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Risk-Bearing in a Winner-Take-All Contest

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Abstract

A parsimonious model is used to explore the risk-bearing decision under a payoff structure that emphasizes relative performance. Equilibrium betting amounts are derived for players who start with unequal endowments and face a lottery that offers either a positive or negative expected return. If the lottery offers negative expected return, disadvantaged players are willing to risk a portion of their endowment, and this induces advantaged players to also gamble, defensively. Although there are equilibria in which the advantaged preemptively gamble more than the disadvantaged, in the robust equilibrium it is the disadvantaged who make the larger bets. If the lottery offers positive expected return, there are equilibria in which the advantaged invest less than the disadvantaged, but full investment by all players is a more robust equilibrium.

1. Introduction

The logic of the risk-bearing decision under a payoff structure that emphasizes relative performance combines stochastic and strategic considerations in a complex way. The complication was lucidly illustrated by Aron and Lazear's (1990) discussion of a sailing race as a metaphor for competition among firms developing new products. In a sailing race, because all nearby boats are affected by the same wind, the leader maximizes probability of winning by imitating the follower's course, even if the leader believes another course has shorter expected time to the finish. Conversely, the follower increases its winning probability by differentiating its course from the leader's, even if it believes the different course has longer expected finish time than the course the leader is on. Applying this insight from sailing, Aron and Lazear study equilibria in which a firm that is behind in given product market leaves it develop a substitute product, and thereby induces the current market leader to also switch to pursuing the risky substitute. Other contests in which strategic risk-bearing has been studied include research portfolio selection by scientists and firms racing to be first (Dasgupta and Maskin, 1987), managers competing for relative performance compensation (Hvide, 2002), gambling by individuals struggling for socioeconomic status (Gregory, 1980; Robson, 1992; Rosen, 1997), and contests embedded in evolutionary settings (Robson, 1996; Dekel and Scotchmer, 1999). The common thread among these different contexts is that the logic of the

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risk-bearing decision under a relative payoff structure may drastically depart from standard risk/return considerations that prevail when only absolute performance matters. In this paper, we abstract away from applied contexts to explore the essence of this phenomenon in a parsimonious setting.

We begin in the next section by describing a model in which two unequally-endowed risk-neutral agents choose a portion of their endowments to risk in a lottery, knowing that top-rank in post-lottery wealth distribution will be allocated a prize. We derive best responses (Section 3) and equilibria (Section 4) for two types of lotteries: positive expected return lotteries (representing investing) and negative expected return lotteries (representing gambling). In Section 5 we generalize the findings to an arbitrary number of players and prizes. Like Aron and Lazear (1990), we find that the worse-endowed agents proactively engage in risk-taking, and this induces the better-endowed agents to also take on risk. Unlike Aron and Lazear, we find this without assuming any correlation in lottery outcomes across agents. We also extend the inquiry to consider the extent of risk-bearing under various timing structures and distinguish between positive and negative expected return cases.

2. The model

Two players indexed by $i=1,2$ have initial monetary endowments A_i such that $A_1>A_2>0$. We will denote the endowment disparity by $\Delta A\equiv A_1-A_2$ and refer to player 1 as “rich” and player 2 as “poor.” Each player may bet any portion of his endowment $z_i\in[0,A_i]$ on a single independent draw of a binary lottery. After both players have placed their bets, two independent draws of the lottery are performed. If a player wins the lottery, he gets back his bet plus wz_i where $w>0$; if he loses he gets back nothing. Probability of a player winning the lottery is $p\in[0,1]$. A_i, p, and w are exogenously fixed parameters and are common knowledge.

The lottery maps players’ endowment vector $A\equiv(A_1,A_2)$ and action vector $z\equiv(z_1,z_2)$ to an “achieved wealth” random variable vector $m\equiv(m_1,m_2)$, where $m_i\in\{A_i+wz_i,A_i-z_i\}$. The expected value of achieved wealth is $E[m_i]=A_i+\theta z_i$, where $\theta\equiv p(w+1)-1$ is the expected rate of return offered by the lottery. We will consider both negative and positive expected returns, referring to the former as “gambling” and the latter as “investing”. We will assume that $(w+1)A_2>A_1$, which ensures that it is possible for the poor player to leapfrog the rich in the achieved wealth distribution.

