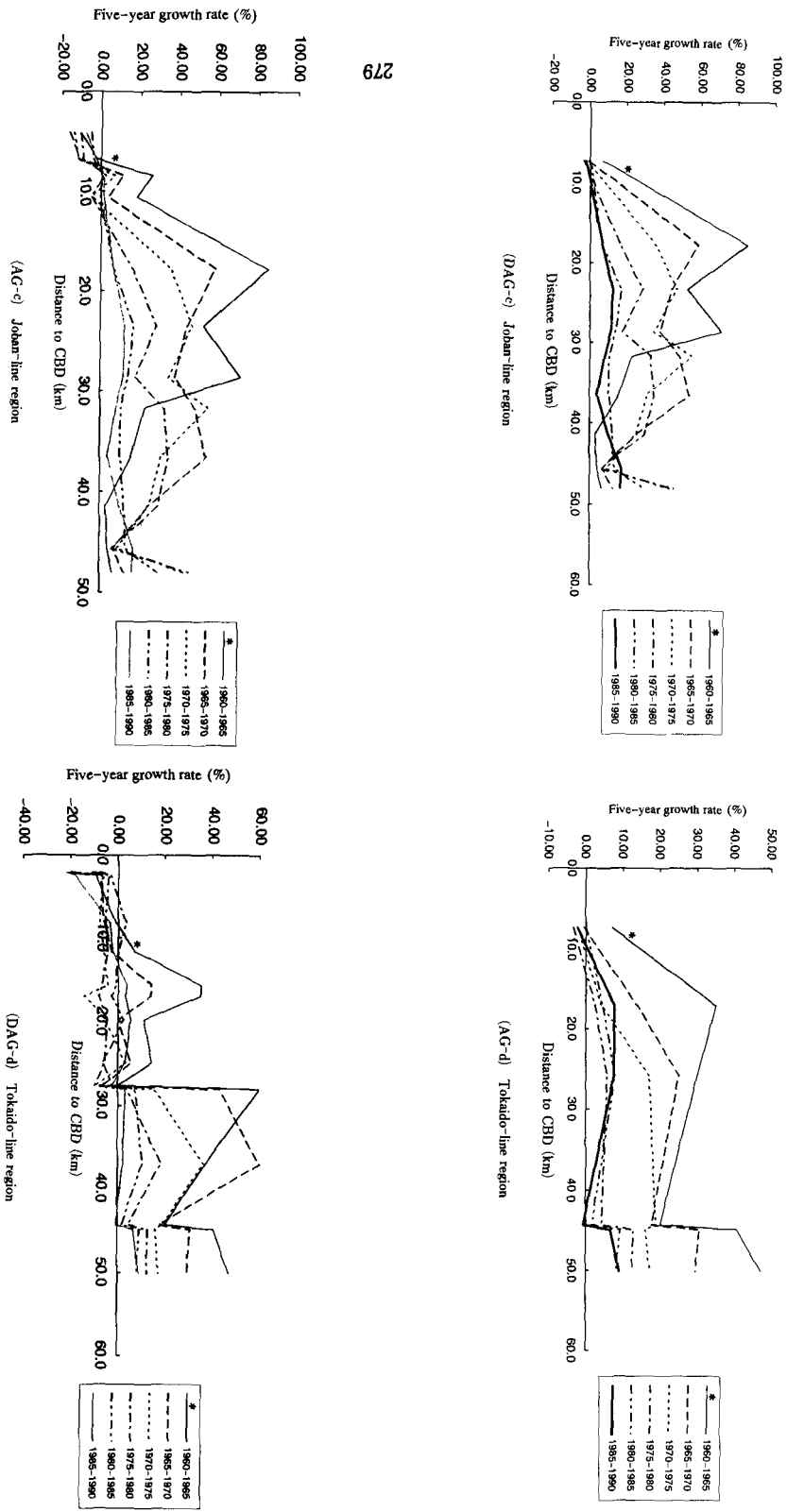


Figure 4 (Continued)

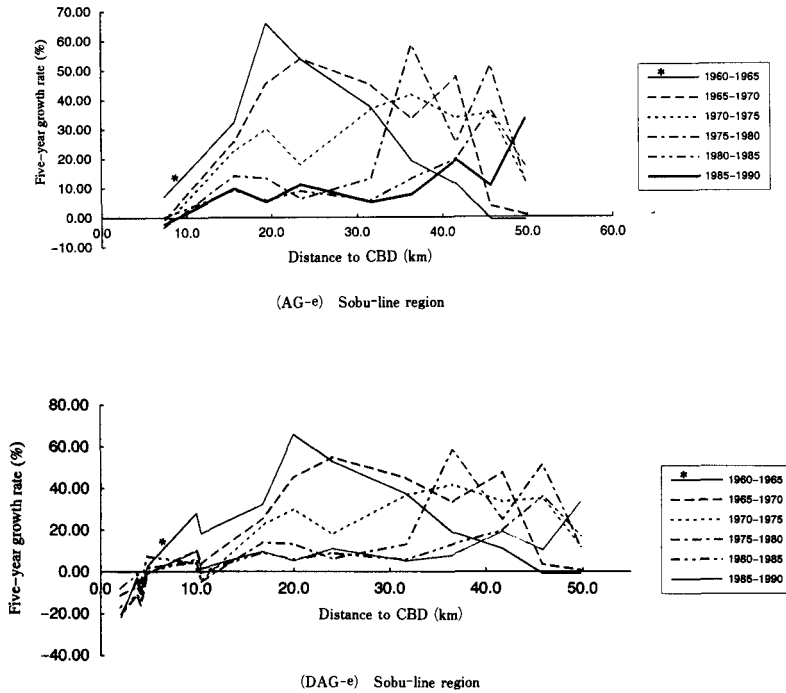


Figure 4 (Continued)

These four tables enable us to produce Figure 5 diagrammatically illustrating the locus of the spatial-cycle paths for the five railway-line regions during the thirty-year period between 1960 and 1990 based on the values of the ROXY indices (i) with a CBD distance for the aggregated case, (ii) with a reversed CBD distance for the aggregated case, (iii) with a CBD distance for the disaggregated case, and (iv) with a reversed CBD distance for the disaggregated case.

For the five railway-line regions for both aggregated and disaggregated cases, Table A-3 shows the average of the CBD distance, average of the reversed CBD distance, and their ratio. If we pick up figures for the Chuo-line region for the aggregated case as an example from this table, the average of the CBD distance is 28.2 while the average of the reversed CBD distance is 34.7. The ratio of these two figures (*i.e.*, RR-ratio) is hence equal to 0.81. From Tables 14 and 15, we know that, for the aggregated case,  $R_d$  is  $-25.45$  and  $R$  is 20.73 for the period 1960-1965. Therefore the absolute value of the ratio of  $R$  to  $R_d$  becomes equal to 20.73 divided by 25.45 which results in also 0.81. Reflecting this, a pair of graphs AG-D-a and AG-RD-a in Figure 5 have the similarity-ratio of 0.81 for both horizontal and vertical directions.

Table 14 Value of ROXY Index and Its Marginal Change for Five Railway-line Regions in Aggregated Case (Weighing Factor: CBD Distance)

	1960-65		1965-70		1970-75		1975-80		1980-85		1985-90	
	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY
Chuo-line region	-25.45	33.28	7.83	27.57	29.69	17.01	41.84	-4.90	19.89	-10.49	20.87	0.98
Tokasaki-line region	-29.49	64.72	35.23	35.68	41.46	5.77	46.97	-6.61	28.65	-14.35	18.07	-10.58
Joban-line region	-84.21	69.78	-14.43	55.54	26.87	35.59	56.75	6.10	39.06	-11.90	32.95	-6.11
Tokaido-line region	67.69	9.49	77.18	-1.54	64.62	-15.07	47.04	-23.09	18.44	-16.10	14.84	-3.60
Sobu-line region	-83.82	65.87	-17.95	66.43	49.63	51.51	85.06	6.87	62.77	-13.95	57.17	-5.60

Table 15 Value of ROXY Index and Its Marginal Change for Five Railway-line Regions in Aggregated Case (Weighing Factor: Reversed CBD Distance)

