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Mathematical Characteristics of the ROXY Index (V): Comparison of the ROXY Index with Other Major Yardsticks Measuring Convergence and Divergence

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CONTENTS

Abstract

In the social sciences, various types of indices and coefficients have been developed in order to measure the phenomena of convergence and divergence of socio-economic activities. This paper compares seven such yardsticks, including the ROXY index, and theoretically categorizes them into five groups according to their kernel, a mathematical factor differentiated with respect to time. The yardsticks are once again grouped, but this time according to the empirical results of the inter-metropolitan analysis of population changes in Japan. It is found that the theoretical groupings are consistent with the empirical results.

Key Words

Coefficient of variation, Concentration, Convergence, Gini coefficient, Herfindahl coefficient, Hoover index, Kernel, Metropolitan area, Population, Rosenbluth coefficient, ROXY index, Spatial cycles, and Theil coefficient

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1 Introduction

A reasonably large number of attempts have been made to develop yardsticks to quantitatively measure the degree of convergence and divergence of socio-economic activities. For example, the *Hoover index* and *ROXY index* have been constructed originally to measure the degree of concentration and deconcentration of population; *the coefficient of variation, Gini coefficient,* and *Theil coefficient* to measure the degree of social inequality in income distribution; and the *Herfindahl coefficient* and *Rosenbluth coefficient* to measure the degree of market share.

The primary goal of this paper is to compare the above seven yardsticks by investigating how the ROXY index differs from the other six yardsticks when they are applied to the same data describing the spatial distribution of population for a system of metropolitan areas in Japan. Section 2 provides definitions for each of the seven yardsticks and discusses their comparable reformulations, while in Section 3 an attempt is made to theoretically categorize them into groups based on their *kernel.* Section 4 shows that empirical results obtained through our inter-metropolitan analysis verify the above categorization. In Section 5, concluding remarks are provided.

2 Definitions for Seven Yardsticks and Their Comparable Reformulations

Among the seven yardsticks to be investigated in this paper, the ROXY index[1] has been developed to identify the dynamic degree[2] of spatial convergence and divergence[3] of socio-economic activities, while the other six yardsticks have been developed to idenfity the static degree[4] of convergence and divergence. Therefore, for comparing the ROXY index with the other six yardsticks, it would be necessary to obtain the derivative for each of the six yardsticks with respect to time t. For each yardstick, the definitional formulation and its derivative use the following notational conventions and functional relationships (1) through (4);

$$\mu = \sum_{i=1}^{n} y_i / n \tag{1}$$

$$S_i = y_i / \sum_{i=1}^{n} y_i \tag{2}$$

$$y_i = n\mu S_i \tag{3}$$

$$\sum_{i=1}^{n} S_i = 1 \tag{4}$$

where n : number of metropolitan areas composing the system of metropolitan areas under consideration,

y_i : population of metropolitan area i, where i is given in descending order in terms of population at time t,

μ : average population for all metropolitan areas, and

s_i : population share of metropolitan area i.

2.1 Coefficient of Variation

The square of *the coefficient of variation* C is defined as the variance σ^2 $(=\sum_{i=1}^{n}(y_i-\mu)^2/n)$ divided by the square of average (μ^2);

$$C^2 = \frac{1}{n}\sum_{i=1}^{n}\frac{(y_i-\mu)^2}{\mu^2}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\frac{(n\mu s_i-\mu)^2}{\mu^2}$$

$$= \frac{1}{n}\sum_{i=1}^{n}(ns_i-1)^2 . \qquad (5)$$

The derivative of C with respect to t, is obtained from Equation (5) as follows;

$$2C\frac{dC}{dt} = \frac{1}{n}\sum_{i=1}^{n}2n(ns_i-1)\frac{ds_i}{dt}$$

$$= 2\sum_{i=1}^{n}(ns_i-1)\frac{ds_i}{dt}$$

$$\therefore \frac{dC}{dt} = \frac{\sum_{i=1}^{n}(ns_i-1)\frac{ds_i}{dt}}{C}$$

$$= \frac{\frac{n}{2}\sum_{i=1}^{n}\frac{ds_i^2}{dt}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(ns_i-1)^2}} . \qquad (6)$$

2.2 Gini Coefficient

The Gini coefficient G is given by;

$$G = 1 + \frac{1}{n} - 2 \sum_{i=1}^{n} \frac{i}{\mu n^2} y_i$$

$$= 1 + \frac{1}{n} - \frac{2}{n} \sum_{i=1}^{n} i s_i \quad . \tag{7}$$

This coefficient measures the relative mean difference corresponding to the area enclosed by the equal-distribution line and the Lorenz curve drawn on the plane with the abscissa indicating the culmulative frequency and the ordinate indicating the cumulative share of population. In the context of our inter-metropolitan analysis, the value of G becomes 0.0 when the population y_i is identical for all n metropolitan areas (i.e., for all i). It turns out to be equal to 1.0 when the total population is concentrated exclusively in one metropolitan area. The derivative of G with respect to t, is obtained from Equation (7);

$$\frac{dG}{dt} = -\frac{2}{n} \sum_{i=1}^{n} i \frac{ds_i}{dt} \quad . \tag{8}$$

Note that Equation (8) holds only when the ranks in population size for all metropolitan areas remain unchanged between time t and time $t+dt$. Accordingly, in general, the difference has to be taken in a discrete manner as follows;

$$\frac{\Delta G}{\Delta t} = -\frac{2}{n} \sum_{i=1}^{n} \frac{\Delta(i s_i)}{\Delta t} \quad . \tag{9}$$

2.3 Herfindahl Coefficient

The Herfindahl coefficient H is defined as the summation of the square of population share;

$$H = \sum_{i=1}^{n} s_i^2 \quad . \tag{10}$$

The value of H is $1/n$ for an equal distribution of population, and 1.0 for the case when population is monopolized by only one metropolitan area. The derivative of H with respect to t, is obatained from Eaquation (10);

$$\frac{dH}{dt} = \sum_{i=1}^{n} \frac{ds_i^2}{dt} \quad . \tag{11}$$

2.4 Hoover Index

When the land-area share of metropolitan area i is given by a_i, *the Hoover index J* is;

$$J = \frac{1}{2}\sum_{i=1}^{n} |S_i - a_i| \times 100 . \tag{12}$$

As can be easily seen, if $S_i = a_i$ for all i, (that is, if the population is uniformly distributed with respect to land-area, making all metropolitan areas have the same population density), then the value of J is equal to 0.0. If only one metropolitan area monopolizes the total population and if its land-area is negligibly small relative to the total land-area of all metropolitan areas, then the value of J approaches 100.0 as its limit. Actually, the value of J which ranges from 0.0 to 100.0, indicates the percentage of the total population which must be spatially resettled in order to equalize population densities for all metropolitan areas. The derivative of J with respect to t, is obtained from Equation (12);

$$\frac{dJ}{dt} = \frac{1}{2}\sum_{i=1}^{n} \frac{d}{dt}|S_i - a_i| \times 100 . \tag{13}$$

In Equation (13), if S_i is not equal to a_i for $\forall i$, then we can divide metropolitan areas into two groups A and B from the viewpoint of differentiability; $A = \{ i \mid S_i - a_i > 0\}$ and $B = \{i \mid S_i - a_i < 0\}$. It is to be noted that if S_i changes crossing over the value of a_i for $\exists i$, then J can not be differentiated.