The utility function $U:R^2\rightarrow R^2$ maps achieved wealth to utility. The baseline case is absolute wealth maximization, represented by utility function

$$U^A(m)=m \tag{1}$$

It is obvious that under this definition of utility, utility-maximizing strategies are [2]:

$$z_i^*=\begin{cases}0 & if\ \theta<0\\ A_i & if\ \theta>0\end{cases}$$

Against this baseline, we will compare a winner-take-all contest in which payoffs are given by the

2 We do not analyze the special case of $\theta=0$.

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utility function

$$U(m)=\begin{cases} m+(V,0) & \text{if } m_1 \geq m_2 \\ m+(0,V) & \text{if } m_1 < m_2 \end{cases} \tag{2}$$

We assume that $V>(w+1)A_1$ is a prize whose utility value exceeds that of any achievable wealth. Under both specifications of utility, higher achieved wealth corresponds to higher utility. However, under (2) achieved wealth can also impact utility through its role in allocating the prize to the relatively rich player.

3. Best-response analysis

3.1 Contest success probabilities

Following the conventional approach to analyzing contests, we first characterize contest success probability $\pi \equiv (Pr(m_1 \geq m_2), Pr(m_2 > m_1))$ as a function of players' actions z. The two players' lottery draws can result in four distinct events: $WW, LL, LW,$ or WL, where each event is a two-letter combination with player 1 listed first, and "W" and "L" representing each player's "win" or "loss" in the lottery. Regardless of the amounts bet, the outcome WL necessarily results in $m_1 > m_2$. Which player achieves more wealth in the other three cases depends on the bets as well as parameters A and w. Figure 1 depicts one possible geometry relating lottery outcomes to relative wealth. Figure 2 and Table 1 show the four possible distinct geometries, enumerated as Cases I through IV, and the contest success probability corresponding to each case. Since there are only 4 distinct contest success probabilities, the contest success function (CSF) in our model is a discrete-valued many-to-one function:

$$\pi:[0,A_1]\times[0,A_2]\rightarrow\{(1,0),(1-p(1-p), p(1-p)),(1-p,p),(p,1-p)\}$$

Figure 1. The geometry of the contest with $A_1=5$, $A_2=3$, $w=2$, $z_1=2$, $z_2=0.5$. The four possible outcomes are the vertices of the rectangle. Outcome LW lies above the 45º line, which implies $m_2 > m_1$ and player 2 obtains the prize. Outcomes LL, WW, and WL lie below the 45º line, which implies $m_1 > m_2$ and player 1 obtains the prize.

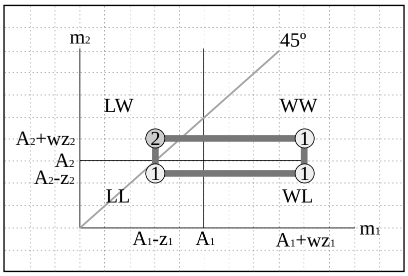

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Figure 2. The four possible geometries relating lottery outcomes and relative wealth.