	1960-65		1965-70		1970-75		1975-80		1980-85		1985-90	
	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY
Chuo-line region	20.73	-27.11	-6.38	-22.46	-24.19	-13.85	-34.08	4.00	-16.20	8.54	-17.00	-0.80
Tokasaki-line region	30.44	-66.81	-36.37	-36.62	-42.79	-5.95	-48.27	6.61	-29.57	14.81	-18.65	10.92
Joban-line region	108.30	-89.77	18.53	-71.40	-34.50	-45.70	-72.86	-7.83	-50.15	15.28	-42.31	7.84
Tokaido-line region	-82.59	-11.57	-94.16	1.88	-78.84	18.39	-57.39	28.18	-22.49	19.64	-18.11	4.38
Sobu-line region	93.61	-73.57	20.04	-74.18	-54.75	-57.52	-95.00	-7.67	-70.10	15.58	-63.84	6.26

**Table 16 Value of ROXY Index and Its Marginal Change for Five Railway-line Regions in Disaggregated Case (Weighing Factor: CBD Distance)**

	1960-65		1965-70		1970-75		1975-80		1980-85		1985-90	
	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY
Chuo-line region	150.12	-11.10	139.02	-16.39	117.34	-20.76	97.51	-30.72	55.90	4.76	107.03	51.13
Tokasaki-line region	36.20	86.75	122.95	35.33	106.86	-16.22	90.51	-25.16	56.55	-20.13	50.25	-6.30
Joban-line region	-11.27	111.71	100.44	78.46	145.65	24.19	148.81	-28.19	89.28	-39.05	70.71	-18.57
Tokaido-line region	171.39	3.69	175.08	-12.16	147.06	-43.33	88.43	-49.85	47.36	3.53	81.38	34.02
Sobu-line region	50.51	109.55	160.06	80.20	210.90	22.29	204.64	-42.08	126.75	-32.12	140.41	13.66

**Table 17 Value of ROXY Index and Its Marginal Change for Five Railway-line Regions in Disaggregated Case (Weighing Factor: Reversed CBD Distance)**

	1960-65		1965-70		1970-75		1975-80		1980-85		1985-90	
	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY	ROXY	$\Delta$ ROXY
Chuo-line region	-90.03	6.66	-83.37	9.83	-70.37	12.45	-58.48	18.43	-33.52	-2.86	-64.19	-30.67
Tokasaki-line region	-32.36	-77.54	-109.90	-31.58	-95.52	14.50	-80.90	22.49	-50.55	17.99	-44.92	5.63
Joban-line region	10.54	-104.47	-93.93	-73.37	-136.20	-22.61	-139.16	26.36	-83.48	36.52	-66.12	17.36
Tokaido-line region	-143.79	-3.10	-146.89	10.21	-123.37	36.35	-74.19	41.82	-39.74	2.96	-68.27	-28.53
Sobu-line region	-35.46	-76.91	-112.37	-56.30	-148.06	-15.65	-143.67	29.54	-88.98	22.54	-98.58	-9.60

Mathematical Characteristics of ROXY Index (I):  
Distance and Reversed Distance Used as Weighing Factors (Kawashima, Hiraoka)

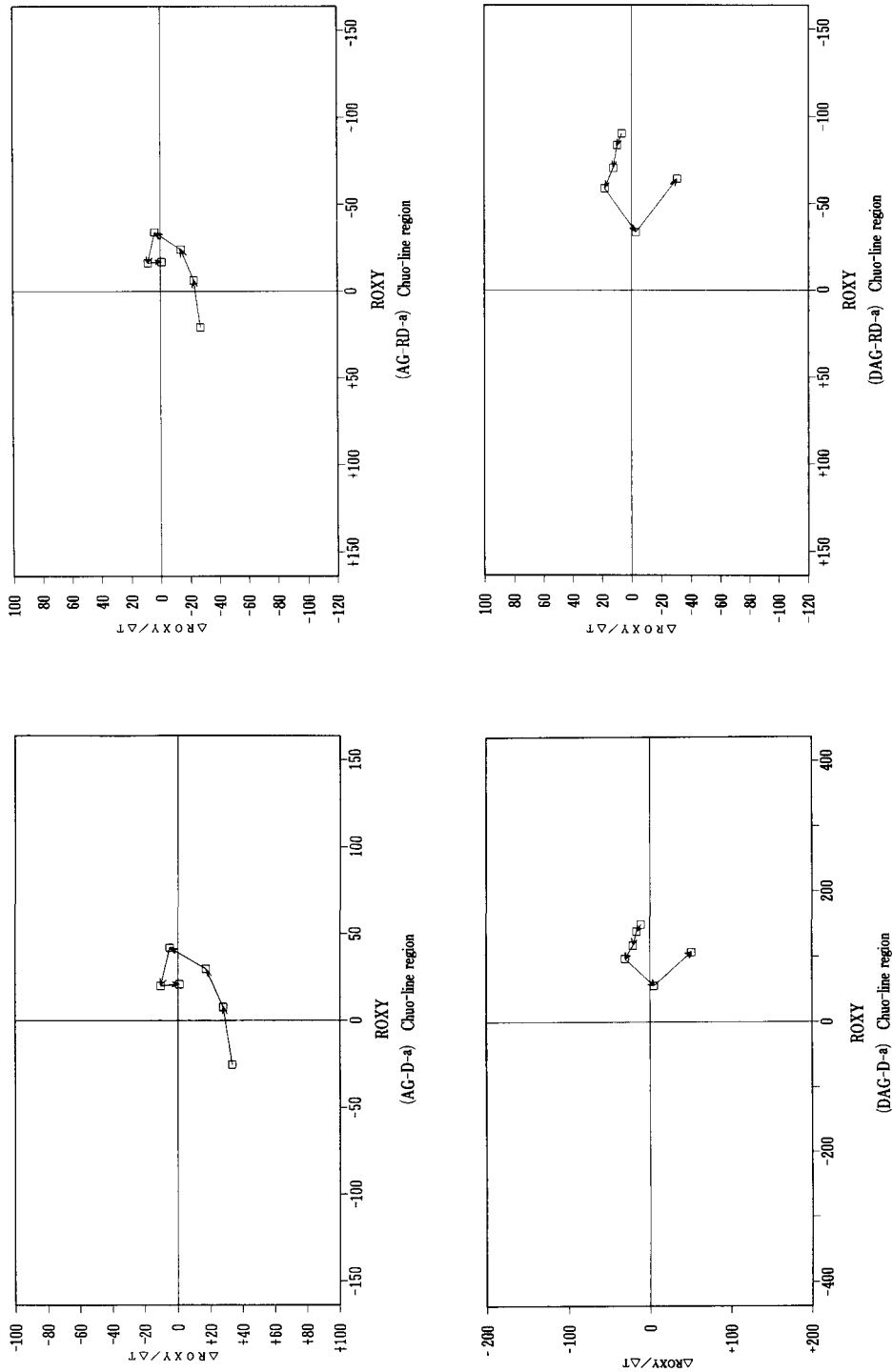


Figure 5 Spatial-cycle Paths for Five Railway-line Regions in the Tokyo Metropolitan Area for Aggregated Case (AG) and Disaggregated Case (DAG): With Weighing Factors of CBD Distance (D) and Reversed CBD Distance (RD)