2.5 Rosenbluth Coefficient

The Rosenbluth coefficient R is defined as follows;

$$R = \frac{1}{2\sum_{i=1}^{n} iS_i - 1} . \tag{14}$$

The value of R is equal to $1/n$ for the case of equal population distribution over n metropolitan areas, and 1.0 for the case when the total population is monopolized by one metropolitan area. The derivative of R with respect to t, is obtained from Equation (14);

$$\frac{dR}{dt} = \frac{-2\sum_{i=1}^{n} i\frac{dS_i}{dt}}{(2\sum_{i=1}^{n} iS_i - 1)^2} . \tag{15}$$

Note that Equation (15) holds only when the ranks in population size for all metropolitan areas remain unchanged between time t and time $t+dt$. Accordingly, in general, the difference has to be taken in a discrete manner as follows;

$$\frac{\Delta R}{\Delta t} = \frac{-2\sum_{i=1}^{n}\frac{\Delta(iS_i)}{\Delta t}}{(2\sum_{i=1}^{n} iS_i - 1)^2} \quad (16)$$

2.6 ROXY Index

The ROXY index[5], ROXY, is defined as follows[6];

$$ROXY = \frac{WA}{SA} - 1, \quad (17)$$

where WA and SA respectively indicate weighted and simple averages of the growth ratio. Usually, the population level of each metropolitan area is employed as a weighting factor for the calculation of weighted averages. Taking this practice, we have the following expressions;

$$SA = \frac{1}{n}\sum_{i=1}^{n}\left\{1 + \frac{1}{y_i}\frac{dy_i}{dt}\right\}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left\{1 + \frac{1}{y_i}\frac{d}{dt}(n\mu S_i)\right\}$$

$$= \frac{1}{n}\left\{n + \frac{n}{\mu}\frac{d\mu}{dt} + \sum_{i=1}^{n}\frac{1}{S_i}\frac{dS_i}{dt}\right\}$$

$$= 1 + \frac{d}{dt}(\ln\mu) + \frac{1}{n}\sum_{i=1}^{n}\frac{d}{dt}(\ln S_i)$$

$$WA = \frac{\sum_{i=1}^{n} n\mu S_i\left\{1 + \frac{1}{y_i}\frac{dy_i}{dt}\right\}}{\sum_{i=1}^{n} n\mu S_i}$$

$$= \sum_{i=1}^{n} S_i \left\{ 1 + \frac{1}{\mu}\frac{d\mu}{dt} + \frac{1}{S_i}\frac{dS_i}{dt} \right\}$$

$$= 1 + \frac{d}{dt}(\ln \mu)$$

$$\therefore \ ROXY_1 = \frac{1+\frac{d}{dt}(\ln \mu)}{1+\frac{d}{dt}(\ln \mu)+\frac{1}{n}\sum_{i=1}^{n}\frac{d}{dt}(\ln S_i)} - 1$$

$$= \frac{-\frac{1}{n}\sum_{i=1}^{n}\frac{d}{dt}(\ln S_i)}{1+\frac{d}{dt}(\ln \mu)+\frac{1}{n}\sum_{i=1}^{n}\frac{d}{dt}(\ln S_i)} \quad . \qquad (18)$$

It should be noted here that Equation (18) includes the time-derivative of average population μ[7], and that this μ is indifferent to population concentration or deconcentration (that is, the shape of the spatial distribution of population) among metropolitan areas. Thus, by designating this conventional type of ROXY index as $ROXY_1$, let us introduce a new type of ROXY index designated as $ROXY_2$, from the viewpoint of mathematical comparability of the ROXY index with other yardsticks. This $ROXY_2$ employs the growth ratio of population share as a principal variable[8] and population share as a weighting factor. For $ROXY_2$, we have the following expressions. Note that the time-derivative of average population μ would not appear in the formulation;

$$SA = \frac{1}{n}\sum_{i=1}^{n}\left\{1 + \frac{1}{S_i}\frac{dS_i}{dt}\right\}$$

$$= 1 + \frac{1}{n}\sum_{i=1}^{n}\frac{d}{dt}(\ln S_i)$$

$$WA = \frac{\sum_{i=1}^{n} S_i\left\{1+\frac{1}{S_i}\frac{dS_i}{dt}\right\}}{\sum_{i=1}^{n} S_i}$$

$$= 1 + \sum_{i=1}^{n} \frac{dS_i}{dt}$$

$$= 1$$

$$ROXY_2 = \frac{1}{1+\frac{1}{n}\sum_{i=1}^{n} \frac{d}{dt}(\ln S_i)} - 1$$

$$= \frac{-\frac{1}{n}\sum_{i=1}^{n} \frac{d}{dt}(\ln S_i)}{1+\frac{1}{n}\sum_{i=1}^{n} \frac{d}{dt}(\ln S_i)} \quad (19)$$

2.7 Theil Coefficient

Theil coefficient T is considered as a measurement carrying a kind of entropy concept and is defined as follows;

$$T = \sum_{i=1}^{n} \{S_i \ln(1/S_i)\}$$
$$= -\sum_{i=1}^{n} S_i \ln S_i \quad (20)$$

The value of T is equal to *ln (n)*, the natural logarithm of *n*, in the case of equal population distribution over *n* metropolitan areas, and 0.0 in the case where the total population is monopolized by only one metropolitan area. The derivative of T with respect t, is obtained from Equation (20);

$$\frac{dT}{dt} = \sum_{i=1}^{n} \frac{d}{dt}(S_i \ln S_i) \quad (21)$$

3 Theoretical Classification of Seven Yardsticks by Kernel

In the attempt to theoretically classify the seven yardsticks so far investigated, let us pay special attention to the *kernel* component in the time-dervative for each yardstick. The kernel is defined as the *time differentiatee* (i. e., what is to be differentiated with respect to time) that depends upon i in equations (6) for C, (9) for G, (11) for H, (13) for J, (16) for R, (18) for $ROXY_1$, (19) for $ROXY_2$ and (21) for T. As shown in Table 1, the kernel is S_i^2 for the coefficient of variation C and for the Herfindahl coefficient H as indicated by Equations (6) and (11) respectively; iS_i for the Gini coefficient G and for the Rosenbluth coefficient R as indicated by Equations (9) and (16) respectively; $|S_i - a_i|$ for the Hoover index J as indicated by Equation (13); lnS_i for the ROXY indices $ROXY_1$ and $ROXY_2$ as indicated by Equations (18) and (19) respectively; and $S_i lnS_i$ for the Theil coefficient T as indicated by Equation (21).

In Table 1, the seven yardsticks are classified into five groupes according to their kernel; $ROXY_1$ and $ROXY_2$ in Type Ⅰ, G and R in Type Ⅱ, T in Type Ⅲ, C and H in Type Ⅳ, and J in Type Ⅴ. The basic features of kernel can be compared diagrammatically, as in Figure 1, among yardsticks belonging to Types Ⅰ, Ⅲ and Ⅳ. Among these three types, the curve of the kernel for Type-Ⅰ is steepest for the domain of smaller population shares, while the curve of the kernel for Type-Ⅳ is steepest for the domain of larger population shares. The curve for Type-Ⅲ is steepest in the domain of middle-size population shares. From this observation, it can be pointed out that the Type-Ⅰ yardsticks (i. e., ROXY indices) are sensitive to dynamic change in the part of smaller population shares, the Type-Ⅲ yardstick in the part of middle-size population shares, and the Type-Ⅳ yardsticks in the part of larger population shares.

Table 1 Classification by Kernel

Type	Kernel	Yardsticks
Ⅰ	$\ln S_i$	ROXY index ($ROXY_1$ and $ROXY_2$)
Ⅱ	iS_i	Gini coefficient (G) Rosenbluth coefficient (R)
Ⅲ	$S_i \ln S_i$	Theil coefficient (T)
Ⅳ	S_i^2	Coefficient of variation (C) Herfindahl coefficient (H)
Ⅴ	$\|S_i - a_i\|$	Hoover index (J)

Figure 2 is given for the examination of the characteristics of the Type-Ⅱ yardstics. The curve for share S_i is expressed by the 45° line in this figure. The curve for population rank i is convex with respect to the origin and monotonically decreasing in case we apply the rank-size rule to our consideration. The kernel iS_i which is the product of the rank i and share S_i, has its maximum slope in the domain of smaller population shares[9]. It therefore follows that the Type-Ⅱ yardsticks are sensitive to change in the part of smaller population shares but in a somewhat larger domain of population shares compared with the Type Ⅰ yardsticks. Type-Ⅱ yardsticks therefore can perhaps be placed between the Type-Ⅰ and Type-Ⅲ yardsticks.

The curve of the kernel for the Type-Ⅴ yardstick is provided in Figure 3. The kernel curve in this figure tells us that, for $S_i - a_i > 0$, the derivative of kernel with respect to S_i is constantly equal to 1.0, and then, for $S_i - a_i < 0$, it is constantly equal to -1.0. Thus, it is indicated that there is no difference existing, with respect to sensitivity of this type of yardstick, between any two metropolitan areas whose population shares are commonly greater than a_i or commonly less than a_i.