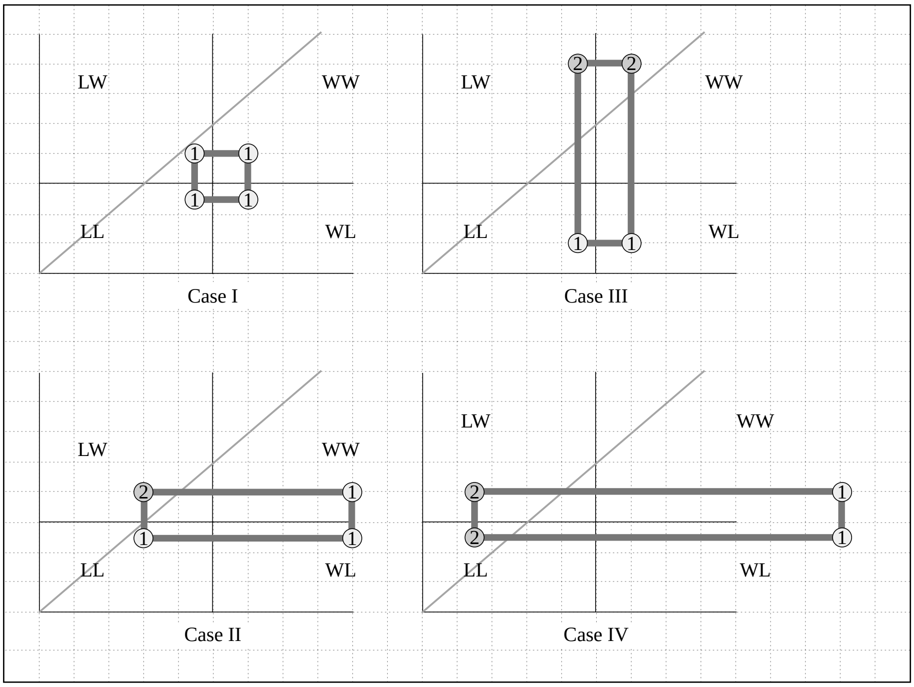

Table 1. The four possible cases of how lottery outcomes relate to relative wealth.

Case	Lottery outcomes resulting in $m_1 \geq m_2$	π_1	Lottery outcomes resulting in $m_2 > m_1$	π_2
I	WL, LL, LW, WW	1		0
II	WL, LL, WW	1-p(1-p)	LW	p(1-p)
III	WL, LL	1-p	LW, WW	p
IV	WL, WW	p	LL, LW	1-p

Figure 3 illustrates how the CSF partitions the domain into the four regions corresponding to four cases in Table 1. Our CSF differs markedly from those in conventional contest models (cf. Skaperdas 1996, Hirshleifer 1989). In a conventional contest model, players compete for a prize by choosing effort level e_i whose cost is given by an increasing function $C(e_i)$. The effort levels of all the players jointly determine how the probability of winning the prize is distributed among the players. CSF is continuous and satisfies $\frac{\partial \pi_i}{\partial e_i} > 0$ (increasing own effort raises own probability of winning) and $\frac{\partial \pi_i}{\partial e_j} < 0 \ i \neq j$ (increase in effort by a player reduces the other player's probability of winning). In contrast, our CSF is discrete-valued and decreasing at some points in its domain. The possibility that π_i may fall as player i raises z_i or may rise as another player $j \neq i$ increases z_j is a consequence of the downward exposure

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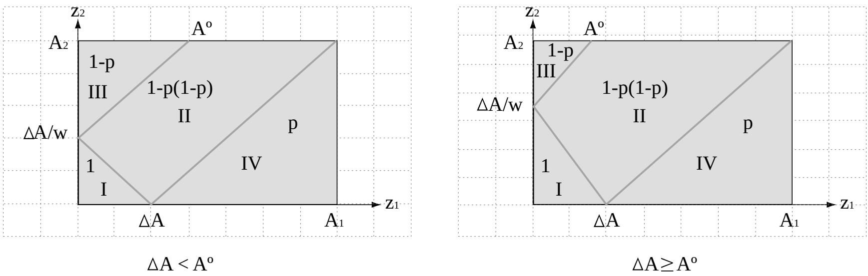
Figure 3. The CSF partitions its domain into four regions, which it maps to distinct success probabilities, corresponding to the four cases in Table 1. Some best responses and equilibria depend on the relative sizes of ΔA and A^o , where $A^o \equiv A_2 - \Delta A/w$.



inherent to risking money in the lottery. For instance, if player 1 increases z_1 by a large enough amount, he may risk losing so much that the situation shifts from Case II to Case IV, reducing his success probability from $1-p(1-p)$ to p.