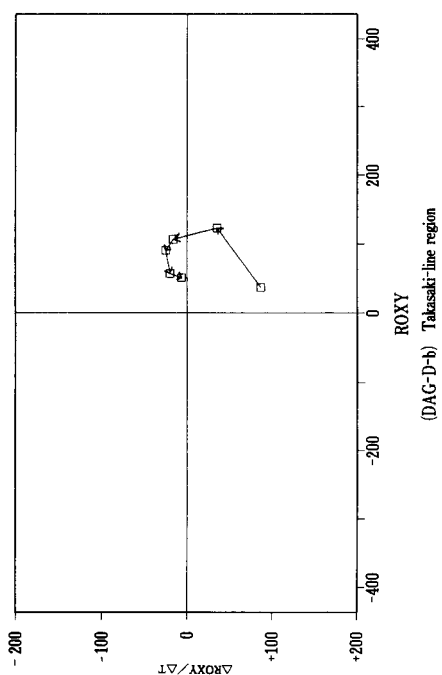
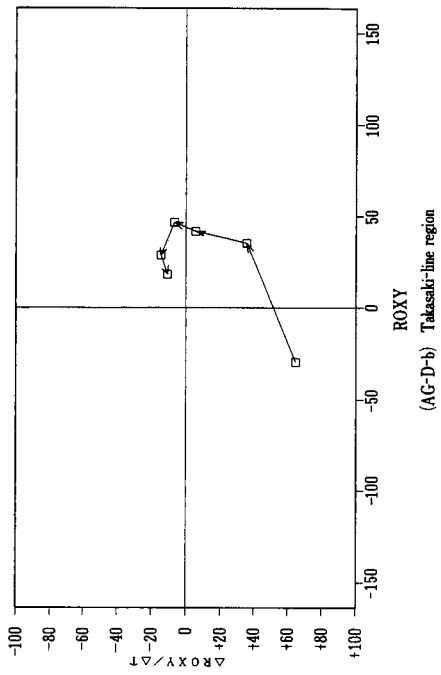
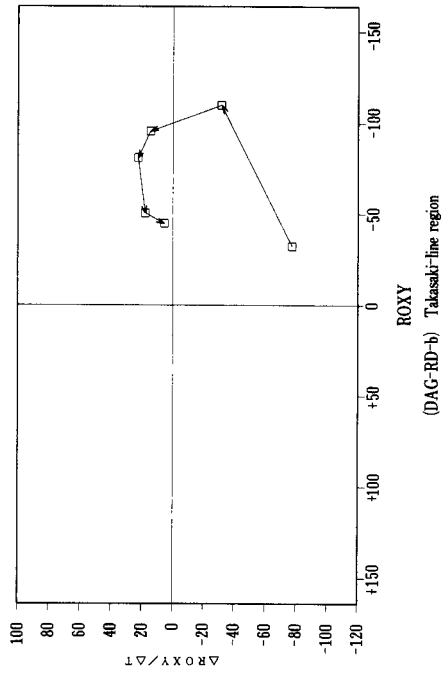
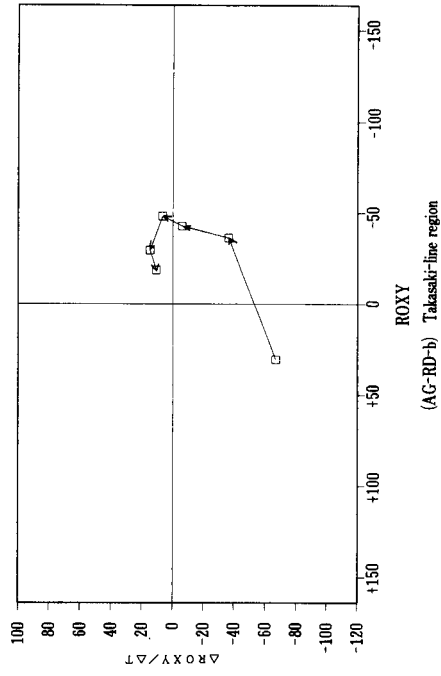


Figure 5 (Continued)

Mathematical Characteristics of ROXY Index (I):  
Distance and Reversed Distance Used as Weighing Factors (Kawashima, Hiraoka)

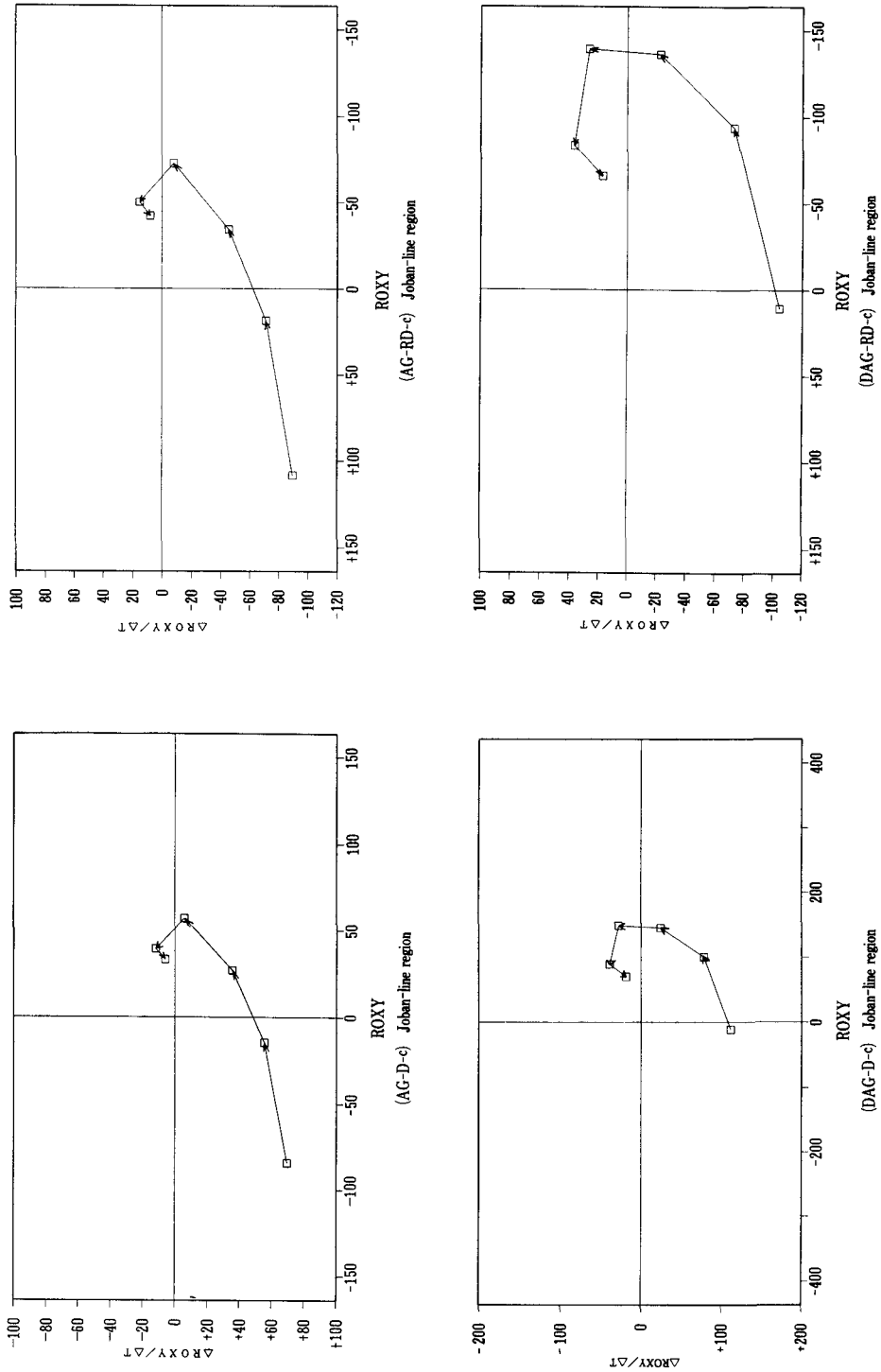


Figure 5 (Continued)