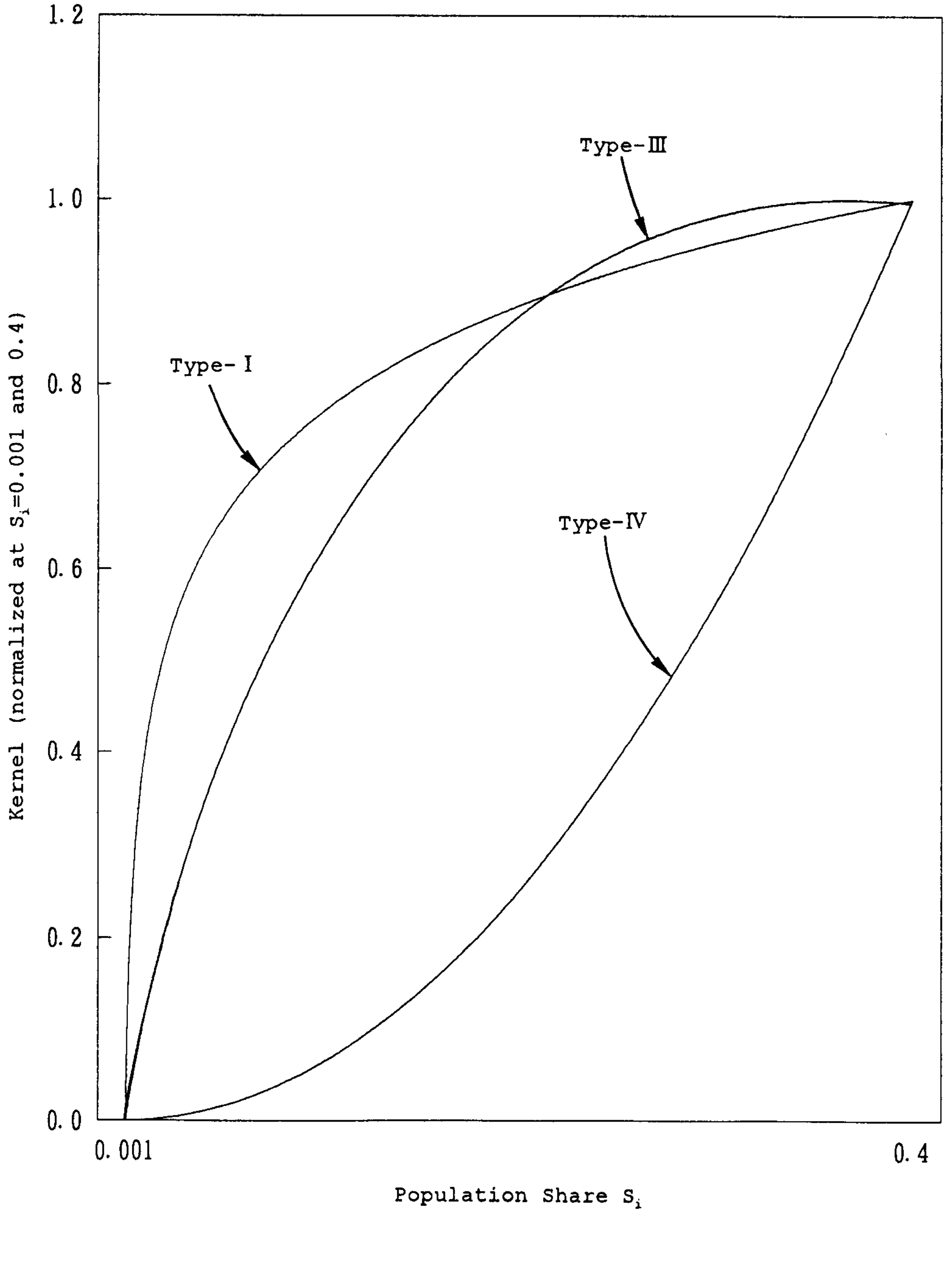


Figure 1 Value of Kernels for Type-Ⅰ, Type-Ⅲ and Type-Ⅳ over Population Share

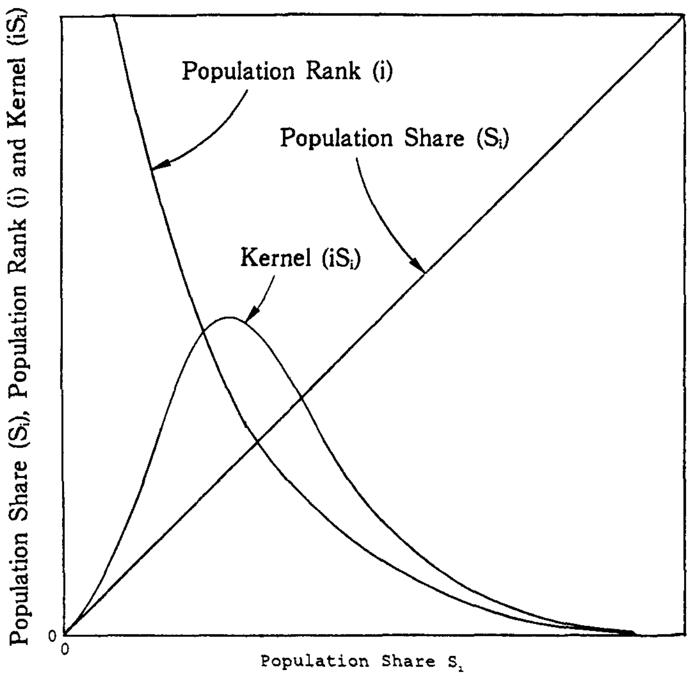


Figure 2 Value of Kernel for Type-II over Population Share

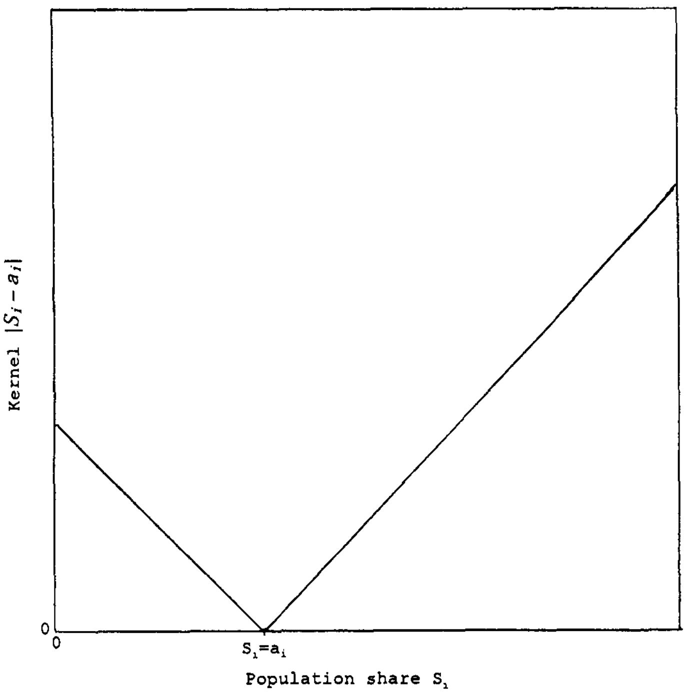


Figure 3 Value of Kernel for Type-V over Population Share

4 Empirical Results Obtained For Each Yardstick

We have examined the direction of changes in population concentration and deconcentration