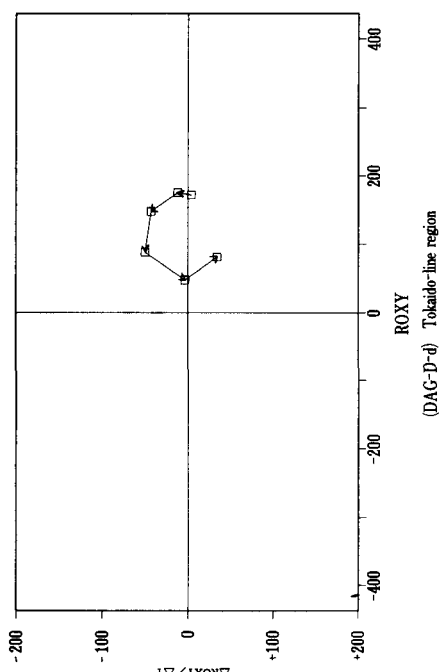
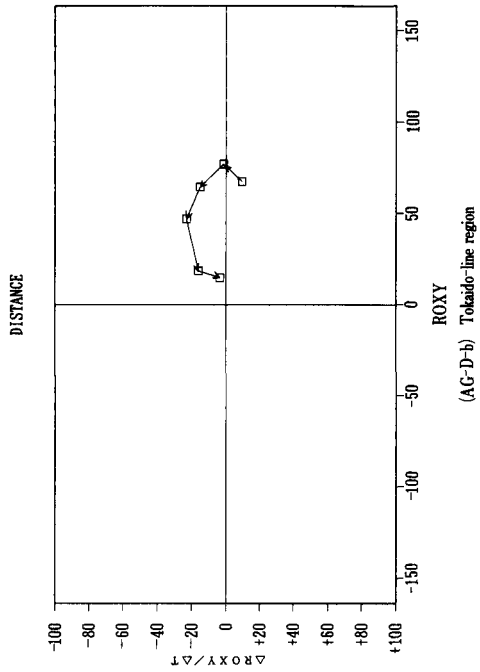
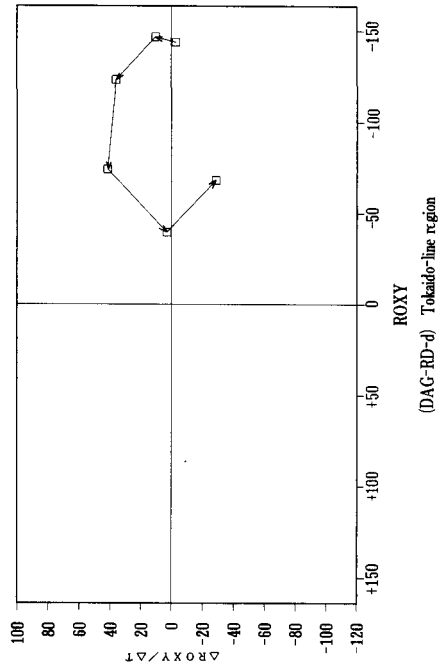
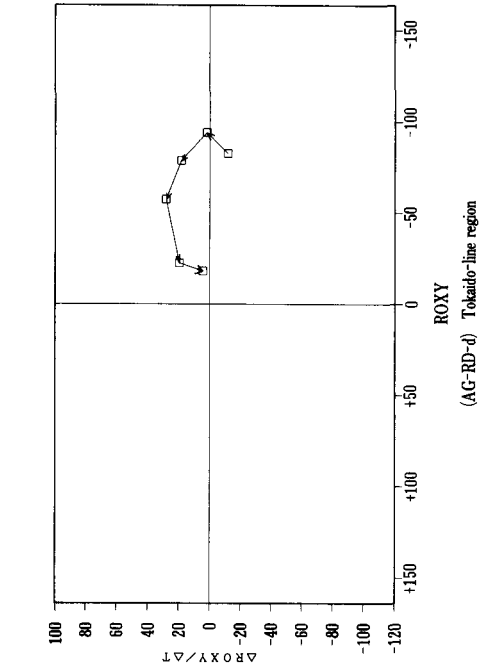


Figure 5 (Continued)



Mathematical Characteristics of ROXY Index (I):  
Distance and Reversed Distance Used as Weighing Factors (Kawashima, Hiraoka)

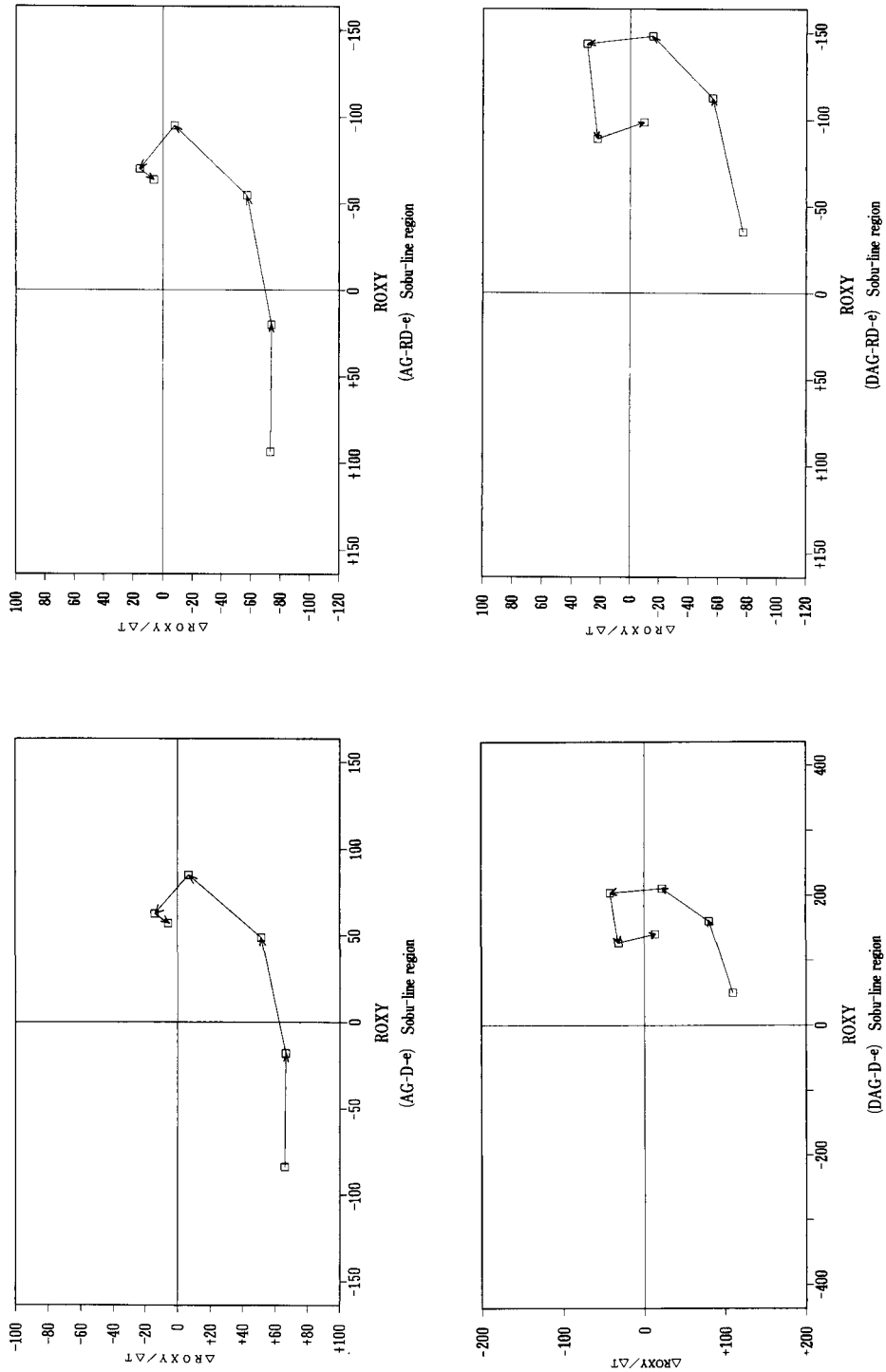


Figure 5 (Continued)

Above observations are certainly consistent with the theoretical conclusion drawn from our mathematical examination discussed in Section 6. This consistency holds, as can be seen from Tables 14, 15, 16, 17 and A-3 as well as from Figure 5, for all five railway-line regions for all five-year periods for both aggregated and disaggregated cases<sup>26)</sup>.

Based on Figure 5, the following points can be made as to the development of the 'spatial-cycle race'<sup>27)</sup> among the five railway-line regions for the disaggregated case<sup>28)</sup>.

- (1) In the early 1960s, the Chuo-line region was taking the lead in the race with its position at the stage of decelerating decentralization. Following the Chuo-line region, were the Tokaido-line region (at the stage of accelerating decentralization), the Takasakai-line region (at the stage of accelerating decentralization), the Sobu-line region (at the stage of accelerating decentralization), and the Joban-line region (at the stage of decelerating centralization) in this order.
- (2) Around the late 1980s, the Chuo-line region whose position was then at the stage of accelerating re-decentralization was leading the race, followed by the Tokaido-line region (at the stage of accelerating re-decentralization), the Sobu-line region (at the stage of accelerating re-decentralization), Takasaki-line region (at the stage of decelerating decentralization), and the Joban-line region (at the stage of decelerating decentralization) in this order.
- (3) The relative order in the spatial-cycle race between the Takasaki-line region and the Sobu-line region was reversed around 1980, resulting in that the Sobu-line region has been taking the lead over the Takasaki-line region since then.

## 8 Conclusion

In this paper we have investigated, based on the values of the two types of ROXY indices ( $R_d$  and  $R_r$ ), the spatial-cycle paths of five railway-line regions, for both aggregated and disaggregated cases, in the Tokyo metropolitan area. In this investigation, one of the focal points is the theoretical and empirical comparisons of the values of the two types of ROXY indices,  $R_d$  and  $R_r$ , where  $R_d$  is the ROXY index with a CBD distance as its weighing factor and  $R_r$  is the ROXY index with a reversed CBD distance as its weighing factor.

The implications of the results of our investigations would be as follows:

- (1) The ROXY-index approach seems to offer a theoretically reasonable and empirically powerful means of conducting systematic inter-metropolitan and intra-metropolitan spatial-cycle researches. One of the reasons for this is that the value of the ROXY index would summarize a great deal of information which tells us about basic properties or performance of the spatial-cycle movements — useful information that might otherwise remain indistinct.
- (2) The ROXY-index approach seems to have introduced new dimensions in quantitatively investigating the spatial-cycle phenomena, within the realm of a simple analytical

device.

- (3) Using  $R$ , would appear to be a better choice than using  $R_d$  when we want to apply the ROXY-index method to studies of the intra-metropolitan spatial redistribution processes of socio-economic activities.

## Notes

- 1) SMSA stands for Standard Metropolitan Statistical Area.
- 2) Those eleven SMSAs are: Cleveland (with a population change of  $-4.7\%$  for the five-year period of 1970-75), New York ( $-4.1\%$ ), Pittsburgh ( $-3.3\%$ ), Newark ( $-2.8\%$ ), St. Louis ( $-1.8\%$ ), Seattle-Everett ( $-1.3\%$ ), Los Angeles-Long Beach ( $-0.8\%$ ), Philadelphia ( $-0.4\%$ ), Boston ( $-0.3\%$ ), Cincinnati ( $-0.3\%$ ), Detroit ( $-0.2\%$ ). See Kawashima (1987a) for a discussion on the disurbanization processes in the United States for the period of 1960-80.
- 3) See Beale (1975), Berry (1978), Gordon (1979) and Alden (1981) for a discussion on the phenomena of net population loss of large US SMSAs in the early 1970s.
- 4) Citation from Kawashima (p.71, 1978a).
- 5) Citation from Kawashima (p.71, 1978a).
- 6) See Klaassen and Paelinck (1979) and Klaassen *et al.* (1981) for the fundamental characteristics of the spatial-cycle framework and the factors leading up to the development of the spatial-cycle hypothesis proposed by Klaassen and his collaborators.
- 7) For a discussion on how the development and applications of either or both of these two frameworks have come about, see for example Kawashima (1985, 1989).
- 8) The connotations of counter-urbanization and reurbanization within the conceptual framework illustrated by Figure 2 are more fully discussed in Kawashima and Hiraoka (1993b).
- 9) There are cases where non-metropolitan areas are included in studies of the inter-metropolitan spatial redistribution processes of socio-economic activities.
- 10) See Kawashima (1977) and Glickman (1979) for a description of functional urban regions in Japan.
- 11) In this paper, the size of a metropolitan area refers to the size of its population. Therefore, a larger metropolitan area is one with a larger population.
- 12) The terms of counter-urbanization and disurbanization are used interchangeably in this paper.
- 13) The foundations for the ROXY index were first conceived by Kawashima in the late 1970s when he was involved in studies of urban growth and decline at the International Institute for Applied Systems Analysis, Austria. See Kawashima (1978, 1981, 1982) for early applications of the ROXY index in his studies on urban changes. Also see Kawashima (1985) for detailed discussions on initial versions of the ROXY index, and Kawashima (1987b) for the application of the ROXY index for both inter- and intra-metropolitan analyses.

- 14) Note that we have thus far limited our discussion to two cases in each of which two spatial units are involved: (i) central city and suburbs for studying a metropolitan area, and (ii) a group of larger metropolitan areas and a group of medium and smaller metropolitan areas for studying a system of metropolitan areas.
- 15) We begin our discussion of ROXY indices with the ROXY index which can be used for studying the phenomena of spatial concentration and deconcentration. This choice reflects the order of the empirical applications of the ROXY index, in which the inter-metropolitan analyses preceded the intra-metropolitan analyses. The first empirical study in which the ROXY index was applied for intra-metropolitan analyses was carried out by Kawashima (1986a) where he compared the speed of suburbanization for major railway-line regions in each of Tokyo, Osaka and Nagoya metropolitan areas.
- 16) In general, the value of the ROXY index would turn out to be (i) greater than, (ii) equal to, or (iii) less than zero when spatial units with relatively heavy weights (in terms of, for example, population, distance, density, production level, or consumption level) attain growth ratios (i) higher than, (ii) equal to, or (iii) lower than spatial units with relatively light weights.
- 17) In this paper, 'CBD distance of each locality' refers to the airline distance from the former Tokyo Metropolitan Government Office (close to Tokyo station in Chiyoda-ku) to the public office (i.e., city hall, ward office, or town hall) of that locality.
- 18) 'Reversed CBD distance of each locality' is defined as 'the sum of the minimum and maximum CBD distances among CBD distances of all localities subtracted by the CBD distance of that locality.'
- 19) See Kawashima (1985, 1986a, 1986b, 1986c) and Kawashima and Hiraoka (1993a) for theoretical discussions and empirical applications of the ROXY index with CBD distance used as its weighing factor.
- 20) See Kawashima (1987b, 1989) for discussions and empirical applications of the ROXY index with reversed CBD distance used as its weighing factor.
- 21) In response to Kawashima's suggestion that there may exist a systematic functional relationship between  $R_d$  (the value of the ROXY index which we calculate by use of a CBD distance as its weighing factor) and  $R_r$  (the value of the ROXY index which we calculate by use of a reversed CBD distance as its weighing factor), Hiraoka came up with a mathematical formulation relating the two values as expressed in Equation-1. Credit for the completion of this mathematical manipulation consequently goes to Hiraoka.
- 22) In this paper, the geographical boundary of the Tokyo metropolitan area is the one delineated as the 1990-version of the Tokyo functional urban region (FUR) by Kawashima *et al.* (1993). For the discussion on the delineation of the FURs in Japan, see Kawashima (1977) as to the 1970-version of FURs, and Kawashima *et al.* (1993) as to the 1970- and 1990-versions of FURs.
- 23) For more details about the grounds for justifying the existence of these three *general*

- tendencies, see Kawashima and Hiraoka (1993a).
- 24) See *ibid.* for the discussion on the *individual* characteristics of growth-rate curves unique to each railway-line region.
  - 25) It is to be noticed that these three points are discussed here in light of the spatial-cycle framework in which we use growth rate (or growth ratio) of population (instead of absolute level of change in population) as its basic reference-variable.
  - 26) It should be additionally noticed that, in Table A-3, the RR-ratio for the disaggregated case is smaller than that for the aggregated case. For a discussion on this subject, see Kawashima and Hiraoka (1993b).
  - 27) 'Spatial-cycle race' implies 'race along the spatial-cycle path.'
  - 28) For the more detailed investigation on the spatial-cycle path which each railway-line region (for disaggregated case) has shown since 1960, see Kawashima and Hiraoka (1993a).

## References

- Alden J, 1981, "A Cross-National Study of Metropolitan Problems in Industrial Countries: Experiences of the USA and West Europe," Institute of Science and Technology, Cardiff (mimeographed).
- Beale C, 1975, "The Revival of Population Growth in Nonmetropolitan America," Economic Research Service Series ERS 605, US Department of Agriculture, Washington, D.C. , U.S.A.
- Berry B J L, 1978, "The Counterurbanization Process: How General?," in N.H.Hansen (ed.) *Human Settlement Systems: International Perspectives on Structure, Change and Public Policy*, Ballinger, Cambridge, Mass. , USA.
- Glickman N, 1979, *The Growth and Management of the Japanese Urban System*, Academic Press, New York, U.S.A.
- Gordon P, 1979, "Deconcentration without a 'Clean Break'," *Environment and Planning A*, Vol. 11, pp.281-290.
- Kawashima T, 1977, "Changes in the Spatial Population Structure of Japan," *Research Memorandum*, 77- 25, International Institute for Applied Systems Analysis, Laxenburg, Austria.
- Kawashima T, 1978, "Recent Urban Evolution Processes in Japan: Analysis of Functional Urban Regions," presented at the Twenty-fifth North American Meetings of the Regional Science Association, Chicago, Illinois, USA, November.
- Kawashima T, 1981, "Analytical Methods for the Phenomena of Urban Changes," *Shin Toshi*, Vol.35, No.8, Toshikeikaku Kyohkai, August, pp.10-21 (in Japanese).
- Kawashima T, 1982, "Recent Urban Trends in Japan: Analysis of Functional Urban Regions" in T. Kawashima and P. Korcelli (eds. ) *Human Settlement Systems: Spatial Patterns and Trends*, International Institute for Applied Systems Analysis, Laxenburg, Austria, pp.21-40.