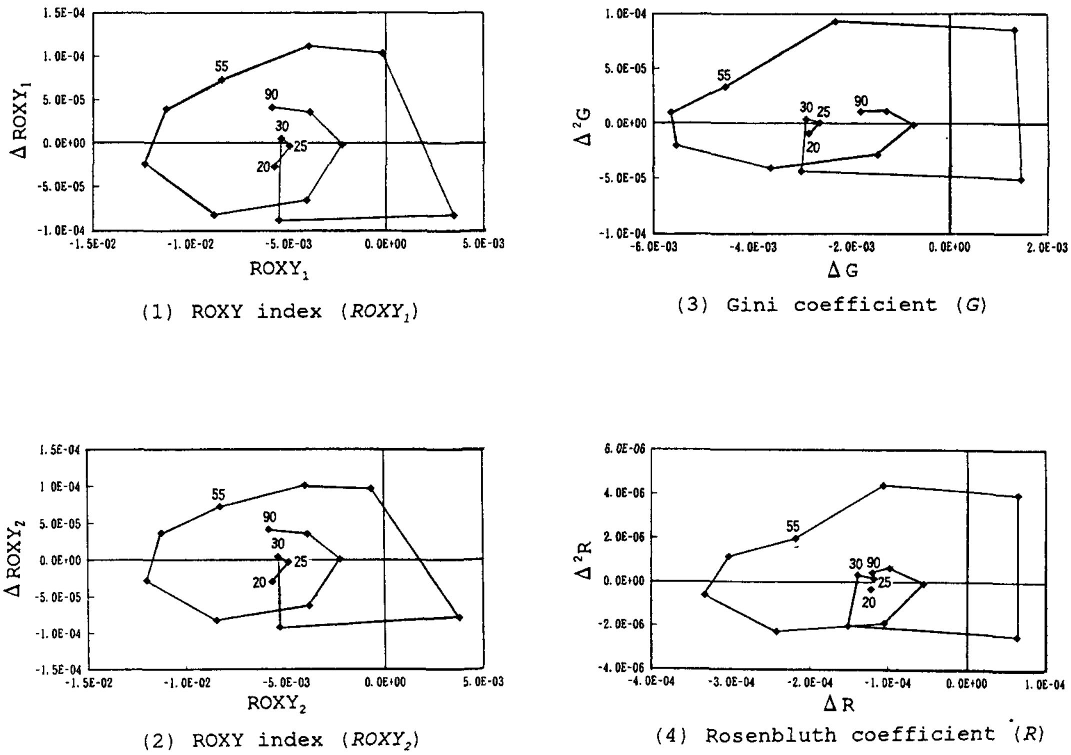
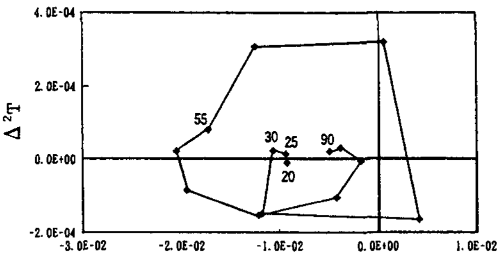


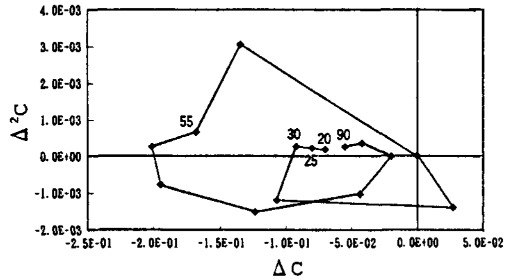
Figure 4 Values and Their Marginal Changes for the Yardsticks (Except J)

of metropolitan areas in Japan in order to compare the theoretical grouping of yardsticks by kernel with the grouping by empirical means. In this examination, 92 functional urban regions (FURs) employed by Hiraoka (1995) have been used as spatial units of analysis. For each FUR, the time-series data covering the period of 1920-90 on the population and the data on the area and its share, are shown in Tables A-1 and A-2 respectively. Results appear in Table 2[9] and Figures 4 and 5.

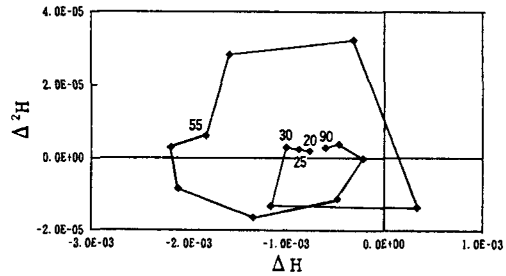
In Figure 4, panels (1) and (2) present the yardsticks which are sensitive to change over smaller values of the domain (population share). Panels (3) and (4), and panel (5), present yardsticks sensitive to change over lower middle and upper middle values, respectively, of the domain. Panels (6) and (7) present the yardsticks sensitive to change over larger values of the dpomain. We can therefore see in Figure 4 that the shape of the trajectory produced by yardsticks almost continuously changes as we go from panels (1) and (2) through panels (6) and (7). For example, the first three points which corresponds to years 1920, 1925 and 1930 respectively, make a triangle in panels (1) and (2), while this triangle shape gradually collapses into a single line as we move to panels (6) and (7) via panels (3) and (4), and panel (5). Another example is that, as we move from panels (1) and (2) through panels (6) and (7),



(5) Theil coefficient (T)



(6) Coefficient of variation (C)



(7) Herfindahl coefficient (H)

Figure 4 (*continued*)

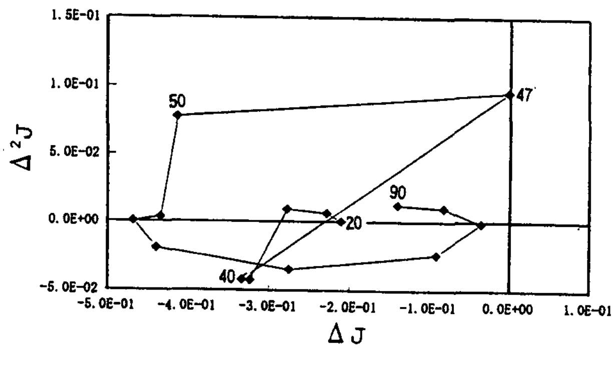


Figure 5 Values and Their Marginal Changes for the Hoover Index (J)

Table 2 Values and Their Marginal Changes for the Seven Yardsticks : Applied to the Direction of Change in Population Concentration in Japanese Metropoitan Areas