- Kawashima T, 1985, "ROXY Index: An Indicative Instrument to Measure the Speed of Spatial Concentration and Deconcentration of Population," *Gakushuin Economic Papers*, Vol.22, No.2, Gakushuin University, Tokyo, September, pp.183-213.
- Kawashima T, 1986a, "Speed of Suburbanization: ROXY Index Analysis for Intrametropolitan Spatial Redistribution of Population in Japan," *Gakushuin Economic Papers*, Vol.22, No.3, Gakushuin University, Tokyo, March, pp.243-304.
- Kawashima T, 1986b, "People Follow Jobs in Japan?: Suburbanization of Job Markets," *Gakushuin Economic Papers*, Vol.23, Nos.1&2, Gakushuin University, Tokyo, October, pp.157-183.
- Kawashima T, 1986c, "Spatial Cycle Race 1985: ROXY Index Analysis of the 1985 Population Census for Three Railway-line Regions in the Tokyo Metropolitan Area," *Gakushuin Economic Papers*, Vol.23, No.3, Gakushuin University, Tokyo, December, pp.53-70.
- Kawashima T, 1987a, "Is Disurbanization Foreseeable in Japan?: A Comparison between U. S. and Japanese Urbanization Processes" in L. van de Berg, L. S. Burns and L. Klaassen (eds.) *Spatial Cycles*, Gower Publishing Company, Hants, England, pp.100-126.
- Kawashima T, 1987b, "ROXY Index Analysis of Population Changes in Japan for 1960-85: Spatial (De)centralization and (De)concentration," *Gakushuin Economic Papers*, Vol.24, No.3, Gakushuin University, Tokyo, December, pp.11-39.
- Kawashima T, 1989, "Basic Concepts of the Nature of ROXY Index," *GEM Bulletin*, Vol.3, Gakushuin University Research Institute of Economics and Management, Tokyo, October, pp.81-94 (in Japanese).
- Kawashima T, *et al.*, 1993, "Metropolitan Analyses: Boundary Delineations and Future Population Changes of Functional Urban Regions," *Gakushuin Economic Papers*, Vol.29, Nos.3&4, Gakushuin University, Tokyo, January, pp.205-248.
- Kawashima T and N. Hiraoka, 1993a, "Centralization and Suburbanization: ROXY Index Analysis for Five Railway-line Regions in Tokyo Metropolitan Area," *Gakushuin Economic Papers*, Vol.30, No.1, Gakushuin University, Tokyo, March, pp.203-230.
- Kawashima T and N. Hiraoka, 1993b "Mathematical Characteristics of ROXY Index (II): Formulation of ROXY Index and Patterns of Spatial-cycles ," *Gakushuin Economic Papers*, Vol. 30, No.3, Gakushuin University, Tokyo, (forthcoming).
- Klaassen L H and J. H. P. Paelinck, 1979, "The Future of Large Towns," *Environment and Planning A*, 10:pp.1095-1104.
- Klaassen L H *et al.*, 1981, *Transport and Reurbanisation*, Gower Publishing Company, Hants, England.

Mathematical Characteristics of ROXY Index (I):  
Distance and Reversed Distance Used as Weighing Factors (Kawashima, Hiraoka)

Appendix Table A-1 CBD Distance, Reversed CBD Distance, and Population for Localities of Five Railway-line Regions in the Tokyo Metropolitan Area

(a) Chuo-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960	1965	1970	1975	1980	1985	1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>							
13100 <sup>(1)</sup>	7.4	55.5	-	8,310,027	8,893,094	8,840,942	8,642,800	8,349,209	8,354,615	8,163,573
13102	1.1	-	55.5	161,299	128,017	103,850	90,097	82,700	79,973	68,041
13101	2.1	-	54.5	116,944	93,047	74,185	61,656	54,801	50,493	39,472
13104	5.7	-	50.9	413,690	413,910	390,657	367,218	343,928	332,722	296,790
13113	6.1	-	50.5	282,687	283,730	274,491	263,815	247,035	242,442	205,625
13114	9.6	-	47.0	351,360	376,697	378,723	373,075	345,733	335,936	319,687
13115	11.7	-	44.9	487,210	536,792	553,016	560,716	542,449	539,842	529,485
13203	18.5	44.4	38.1	120,337	133,516	136,959	139,493	136,895	138,783	139,077
13204	18.5	44.4	38.1	98,038	135,873	155,693	164,852	164,449	166,252	165,564
13210	23.7	39.2	32.9	45,734	76,350	94,448	102,703	102,412	104,642	105,899
13206	25.8	37.1	30.8	82,098	126,235	163,173	182,379	191,980	201,972	209,396
13214	27.5	35.4	29.1	39,098	64,911	81,259	88,155	91,014	95,467	100,982
13215	29.2	33.7	27.4	32,609	43,477	59,709	64,404	64,154	64,881	65,833
13202	31.0	31.9	25.6	81,951	100,699	117,057	138,097	142,600	146,523	152,824
13212	33.2	29.7	23.4	43,394	67,979	98,557	126,754	145,417	156,031	165,928
13201	40.3	22.6	16.3	164,622	207,753	253,527	322,558	387,162	426,654	466,347
14424	55.5	7.4	1.1	8,659	8,473	8,295	8,571	9,470	10,186	10,729

(b) Takasaki-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960	1965	1970	1975	1980	1985	1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>							
13100 <sup>(1)</sup>	7.4	58.0	-	8,310,027	8,893,094	8,840,942	8,642,800	8,349,209	8,354,615	8,163,573
13106	4.2	-	58.0	318,889	286,324	240,769	207,649	186,048	176,804	162,969
13118	6.7	-	55.5	285,480	278,412	247,013	217,905	198,126	190,061	184,809
13117	8.9	-	53.3	418,603	452,064	431,219	419,996	387,458	367,579	354,647
11203	14.8	50.6	47.4	173,692	249,112	305,886	345,547	379,357	403,015	438,680
11223	18.0	47.4	44.2	50,952	69,715	77,225	76,312	70,876	70,408	73,620
11204	23.2	42.2	39.0	174,437	221,323	269,397	331,145	358,180	377,235	418,271
11220	26.0	39.4	36.2	40,840	51,746	62,802	71,045	72,326	70,597	79,060
11205	28.0	37.4	34.2	169,996	215,646	268,777	327,696	354,082	373,022	403,776
11219	36.5	28.9	25.7	38,889	54,776	110,792	146,359	166,244	178,587	194,947
11231	40.2	25.2	22.0	21,309	28,108	38,717	48,034	55,746	61,499	69,029
11233	44.0	21.4	18.2	15,483	20,576	31,699	46,632	50,888	58,114	63,929
11217	48.0	17.4	14.2	31,868	36,526	41,990	51,632	57,085	60,565	72,435
11304	54.5	10.9	7.7	12,095	14,482	17,247	18,775	22,606	24,990	26,928
11206	58.0	7.4	4.2	54,746	56,152	60,135	66,069	73,205	79,359	83,181

(c) Joban-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960	1965	1970	1975	1980	1985	1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>							
13100 <sup>(1)</sup>	7.4	48.0	-	8,310,027	8,893,094	8,840,942	8,642,800	8,349,209	8,354,615	8,163,573
13106	4.2	-	48.0	318,889	286,324	240,769	207,649	186,048	176,804	162,969
13118	6.7	-	45.5	285,480	278,412	247,013	217,905	198,126	190,061	184,809
13121	8.4	-	43.8	408,768	514,717	571,791	609,025	619,961	622,640	631,163
13122	10.5	-	41.7	376,724	446,059	462,954	442,328	420,187	419,017	424,801
12207	17.8	37.6	34.4	86,372	160,001	253,591	344,552	400,870	427,443	456,210
12220	23.5	31.9	28.7	25,672	39,166	56,485	82,936	106,635	124,682	140,059
12217	28.6	26.8	23.6	63,745	109,239	150,635	203,063	239,199	273,128	305,058
12222	31.7	23.7	20.5	27,063	33,216	49,240	76,218	101,061	111,659	120,628
8217	36.5	18.9	15.7	22,582	26,179	40,287	52,821	71,246	78,608	81,665
8563	41.4	14.0	10.8	12,606	13,002	16,309	20,407	26,464	29,757	32,744
8208	45.6	9.8	6.6	33,581	34,917	37,267	40,569	43,131	48,857	57,238
8219	48.0	7.4	4.2	16,131	17,203	19,372	27,674	40,170	51,926	60,693