Indices	1920	1925	1930	1935	1940	1947	1950	1955	1960	1965	1970	1975	1980	1985	1990
$ROXY_1$	5.62E-03	4.80E-03	5.33E-03	5.24E-03	-3.82E-03	6.57E-04	3.97E-03	8.26E-03	1.12E-02	1.20E-02	8.44E-03	3.76E-03	2.23E-03	3.86E-03	5.80E-03
Δ $ROXY_1$	-2.97E-04	-2.88E-05	4.37E-05	-9.15E-04	-7.91E-04	9.64E-04	1.01E-03	7.27E-04	3.72E-04	-2.79E-04	-8.22E-04	-6.21E-04	9.78E-06	3.57E-04	4.18E-04
$ROXY_2$	5.67E-03	4.92E-03	5.35E-03	5.47E-03	-3.47E-03	1.56E-04	3.94E-03	8.39E-03	1.12E-02	1.23E-02	8.80E-03	4.07E-03	2.23E-03	3.89E-03	5.81E-03
Δ $ROXY_2$	-2.70E-04	-3.28E-05	5.49E-05	-8.82E-04	-8.27E-04	1.04E-03	1.12E-03	7.28E-04	3.94E-04	-2.42E-04	-8.26E-04	-6.57E-04	-1.78E-05	3.58E-04	4.11E-04
G	4.92E-01	5.05E-01	5.18E-01	5.35E-01	5.48E-01	4.97E-01	5.01E-01	5.21E-01	5.46E-01	5.78E-01	6.02E-01	6.14E-01	6.16E-01	6.21E-01	6.29E-01
Δ G	2.87E-03	2.64E-03	2.92E-03	3.01E-03	-1.47E-03	-1.33E-03	2.31E-03	4.55E-03	5.64E-03	5.54E-03	3.63E-03	1.46E-03	7.32E-04	1.27E-03	1.80E-03
Δ^2G	-9.45E-05	5.45E-06	3.67E-05	-4.39E-04	-5.14E-04	8.55E-04	9.26E-04	3.33E-04	9.93E-05	-2.01E-04	-4.08E-04	-2.90E-04	-1.88E-05	1.07E-04	1.03E-04
R	2.14E-02	2.20E-02	2.26E-02	2.34E-02	2.41E-02	2.16E-02	2.18E-02	2.27E-02	2.40E-02	2.57E-02	2.73E-02	2.82E-02	2.83E-02	2.87E-02	2.93E-02
Δ R	1.21E-04	1.17E-04	1.38E-04	1.50E-04	-6.46E-05	-6.54E-05	1.06E-04	2.18E-04	3.03E-04	3.33E-04	2.42E-04	1.04E-04	5.44E-05	9.73E-05	1.19E-04
Δ^2R	-3.15E-06	1.70E-06	3.30E-06	-2.03E-05	-2.51E-05	3.99E-05	4.41E-05	1.98E-05	1.15E-05	-6.11E-06	-2.29E-05	-1.88E-05	-6.68E-07	6.48E-06	4.39E-06
T	3.94E+00	3.89E+00	3.84E+00	3.78E+00	3.73E+00	3.91E+00	3.88E+00	3.80E+00	3.71E+00	3.60E+00	3.51E+00	3.47E+00	3.47E+00	3.46E+00	3.43E+00
Δ T	-9.19E-03	-9.31E-03	-1.06E-02	-1.17E-02	4.23E-03	5.48E-04	-1.25E-02	-1.72E-02	-2.05E-02	-1.95E-02	-1.21E-02	-4.12E-03	-1.70E-03	-3.74E-03	-4.84E-03
Δ^2T	9.22E-05	-1.42E-04	-2.40E-04	1.48E-03	1.64E-03	-3.20E-03	-3.07E-03	-8.04E-04	-2.22E-04	8.46E-04	1.53E-03	1.04E-03	3.81E-05	-3.15E-04	-2.21E-04
C	3.00E+00	3.37E+00	3.80E+00	4.29E+00	4.87E+00	3.28E+00	3.65E+00	4.41E+00	5.33E+00	6.43E+00	7.27E+00	7.66E+00	7.71E+00	7.86E+00	8.13E+00
Δ C	6.98E-02	7.99E-02	9.19E-02	1.07E-01	-2.77E-02	1.84E-02	1.35E-01	1.68E-01	2.01E-01	1.95E-01	1.23E-01	4.36E-02	1.95E-02	4.18E-02	5.47E-02
Δ^2C	1.83E-03	2.21E-03	2.69E-03	-1.20E-02	-1.30E-02	2.91E-02	2.67E-02	6.64E-03	2.64E-03	-7.77E-03	-1.51E-02	-1.04E-02	-1.72E-04	3.52E-03	2.59E-03
H	4.35E-02	4.75E-02	5.22E-02	5.75E-02	6.38E-02	4.59E-02	5.05E-02	5.89E-02	6.88E-02	8.07E-02	8.99E-02	9.41E-02	9.47E-02	9.63E-02	9.92E-02
Δ H	7.59E-04	8.69E-04	9.99E-04	1.16E-03	-3.33E-04	3.03E-04	1.58E-03	1.83E-03	2.19E-03	2.11E-03	1.34E-03	4.73E-04	2.12E-04	4.55E-04	5.95E-04
Δ^2H	1.99E-05	2.40E-05	2.93E-05	-1.33E-04	-1.36E-04	3.25E-04	2.84E-04	6.09E-05	2.87E-05	-8.45E-05	-1.64E-04	-1.13E-04	-1.87E-06	3.82E-05	2.81E-05
J	2.78E+01	2.89E+01	3.01E+01	3.17E+01	3.33E+01	2.76E+01	2.89E+01	3.10E+01	3.32E+01	3.57E+01	3.76E+01	3.85E+01	3.85E+01	3.88E+01	3.94E+01
Δ J	2.10E-01	2.28E-01	2.78E-01	3.23E-01	-1.50E-01	6.23E-02	4.34E-01	4.34E-01	4.68E-01	4.39E-01	2.75E-01	9.06E-02	3.42E-02	8.14E-02	1.40E-01
Δ^2J	4.60E-04	6.72E-03	9.47E-03	-4.28E-02	-4.26E-02	9.59E-02	7.74E-02	3.42E-03	4.92E-04	-1.94E-02	-3.48E-02	-2.41E-02	-9.19E-04	1.06E-02	1.28E-02

the point for the year 1955 changes its position from the location with a slight convexity along the spatial-cycle path to that with a relatively sharp concavity.

Figure 5, in the meantime, shows the trajectory produced by the Hoover index whose shape is rather unique as compared with panels (1) through (7) in Figure 4. It can be seen that the position of the point for the year 1940 is much different form that of trajectories produced by other yardsticks, and that the change from 1947 to 1950 is much more pronounced than that of other trajectories.

5 Conclusion

In the light of the preceding investigation, we make the following general and specific remarks.

(1) General remarks

(i) The grouping of yardsticks based on our empirical results seems to be consistent with the theoretical grouping by kernel.

(ii) Depending upon the position over the population share domain, the sensivity to dynamic change varies according to the yardstick employed. For measuring the tendency of change in popoulation levels with special emphasis on relatively smaller metropolitan areas in terms of population size, the ROXY indices appear to be appropriate since the ROXY indices are sensitive to dynamic changes in the domain of smaller population shares. For measuring the tendency of change in population levels of relatively larger metropolitan areas, meanwhile, yardsticks of Type-Ⅳ appear to be appropriate since the coefficient of variation and the Herfindahl coefficient are sensitive to dynamic changes in the domain of larger population shares.

(2) Specific remarks

(i) Empirical results obtained through each yardstick except the yardstick of Type-V (i. e., Hoover index), indicate that the system of Japanese functional urban regions (Japanese FUR system) was (a) at the decelerating concentration stage in 1935, (b) at the accelerating deconcentration stage in 1940, (c) at the accelerating concentration stage in 1950, 55, and 60, (d) at the decelerating concentration stage in 1965, 70, and 75, and (e) at the accelerating concentration stage in 1985 and 90.

(ii) Empirical results obtained through the Hoover index indicate that the Japanese FUR system was (a) at the decelerating concentration stage in 1935 and 40, (b) at the accelerating concentration stage in 1950 and 55, (c) at the decelerating concentration stage in 1965, 70, and 75, and (d) at the accelerating concentration stage in 1985 and 90.

Based on the aforementioned remarks (1) and (2), it is noticed that all of the seven kinds of yardsticks would generate almost the same empirical results in identifying the stage or direction of spatial-cycle path for the Japanese FUR system in the past forty years from

1950 through 90, though each type of yardsticks differs from one another with respect to sensitivities to dynamic change in the value of the domain of population shares, and though they differ with respect to magnitude. This would imply the two-fold features of the ROXY index: (i) The ROXY index has a basic attribute common to other major yardsticks in a sense that all the yardsticks but the Hoover index would provide us with roughly parallel results in measuring the phenomena of convergence and divergence, and (ii) the ROXY index has more straightforward conceptual features to measure dynamic changes in convergence and divergence as compared with other six major yardsticks since the ROXY index has been developed to directly identify the changes which take place for a given time period with respect to the phenomena of convergence and divergence, and since the other six yardsticks have been developed to identify the static situation of the phenomena of convergence and divergence.

Notes

1) The basic concept of the ROXY index was originally constructed and applied in an empirical study by Kawashima (1978, pp.9, 13 and 14) as an analytical instrument to empirically investigate the Klaassen's spatial-cycle hypothesis. Since then, the ROXY-index method has been applied in a number of empirical studies to examine spatial-cycle processes of population redistribution in both inter-metropolitan and intra-metropolitan scopes. At the same time, a series of theoretical examinations on the basic characteristics of the ROXY index have also been carried out. This paper falls in this category of studies.
2) The dynamic degree here means the direction *and* magnitude of the change taking place for a given time period.
3) In case the ROXY-index method is applied to the study of changes in the level of metropolitan socio-economic activities, the terminology of spatial convergence and divergence has two different implications: (i) one corresponding to the phenomena of centralization (or urbanization) and decentralization (or suburbanization) often examined in intra-metropolitan analyses, and (ii) the other corresponding to the phenomena of concentration and deconcentration often examined in inter-metropolitan analyses. The phenomena of centralization or decentralization imply the tendency of activities to converge towards or diverge out of the central part of a given single metropolitan area respectively. These phenomena are divided into four stages in the context of spatial-cycle process: accelerating centralization, decelerating centralization, accelerating decentralization, and decelerating decentralization stages. On the other hand, the phenomena of concentration or deconcentration imply the tendency of activities to converge towards or diverge out of the larger metropolitan areas in a given system of metropolitan areas respectively. These phenomena are divided into four stages in the context of spatial-cycle process: accelerating concentration, decelerating concentration, accelerating deconcentration, and decelerating deconcentration stages. On the above

two kinds of spatial-cycle processes, see for example Kawashima (1987, pp. 15-16).