## (d) Tokaido-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960	1965	1970	1975	1980	1985	1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>							
13100 <sup>(1)</sup>	7.4	50.1	-	8,310,027	8,893,094	8,840,942	8,642,800	8,349,209	8,354,615	8,163,573
13101	2.1	-	50.1	116,944	93,047	74,185	61,656	54,801	50,493	39,472
13103	2.4	-	49.8	267,024	241,539	223,978	209,492	201,257	194,591	158,499
13109	8.1	-	44.1	427,859	423,015	397,302	366,058	346,247	357,732	344,611
13111	11.6	-	40.6	706,219	755,535	734,990	691,337	661,147	662,814	647,914
14130 <sup>(4)</sup>	17.2	40.3	-	632,975	854,866	973,486	1,014,951	1,040,802	1,088,624	1,173,603
14132	15.6	-	36.6	632,975 <sup>(6)</sup>	854,866 <sup>(6)</sup>	155,549	148,756	138,585	137,306	142,320
14131	16.9	-	35.3	632,975 <sup>(6)</sup>	854,866 <sup>(6)</sup>	251,906	216,569	199,148	193,954	200,056
14100 <sup>(5)</sup>	25.8	31.7	-	1,375,710	1,470,316	1,935,412	2,621,771	2,773,674	2,992,926	3,220,311
14101	19.8	-	32.4	230,377	255,755	256,403	242,808	231,477	237,083	250,100
14102	24.9	-	27.3	172,068	196,559	207,319	213,654	201,794	188,952	194,506
14103	27.6	-	24.6	104,173	104,352	97,906	89,015	80,539	78,858	76,978
14106	28.0	-	24.2	143,804	229,724	327,953	377,337	390,747	419,468	432,585
14110	37.1	-	15.1	113,514	155,645	248,696	339,420	401,973	444,116	453,773
14204	44.3	13.2	7.9	98,617	118,329	139,249	165,552	172,629	175,495	174,307
14205	44.9	12.6	7.3	124,601	175,183	228,978	265,975	300,248	328,387	350,330
14207	50.1	7.4	2.1	68,054	100,081	129,621	152,023	171,016	185,030	201,675

## (e) Sobu-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960	1965	1970	1975	1980	1985	1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>							
13100 <sup>(1)</sup>	7.4	49.8	-	8,310,027	8,893,094	8,840,942	8,642,800	8,349,209	8,354,615	8,163,573
13101	2.1	-	49.8	116,944	93,047	74,185	61,656	54,801	50,493	39,472
13107	3.8	-	48.1	331,843	317,856	281,237	250,714	232,796	229,986	222,944
13106	4.2	-	47.7	318,889	286,324	240,769	207,649	186,048	176,804	162,969
13108	4.9	-	47.0	351,053	359,672	355,835	355,382	362,270	388,927	385,159
13123	10.0	-	41.9	316,593	405,139	446,758	473,656	495,231	514,812	565,939
13122	10.5	-	41.4	376,724	446,059	462,954	442,328	420,187	419,017	424,801
12203	16.8	40.4	35.1	157,301	207,988	261,055	319,272	364,244	397,822	436,596
12204	20.0	37.2	31.9	135,038	223,989	325,426	423,160	479,437	506,966	533,270
12216	24.0	33.2	27.9	42,167	64,477	99,951	117,851	125,154	136,365	151,471
12201	31.7	25.5	20.2	241,615	332,188	482,133	659,356	746,430	788,930	829,455
12228	36.5	20.7	15.4	16,623	19,778	26,375	37,401	59,236	67,008	72,157
12212	41.7	15.5	10.2	36,869	40,941	60,433	80,804	101,180	121,213	144,688
12322	45.8	11.4	6.1	6,093	6,040	6,259	8,465	12,807	17,463	19,298
12323	49.8	7.4	2.1	25,387	25,173	25,357	28,511	31,939	37,532	50,036

## Notes

- (1) Code 13100 is for Tokyo city with 23 wards.
- (2) Localities under type a are those for the aggregated case.
- (3) Localities under type d are those for the disaggregated case.
- (4) Code 14130 is for Kawasaki city.
- (5) Code 14100 is for Yokohama city.
- (6) These figures represent the population of Kawasaki-city. Saiwai-ku (14132) and Kawasaki-ku (14131) were designated as *ku* (i.e., ward) in April of 1972. Before that, each of them was simply a part of Kawasaki city, which makes their population statistics unavailable from the national population census for the years 1960 and 1965.



Mathematical Characteristics of ROXY Index (I):  
Distance and Reversed Distance Used as Weighing Factors (Kawashima, Hiraoka)

**Table A-2 Annual Growth Ratio of Population for Localities of Five Railway-line Regions in the Tokyo Metropolitan Area**

(a) Chuo-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960-1965	1965-1970	1970-1975	1975-1980	1980-1985	1985-1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>						
13100 <sup>(1)</sup>	7.4	55.5	-	1.0137	0.9988	0.9955	0.9931	1.0006	0.9954
13102	1.1	-	55.5	0.9548	0.9590	0.9720	0.9830	0.9933	0.9682
13101	2.1	-	54.5	0.9553	0.9557	0.9637	0.9767	0.9838	0.9519
13104	5.7	-	50.9	1.0001	0.9885	0.9877	0.9870	0.9934	0.9774
13113	6.1	-	50.5	1.0007	0.9934	0.9921	0.9869	0.9963	0.9676
13114	9.6	-	47.0	1.0140	1.0011	0.9970	0.9849	0.9943	0.9901
13115	11.7	-	44.9	1.0196	1.0060	1.0028	0.9934	0.9990	0.9961
13203	18.5	44.4	38.1	1.0210	1.0051	1.0037	0.9962	1.0027	1.0004
13204	18.5	44.4	38.1	1.0675	1.0276	1.0115	0.9995	1.0022	0.9992
13210	23.7	39.2	32.9	1.1079	1.0435	1.0169	0.9994	1.0043	1.0024
13206	25.8	37.1	30.8	1.0899	1.0527	1.0225	1.0103	1.0102	1.0072
13214	27.5	35.4	29.1	1.1067	1.0459	1.0164	1.0064	1.0096	1.0113
13215	29.2	33.7	27.4	1.0592	1.0655	1.0153	0.9992	1.0023	1.0029
13202	31.0	31.9	25.6	1.0421	1.0306	1.0336	1.0064	1.0054	1.0085
13212	33.2	29.7	23.4	1.0939	1.0771	1.0516	1.0279	1.0142	1.0124
13201	40.3	22.6	16.3	1.0476	1.0406	1.0493	1.0372	1.0196	1.0180
14424	55.5	7.4	1.1	0.9957	0.9958	1.0066	1.0201	1.0147	1.0104

(b) Takasaki-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960-1965	1965-1970	1970-1975	1975-1980	1980-1985	1985-1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>						
13100 <sup>(1)</sup>	7.4	58.0	-	1.0137	0.9988	0.9955	0.9931	1.0006	0.9954
13106	4.2	-	58.0	0.9787	0.9659	0.9708	0.9783	0.9899	0.9838
13118	6.7	-	55.5	0.9950	0.9764	0.9752	0.9811	0.9917	0.9944
13117	8.9	-	53.3	1.0155	0.9906	0.9947	0.9840	0.9895	0.9929
11203	14.8	50.6	47.4	1.0748	1.0419	1.0247	1.0188	1.0122	1.0171
11223	18.0	47.4	44.2	1.0647	1.0207	0.9976	0.9853	0.9987	1.0090
11204	23.2	42.2	39.0	1.0488	1.0401	1.0421	1.0158	1.0104	1.0209
11220	26.0	39.4	36.2	1.0485	1.0395	1.0250	1.0036	0.9952	1.0229
11205	28.0	37.4	34.2	1.0487	1.0450	1.0404	1.0156	1.0105	1.0160
11219	36.5	28.9	25.7	1.0709	1.1513	1.0573	1.0258	1.0144	1.0177
11231	40.2	25.2	22.0	1.0569	1.0661	1.0441	1.0302	1.0198	1.0234
11233	44.0	21.4	18.2	1.0585	1.0903	1.0803	1.0176	1.0269	1.0193
11217	48.0	17.4	14.2	1.0277	1.0283	1.0422	1.0203	1.0119	1.0364
11304	54.5	10.9	7.7	1.0367	1.0356	1.0171	1.0378	1.0203	1.0151
11206	58.0	7.4	4.2	1.0051	1.0138	1.0190	1.0207	1.0163	1.0095

(c) Joban-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960-1965	1965-1970	1970-1975	1975-1980	1980-1985	1985-1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>						
13100 <sup>(1)</sup>	7.4	48.0	-	1.0137	0.9988	0.9955	0.9931	1.0006	0.9954
13106	4.2	-	48.0	0.9787	0.9659	0.9708	0.9783	0.9899	0.9838
13118	6.7	-	45.5	0.9950	0.9764	0.9752	0.9811	0.9917	0.9944
13121	8.4	-	43.8	1.0472	1.0213	1.0127	1.0036	1.0009	1.0027
13122	10.5	-	41.7	1.0344	1.0075	0.9909	0.9898	0.9994	0.0027
12207	17.8	37.6	34.4	1.1312	1.0965	1.0632	1.0307	1.0129	1.0131
12220	23.3	31.9	28.7	1.0882	1.0760	1.0798	1.0516	1.0318	1.0235
12217	28.6	26.8	23.6	1.1137	1.0664	1.0616	1.0333	1.0269	1.0224
12222	31.7	23.7	20.5	1.0418	1.0819	1.0913	1.0580	1.0201	1.0156
8217	36.5	18.9	15.7	1.0300	1.0900	1.0557	1.0617	1.0199	1.0077
8563	41.4	14.0	10.8	1.0062	1.0464	1.0459	1.0534	1.0237	1.0193
8208	45.6	9.8	6.6	1.0078	1.0131	1.0171	1.0123	1.0252	1.0322
8219	48.0	7.4	4.2	1.0130	1.0240	1.0739	1.0774	1.0527	1.0317

## (d) Tokaido-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960-1965	1965-1970	1970-1975	1975-1980	1980-1985	1985-1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>						
13100 <sup>(1)</sup>	7.4	50.1	-	1.0137	0.9988	0.9955	0.9931	1.0006	0.9954
13101	2.1	-	50.1	0.9553	0.9557	0.9637	0.9767	0.9838	0.9519
13103	2.4	-	49.8	0.9801	0.9850	0.9867	0.9920	0.9933	0.9598
13109	8.1	-	44.1	0.9977	0.9875	0.9838	0.9889	1.0065	0.9926
13111	11.6	-	40.6	1.0136	0.9945	0.9878	0.9911	1.0005	0.9955
14130 <sup>(4)</sup>	17.2	40.3	-	1.0619	1.0263	1.0084	1.0050	1.0090	1.0152
14132	15.6	-	36.6	1.0619 <sup>(8)</sup>	1.0263 <sup>(8)</sup>	0.9911	0.9859	0.9981	1.0072
14131	16.9	-	35.3	1.0619 <sup>(8)</sup>	1.0263 <sup>(8)</sup>	0.9702	0.9834	0.9947	1.0062
14100 <sup>(8)</sup>	25.8	31.7	-	1.0134	1.0557	1.0627	1.0113	1.0153	1.0148
14101	19.8	-	32.4	1.0211	1.0005	0.9892	0.9905	1.0048	1.0107
14102	24.9	-	27.3	1.0270	1.0107	1.0060	0.9886	0.9869	1.0058
14103	27.6	-	24.6	1.0003	0.9873	0.9811	0.9802	0.9958	0.9952
14106	28.0	-	24.2	1.0982	1.0738	1.0285	1.0070	1.0143	1.0062
14110	37.1	-	15.1	1.0652	1.0983	1.0642	1.0344	1.0201	1.0043
14204	44.3	13.2	7.9	1.0371	1.0331	1.0352	1.0084	1.0033	0.9986
14205	44.9	12.6	7.3	1.0705	1.0550	1.0304	1.0245	1.0181	1.0130
14207	50.1	7.4	2.1	1.0802	1.0531	1.0324	1.0238	1.0159	1.0174

## (e) Sobu-line region

(unit of distance: km)

Code	Distance	Reversed distance		1960-1965	1965-1970	1970-1975	1975-1980	1980-1985	1985-1990
		Type a <sup>(2)</sup>	Type d <sup>(3)</sup>						
13100 <sup>(1)</sup>	7.4	49.8	-	1.0137	0.9988	0.9955	0.9931	1.0006	0.9954
13101	2.1	-	49.8	0.9553	0.9557	0.9637	0.9767	0.9838	0.9519
13107	3.8	-	48.1	0.9914	0.9758	0.9773	0.9853	0.9976	0.9938
13106	4.2	-	47.7	0.9787	0.9659	0.9708	0.9783	0.9899	0.9838
13108	4.9	-	47.0	1.0049	0.9979	0.9997	1.0038	1.0143	0.9981
13123	10.0	-	41.9	1.0506	1.0197	1.0118	1.0089	1.0078	1.0191
13122	10.5	-	41.4	1.0344	1.0075	0.9909	0.9898	0.9994	1.0027
12203	15.7	40.4	35.1	1.0575	1.0465	1.0411	1.0267	1.0178	1.0188
12204	19.5	37.2	31.9	1.1065	1.0776	1.0539	1.0253	1.0112	1.0102
12216	23.5	33.2	27.9	1.0901	1.0902	1.0335	1.0121	1.0173	1.0212
12201	31.7	25.5	20.2	1.0657	1.0773	1.0646	1.0251	1.0111	1.0101
12228	36.5	20.7	15.4	1.0354	1.0593	1.0724	1.0963	1.0250	1.0149
12212	41.7	15.5	10.2	1.0212	1.0810	1.0598	1.0460	1.0368	1.0360
12322	45.8	11.4	6.1	0.9983	1.0071	1.0622	1.0863	1.0640	1.0202
12323	49.8	7.4	2.1	0.9983	1.0015	1.0237	1.0230	1.0328	1.0592

## Notes

- (1) The code 13100 is for Tokyo city with 23 wards.
- (2) Localities under type a are those for the aggregated case.
- (3) Localities under type b are those for the disaggregated case.
- (4) The code 14130 is for Kawasaki city.
- (5) The code 14100 is for Yokohama city.
- (6) These figures represent the population of Kawasaki-city. Saiwai-ku (14132) and Kawasaki-ku (14131) were designated as *ku* (ward) in April 1972. Before that, each of them was simply a part of Kawasaki city, which makes their population statistics unavailable from the national population census for years 1960 and 1965. It should be, however, noted that unavailability of these data would not seem to cause serious distortions in the results of our analyses, mainly because of the fact that we calculate the value of our ROXY index in terms of the annual growth ratio of the population (instead of the annual increment or decrement of the population).

**Table A-3 Average of CBD Distance, Average of Reversed CBD Distance, and RR-ratio: Aggregated and Disaggregated Cases for Five Railway-line Regions**

(unit of distance: km)

Railway-line region	Aggregated case			Disaggregated case		
	Average of CBD distance (A)	Average of reversed CBD distance (B)	RR-ratio (A/B)	Average of CBD distance (A)	Average of reversed CBD distance (B)	RR-ratio (A/B)
Chuo-line region	28.2	34.7	0.81	21.2	35.4	0.60
Takasaki-line region	33.2	32.2	1.03	29.4	32.8	0.90
Joban-line region	31.2	24.2	1.29	25.2	27.0	0.93
Tokaido-line region	31.6	25.9	1.22	23.8	28.4	0.84
Sobu-line region	30.4	26.8	1.13	21.6	30.3	0.71

*Note*

RR-ratio refers to the absolute value of the ratio of  $R_1$  to  $R_2$  which is equal to the average of CBD distance divided by the average of reversed CBD distance.