4) The static degree here means the magnitude of the state of relative share at a given point in time.

5) Since the ROXY index method had been proposed towards the end of the 1970s, a series of studies have been conducted on mathematical characteristics of this index. For example, Kawashima and Hiraoka (1993b) show that, for a one-dimensional discrete-linear region, there exists a straightforward functional relationship *between* the ROXY index value, for which the reversed CBD distance is used as a weighting factor, and the ROXY index value for which the reversed CBD distance is used as a weighting factor. Hiraoka and Kawashima (1993) propose two types of theoretically-ideal formulations of the ROXY index: (one for a one-dimensional continuous-linear region, and the other for a two-dimensional fan-shaped region. Each of these two types of theoretically-ideal formulations is examined in Hiraoka and Kawashima (1994) which specifies a functional relationship between the ROXY-index value calculated by use of the CBD distance as its weighting factor and the ROXY-index value calculated by the reversed CBD distance as its weighting factor. In Asami et.al. (1994), both (i) the mathematical characteristics commonly shared by the ROXY index and the correlation coefficient and (ii) the mathematical characteristics of the ROXY index that are different from those of the correlation coefficient are discussed are dicsussed.

6) In Equation (17), the multiplier parameter 10^4 is eliminated from the definitional formulation of the ROXY index for the sake of mathematical tractability. This treatment would not cause any inappropriateness in the investigation of this paper. The precise definition of *ROXY* is; $(WA/SA-1)\times 10^4$.

7) More precisely speaking, the time-derivative of the natural-log of average population.

8) The principal variable is the variable for which the value of the ROXY index is calculated.

9) "The domain of smaller population shares" means "the domain of population shares which is smaller than a half of the population share of the largest metropolitan area in terms of population."

10) Since the 1947 values for Naha and Okinawa FURs are not available in Table A-1, we calculated the values for the yardsticks as follows.

(i) For $ROXY_1$ and $ROXY_2$: We used the estimated population level for each of Naha and Okinawa FURs in year 1947 which was obtained through the formulation of

$$\left\{\frac{\text{Pop (1950)}}{\text{Pop (1940)}}\right\}^{\frac{7}{10}} \times \text{Pop (1940)}$$

where Pop (t) indicates the population level in year t.

(ii) For G, R, T, C, H and J:
We used ninety, instead of ninety-two, FURs excluding Naha and Okinawa FURs by

expecting that this expedient means can be reasonably justified since the population level is relatively small, among the FURs listed in Table A-1, in years 1940 and 1950 for both Naha and Okinawa FURs.

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Appendix Table A-1 Population of Functional Urban Regions (FURs)

No	FUR	1920	1925	1930	1935	1940	1947	1950	1955	1960	1965	1970	1975	1980	1985	1990
1	Sapporo	334,726	411,532	461,508	511,743	526,741	615,654	694,141	836,244	972,841	1,192,728	1,405,689	1,668,650	1,872,247	2,035,511	2,186,324
2	Hakodate	204,896	229,663	269,232	290,011	278,949	307,156	326,178	343,651	342,298	349,937	360,438	378,408	395,484	395,722	383,456
3	Asahikawa	161,553	175,999	195,261	204,451	200,265	239,479	263,832	297,455	320,478	345,869	362,253	378,547	409,393	418,216	410,073
4	Muroran	75,798	70,739	77,404	88,592	135,707	138,171	154,767	173,048	201,221	227,200	238,137	242,935	241,407	229,402	207,933
5	Kushiro	57,836	65,724	79,116	88,009	105,694	108,410	135,689	165,177	199,728	218,458	223,520	239,858	250,479	252,519	245,220
6	Obihiro	71,818	75,259	94,440	109,633	109,865	134,705	143,879	174,507	181,622	195,595	207,501	219,106	237,346	250,259	253,419
7	Kitami	48,924	48,448	56,489	62,253	66,124	82,330	90,076	98,843	105,692	110,154	113,002	118,932	129,787	133,254	131,266
8	Yubari	51,022	48,657	51,924	42,473	64,945	82,055	99,448	107,244	107,883	85,071	69,871	50,131	41,715	31,665	20,969
9	Tomakomai	28,635	31,357	32,303	33,685	36,536	46,411	56,320	69,899	84,082	106,433	128,145	161,409	182,369	188,174	188,821
10	Aomori	138,445	155,418	171,546	191,376	193,951	211,346	235,855	260,887	272,118	281,292	294,737	317,112	340,554	346,125	336,472
11	Hirosaki	198,460	211,336	228,893	246,596	254,889	299,660	318,329	333,281	332,545	323,351	321,063	325,527	338,204	336,055	329,629
12	Hachinohe	162,261	175,203	191,830	212,470	227,074	275,595	304,313	336,069	367,572	376,358	389,482	407,111	425,040	430,940	428,857
13	Morioka	233,394	255,660	278,343	299,716	314,524	365,061	398,918	429,921	450,131	458,920	469,618	504,213	547,803	575,076	590,652
14	Sendai	526,497	582,836	643,584	697,336	714,861	883,621	950,257	1,000,027	1,035,016	1,085,065	1,175,808	1,321,474	1,442,952	1,537,479	1,624,932
15	Ishinomaki	126,837	137,427	150,561	161,437	168,266	214,199	224,753	225,787	225,482	220,519	221,164	226,400	232,153	234,174	230,493
16	Akita	277,169	293,470	309,699	323,646	327,712	393,644	418,479	440,054	447,400	446,340	455,580	475,953	502,541	512,620	513,778
17	Yamagata	367,172	398,595	424,695	437,589	438,081	524,669	534,241	525,038	519,456	508,888	509,245	522,679	547,416	561,280	566,810
18	Tsuruoka	135,430	139,989	146,046	151,568	151,828	178,613	183,060	188,610	178,133	168,933	162,037	158,731	161,809	161,147	159,103
19	Sakata	133,659	139,723	146,892	149,564	147,949	179,529	185,421	187,651	184,317	176,342	170,361	168,870	173,832	171,552	169,260
20	Fukushima	282,528	299,538	313,301	322,976	324,934	388,211	402,451	411,010	406,011	407,202	414,465	430,776	448,289	456,835	461,898
21	Aizuwakamatsu	188,890	204,507	211,420	222,203	220,593	264,735	266,888	269,816	263,201	252,390	242,059	239,537	245,287	248,233	248,328
22	Koriyama	242,816	262,532	286,453	302,818	311,762	384,200	396,899	406,822	407,538	409,984	420,469	442,133	467,138	484,248	499,325
23	Iwaki	222,763	219,335	228,925	247,286	273,198	331,222	358,185	368,712	361,544	348,134	340,318	342,893	355,775	364,314	369,725
24	Mito	326,995	348,181	372,161	388,093	398,658	504,687	517,665	528,254	534,066	548,166	577,712	630,129	675,217	709,480	729,738
25	Hitachi	175,504	179,637	188,379	202,451	244,467	266,990	278,703	295,674	323,460	332,414	331,631	336,937	344,383	351,939	350,816
26	Tsuchiura-Tsukuba	331,493	350,251	369,774	378,780	385,091	488,351	486,426	484,194	459,404	454,050	467,485	514,558	580,214	620,971	658,516
27	Utsunomiya	454,242	483,690	508,592	529,039	530,677	696,626	705,876	706,828	695,102	705,435	744,500	821,883	883,555	934,074	978,301
28	Ashikaga	175,270	187,382	191,959	205,650	208,294	245,176	248,675	246,915	244,341	248,071	256,192	267,567	273,961	278,585	281,593
29	Oyama	189,716	201,720	211,942	217,416	220,255	290,255	289,984	289,139	283,668	293,011	317,419	349,807	373,007	389,366	403,210
30	Maebashi-Takasaki-Isesaki	659,646	706,909	754,121	774,555	789,969	968,076	986,921	994,849	977,439	1,012,856	1,063,747	1,137,151	1,203,346	1,255,446	1,295,705
31	Kiryu	242,568	254,663	271,353	303,121	340,096	400,052	406,167	407,854	399,790	408,979	433,878	469,417	501,265	533,295	539,703
32	Kumagaya	217,549	230,333	237,894	243,968	249,774	315,791	316,734	322,447	318,507	331,269	355,408	385,275	413,878	442,741	465,752
33	Tokyo	6,355,850	7,316,559	8,531,697	9,805,191	11,239,839	9,568,285	11,172,003	13,536,749	16,008,664	19,120,214	22,165,352	24,908,173	26,483,892	27,970,346	29,420,554
34	Hiratsuka-Odawara-Atsugi	310,188	337,802	352,382	378,191	398,658	510,479	524,763	559,245	593,196	689,836	806,735	952,560	1,061,160	1,157,915	1,242,246
35	Niigata	504,802	536,239	573,535	600,545	631,701	755,272	782,555	803,740	815,278	835,475	860,089	907,429	962,506	993,771	1,011,402
36	Nagaoka	277,706	291,013	301,790	309,541	319,606	350,360	361,089	361,113	357,623	354,690	351,227	354,895	362,478	364,614	364,298
37	Joetsu	247,569	252,797	257,427	258,869	265,353	312,183	309,414	305,856	295,325	284,255	269,078	263,463	264,335	263,585	258,386
38	Toyama	651,358	672,010	699,368	710,833	740,610	881,776	911,096	926,104	939,871	936,208	948,271	991,518	1,025,600	1,041,008	1,045,466
39	Kanazawa	367,677	372,448	377,444	384,362	381,071	455,626	477,401	493,449	507,639	531,626	561,631	622,756	668,323	701,178	725,316
40	Komatsu	125,417	123,533	124,585	129,248	130,504	160,463	164,948	162,349	164,099	168,318	176,621	188,020	196,855	202,772	203,702
41	Fukui	455,922	453,070	471,211	500,249	497,556	557,272	582,230	583,588	587,103	591,140	590,826	616,843	636,450	654,958	658,707
42	Kofu	404,719	419,544	447,740	454,860	456,738	558,293	557,883	551,471	532,773	520,315	523,916	544,698	567,438	593,500	610,640
43	Nagano	393,052	409,345	426,972	434,365	438,252	542,802	547,406	543,119	535,459	535,888	545,475	569,889	593,005	607,558	614,884
44	Matsumoto	282,655	297,998	313,907	317,504	319,358	387,688	390,431	387,362	385,776	384,870	392,358	413,496	433,217	447,159	454,513
45	Ueda	194,906	200,557	210,536	206,452	204,868	256,652	254,609	246,966	240,954	237,896	238,584	246,146	256,480	264,429	269,648
46	Gifu	564,522	604,673	634,548	660,708	684,522	800,363	835,615	868,259	918,752	996,090	1,072,017	1,159,347	1,222,013	1,263,849	1,287,096
47	Shizuoka-Shimizu	408,409	456,898	498,337	540,880	570,779	634,219	685,090	749,324	807,588	874,745	940,111	1,005,437	1,043,081	1,071,168	1,087,825

Appendix　Table A-1 (*continued*)

No	FUR	1920	1925	1930	1935	1940	1947	1950	1955	1960	1965	1970	1975	1980	1985	1990
48	Hamamatsu	456,458	492,971	532,797	575,680	593,581	660,200	695,678	746,706	771,838	805,907	853,363	917,811	973,545	1,024,729	1,067,398
49	Numazu-Fuji-Mishima	320,513	344,987	370,304	396,445	430,631	537,925	557,014	595,193	631,027	697,271	772,328	851,284	893,402	934,694	966,836
50	Nagoya	1,529,660	1,735,929	1,939,866	2,216,060	2,493,647	2,289,733	2,517,345	2,859,915	3,278,820	3,773,989	4,203,623	4,601,116	4,812,714	4,980,918	5,146,744
51	Toyohashi	308,730	329,931	355,653	361,651	369,517	450,620	472,297	497,459	513,431	552,074	585,415	634,982	670,292	702,628	726,699
52	Kariya-Toyota-Anjo	340,188	378,028	403,447	433,599	459,658	564,093	589,994	611,669	641,601	727,111	870,522	993,132	1,086,633	1,163,660	1,242,716
53	Tsu-Matsusaka-Ise	473,830	491,991	514,621	523,456	523,170	603,303	620,515	630,830	621,479	623,075	632,107	660,365	677,738	693,392	699,005
54	Yokkaichi	249,782	263,106	276,692	285,864	300,867	367,573	381,063	390,888	409,285	444,919	473,360	518,822	543,705	564,714	588,758
55	Kyoto	1,158,724	1,295,803	1,454,654	1,613,146	1,629,191	1,647,245	1,747,809	1,853,229	1,934,328	2,085,116	2,283,938	2,512,582	2,672,147	2,789,667	2,861,101
56	Osaka	3,426,334	3,982,818	4,544,247	5,425,715	6,085,710	4,795,624	5,386,989	6,282,269	7,327,858	8,797,537	10,075,474	11,032,245	11,436,841	11,793,746	12,013,470
57	Kobe	979,644	1,068,023	1,178,036	1,334,898	1,432,130	1,053,180	1,201,382	1,384,311	1,524,612	1,668,326	1,818,488	1,988,724	2,073,824	2,155,045	2,246,179
58	Himeji	517,242	525,472	545,937	563,761	606,630	731,523	751,076	769,044	786,264	830,243	875,419	931,730	963,501	987,334	988,403
59	Wakayama	387,624	415,536	445,116	466,782	464,478	490,432	507,862	529,826	541,226	581,608	617,446	646,199	662,204	665,184	660,198
60	Tottori	193,569	197,305	203,296	203,647	200,610	237,930	244,532	250,077	244,126	235,793	231,803	235,653	243,508	249,296	252,139
61	Yonago	179,439	189,000	196,350	197,293	198,157	241,117	247,420	256,279	251,715	248,515	248,297	259,070	272,487	277,765	275,803
62	Matsue	248,500	257,096	270,352	279,176	278,626	322,569	334,399	343,232	338,603	332,671	333,985	343,795	361,118	372,127	375,314
63	Okayama-Kurashiki	747,248	764,148	800,083	843,398	845,599	1,008,480	1,043,949	1,082,292	1,097,992	1,128,242	1,228,735	1,346,700	1,408,663	1,454,094	1,472,284
64	Hiroshima-Kure	774,367	835,029	892,516	986,413	1,048,801	995,970	1,053,803	1,126,665	1,180,196	1,298,064	1,400,886	1,566,401	1,629,900	1,790,584	1,840,281
65	Fukuyama-Onomichi	471,768	490,128	504,501	522,169	516,501	632,852	648,174	654,837	660,016	675,910	729,183	781,749	797,215	809,108	803,045
66	Shimonoseki	206,748	228,102	238,742	254,384	281,079	286,676	307,942	336,956	344,335	342,507	338,098	345,540	348,987	348,354	338,633
67	Ube	173,159	187,184	204,775	224,032	271,850	292,535	319,105	336,684	344,132	319,062	312,358	327,966	344,494	361,489	366,863
68	Tokuyama	213,139	220,159	225,946	247,255	272,192	313,595	322,240	331,759	333,356	344,521	360,393	392,246	407,130	414,782	408,236
69	Iwakuni	142,758	147,642	154,974	157,922	162,250	200,124	203,860	216,302	217,572	213,247	207,919	212,466	214,120	210,808	205,857
70	Tokushima	460,682	476,776	499,580	511,649	505,500	596,248	615,574	617,394	606,793	599,901	601,604	630,353	658,479	673,768	679,202
71	Takamatsu	500,817	520,702	544,712	557,379	541,962	673,488	698,321	705,187	694,724	691,964	709,610	762,919	800,398	823,712	828,568
72	Matsuyama	249,571	268,143	279,572	277,899	278,263	345,613	365,778	387,634	402,230	424,022	454,873	506,946	549,756	578,819	596,218
73	Imabari	124,732	135,192	142,586	147,234	146,140	176,194	185,430	186,584	185,950	184,968	188,502	198,910	203,470	205,328	201,230
74	Niihama	102,825	109,322	117,910	132,443	149,051	188,598	200,039	205,895	207,305	203,996	202,739	210,615	213,573	216,575	213,703
75	Kochi	366,168	379,128	398,645	395,359	390,541	460,958	477,227	487,642	481,737	486,956	497,506	531,253	556,857	574,065	572,865
76	Kitakyushu	742,621	788,169	891,100	1,017,537	1,212,683	1,081,748	1,225,871	1,388,663	1,494,217	1,501,494	1,484,441	1,523,904	1,567,605	1,577,849	1,541,920
77	Fukuoka	505,233	547,479	609,437	669,483	706,081	810,362	898,727	1,021,316	1,105,380	1,218,854	1,382,127	1,623,726	1,820,574	1,967,826	2,112,804
78	Omuta	202,930	215,646	228,691	238,119	275,893	297,522	336,293	347,099	345,890	325,751	297,188	290,578	290,763	289,542	275,564
79	Kurume	412,093	440,243	464,660	483,625	479,541	566,770	589,742	614,768	607,836	596,384	598,454	607,396	630,238	641,813	644,645
80	Iizuka-Tagawa	365,128	358,439	385,072	398,936	473,561	508,579	577,948	590,747	551,303	419,369	361,262	355,160	364,678	370,219	364,144
81	Saga	312,797	320,378	322,081	326,773	327,645	432,850	445,210	462,556	451,591	424,410	407,799	408,080	423,937	432,737	434,544
82	Nagasaki	372,955	390,608	408,202	418,474	446,109	475,959	529,503	582,911	618,229	630,684	649,430	690,470	720,789	740,699	746,367
83	Sasebo	218,567	233,476	269,523	323,202	361,675	345,076	379,700	417,715	410,148	356,665	339,277	340,970	343,564	344,167	337,184
84	Kumamoto	532,560	567,103	598,200	616,817	600,141	762,204	787,465	828,963	828,677	827,681	842,210	883,348	954,129	1,006,573	1,038,296
85	Yatsushiro	108,334	115,905	118,673	124,747	126,168	164,776	172,673	182,190	182,274	173,634	166,735	164,857	168,229	167,371	163,493
86	Oita	330,837	361,490	377,119	390,979	393,916	502,055	511,741	532,944	532,795	544,349	567,675	629,797	672,620	699,709	707,192
87	Miyazaki	190,242	202,415	227,407	234,122	237,424	306,878	324,649	337,354	345,236	344,646	357,473	392,719	434,882	457,597	472,077
88	Miyakonojo	144,455	158,829	173,942	181,328	181,472	227,308	236,968	242,965	235,410	220,967	207,860	208,502	223,369	227,624	226,177
89	Nobeoka	94,305	99,111	110,060	146,024	150,201	158,104	176,754	194,180	200,333	200,786	206,457	217,955	226,195	226,511	219,204
90	Kagoshima	456,592	490,099	518,968	535,126	530,638	642,845	670,740	697,740	685,988	693,102	703,272	754,284	814,370	846,156	851,840
91	Naha	237,801	229,838	239,906	249,788	242,478	–	254,540	336,244	403,619	452,062	490,092	570,851	617,119	665,351	698,876
92	Okinawa	101,970	97,358	100,103	101,676	97,033	–	153,203	179,949	195,501	209,915	216,898	238,963	254,990	274,430	286,808
	Total	39,222,903	42,666,436	46,656,573	50,923,767	54,560,135	56,003,847	60,904,607	66,639,663	71,659,164	78,149,248	85,071,849	92,841,913	98,019,431	102,221,066	105,273,643

Appendix Table A-2 Area and Its Share of Functional Urban Regions (FURs)

No.	FUR	Area(km²)	Share	No.	FUR	Area(km²)	Share
1	Sapporo	3474.7	2.44E-02	47	Shizuoka-Shimizu	1856.5	1.31E-02
2	Hakodate	1235.6	8.69E-03	48	Hamamatsu	1354.7	9.53E-03
3	Asahikawa	2421.4	1.70E-02	49	Numazu-Fuji-Mishima	1548.2	1.09E-02
4	Muroran	462.8	3.26E-03	50	Nagoya	2585.4	1.82E-02
5	Kushiro	1987.0	1.40E-02	51	Toyohashi	1016.1	7.15E-03
6	Obihiro	2604.2	1.83E-02	52	Kariya-Toyota-Anjo	1490.5	1.05E-02
7	Kitami	1340.4	9.43E-03	53	Tsu-Matsusaka-Ise	2315.0	1.63E-02
8	Yubari	763.4	5.37E-03	54	Yokkaichi	908.0	6.39E-03
9	Tomakomai	1140.9	8.03E-03	55	Kyoto	2575.0	1.81E-02
10	Aomori	1286.3	9.05E-03	56	Osaka	4614.6	3.25E-02
11	Hirosaki	1597.8	1.12E-02	57	Kobe	1281.6	9.01E-03
12	Hachinohe	1551.9	1.09E-02	58	Himeji	2603.4	1.83E-02
13	Morioka	3688.4	2.59E-02	59	Wakayama	901.4	6.34E-03
14	Sendai	3092.2	2.18E-02	60	Tottori	1533.0	1.08E-02
15	Ishinomaki	738.5	5.19E-03	61	Yonago	1083.6	7.62E-03
16	Akita	2382.7	1.68E-02	62	Matsue	1135.3	7.99E-03
17	Yamagata	2130.9	1.50E-02	63	Okayama-Kurashiki	2785.3	1.96E-02
18	Tsuruoka	1344.7	9.46E-03	64	Hiroshima-Kure	3016.3	2.12E-02
19	Sakata	1060.4	7.46E-03	65	Fukuyama-Onomichi	1494.1	1.05E-02
20	Fukushima	1487.7	1.05E-02	66	Shimonoseki	805.2	5.66E-03
21	Aizuwakamatsu	1082.0	7.61E-03	67	Ube	712.8	5.01E-03
22	Koriyama	1785.0	1.26E-02	68	Tokuyama	1076.5	7.57E-03
23	Iwaki	1392.9	9.80E-03	69	Iwakuni	1022.0	7.19E-03
24	Mito	1429.5	1.01E-02	70	Tokushima	1465.8	1.03E-02
25	Hitachi	915.9	6.44E-03	71	Takamatsu	1268.5	8.92E-03
26	Tsuchiura-Tsukuba	1284.8	9.04E-03	72	Matsuyama	919.0	6.46E-03
27	Utsunomiya	2335.0	1.64E-02	73	Imabari	382.6	2.69E-03
28	Ashikaga	442.1	3.11E-03	74	Niihama	554.4	3.90E-03
29	Oyama	538.2	3.79E-03	75	Kochi	1909.0	1.34E-02
30	Maebashi-Takasaki-Isesaki	1988.2	1.40E-02	76	Kitakyushu	1327.7	9.34E-03
31	Kiryu	791.1	5.56E-03	77	Fukuoka	1524.2	1.07E-02
32	Kumagaya	459.2	3.23E-03	78	Omuta	290.8	2.05E-03
33	Tokyo	7805.9	5.49E-02	79	Kurume	784.5	5.52E-03
34	Hiratsuka-Odawara-Atsugi	1162.9	8.18E-03	80	Iizuka-Tagawa	733.0	5.16E-03
35	Niigata	1916.6	1.35E-02	81	Saga	907.0	6.38E-03
36	Nagaoka	1065.8	7.50E-03	82	Nagasaki	820.3	5.77E-03
37	Joetsu	1439.9	1.01E-02	83	Sasebo	660.6	4.65E-03
38	Toyama	3043.1	2.14E-02	84	Kumamoto	660.6	4.65E-03
39	Kanazawa	1475.4	1.04E-02	85	Yatsushiro	479.6	3.37E-03
40	Komatsu	549.5	3.86E-03	86	Oita	1763.6	1.24E-02
41	Fukui	2467.1	1.74E-02	87	Miyazaki	1414.2	9.95E-03
42	Kofu	2134.2	1.50E-02	88	Miyakonojo	1008.1	7.09E-03
43	Nagano	1826.2	1.28E-02	89	Nobeoka	1002.2	7.05E-03
44	Matsumoto	2453.5	1.73E-02	90	Kagoshima	1649.8	1.16E-02
45	Ueda	1036.7	7.29E-03	91	Naha	262.7	1.85E-03
46	Gifu	1841.5	1.30E-02	92	Okinawa	207.8	1.46E-03
					Total	142164.0	1.00E+00