3.2 Maximization of expected utility

To maximize expected utility (2), player 1 solves:

$$z_1^* = \arg \max_{x \in [0, A_1]} (\theta x + V\pi_1(x, z_2))$$

Player 2's maximization problem is analogous. Since $\pi \in \{1, 1-p(1-p), 1-p, p\}$, for V large enough, the second summand dominates. If $\theta<0$, maximum expected utility for player 1 occurs at the smallest x that yields the highest π_1 . Thus, assuming the prize carries sufficient utility value, the problem of maximizing expected utility facing each player boils down to the following: bet the least amount that maximizes the probability of achieving more wealth than the other player.[3] Formally, for player 1:

$$\mathit{if}\ \theta<0\quad z_1^* = \inf\{y: \pi_1(y, z_2) \geq \pi_1(x, z_2)\ \forall x \in [0, A_1]\}$$

If the lottery offers positive expected return, a player maximizes expected utility by risking the largest amount that maximizes the probability of achieving more wealth than the other player. Formally,

$$\mathit{if}\ \theta>0\quad z_1^* = \sup\{y: \pi_1(y, z_2) \geq \pi_1(x, z_2)\ \forall x \in [0, A_1]\}$$

Regardless of whether expected returns are positive or negative, the preferences of player 1 over the four possible outcomes in listed in Table I and depicted in Figure 2 are:

3 Geometrically, in Figure 1 player 1 maximizes expected utility by finding the narrowest rectangle that includes the largest possible number of vertices below the 45º line whereas player 2 seeks the shortest rectangle that includes the largest possible number of vertices above the 45º line.

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$$I \succ II \succ III \succ IV \ \ if \ \ p < \frac{1}{2}$$

$$I \succ II \succ IV \succ III \ \ if \ \ p > \frac{1}{2}$$

Since the competition for success probability is a zero-sum game, player 2 holds the opposite preferences.

3.3 Best responses

3.3.1 Negative expected return case

The rich player's best response depends on whether the poor player gambles more or less than $\Delta A / w$. If $z_2 \leq \Delta A / w$, then even if the poor wins the lottery he will not leapfrog the rich in the achieved wealth distribution. The rich is not threatened and his best response is to abstain. However, if the poor gambles $z_2 > \Delta A / w$ and the rich abstains, then with probability p the poor will leapfrog the rich. Although the rich cannot completely defend against this possibility, he can reduce its probability from p down to $p(1-p)$ by gambling enough to establish Case II. The smallest such defensive gamble is $z_1 = z_2 - \Delta A / w$. Thus, rich player's best response is (see Figure 4a):

$$z_1^*(z_2) = \begin{cases} 0 & if \ z_2 \in [0, \frac{\Delta A}{w}] \\ z_2 - \frac{\Delta A}{w} & if \ z_2 \in (\frac{\Delta A}{w}, A_2] \end{cases} \tag{1}$$

We next turn to poor player's best response when $p < 1/2$. If the rich abstains, the poor is best-off gambling $z_2 = \Delta A / w + \alpha$, where α is an arbitrarily small positive constant. If $z_1 > \Delta A$, then the poor is best-off abstaining, hoping that the rich will lose his endowment advantage in his own gamble. If the rich gambles $0 < z_1 \leq \Delta A$, then the poor is best-off matching z_1 and betting an additional $\Delta A / w + \alpha$ to obtain Case III. However, if the poor cannot afford this bet, then he is best-off aiming for Case II by gambling as if the endowment gap has been reduced by the potential loss z_1 of the rich. Specifically, the poor player's best response function is (see Figure 4b):

$$If \ \ p < 1/2 \quad z_2^*(z_1) = \begin{cases} 0, & if \ z_1 \in (\Delta A, A_1] \\ \frac{\Delta A - z_1}{w} + \alpha, & if \ z_1 \in [A^0, \Delta A] \\ \frac{\Delta A}{w} + z_1 + \alpha, & if \ z_1 \in [0, \min(A^0, \Delta A)] \end{cases}$$

where $A^0 \equiv A_2 - \Delta A / w$.

If $p > 1/2$, the poor player's best response is given by (see Figure 4c)

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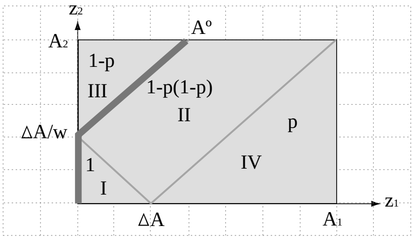
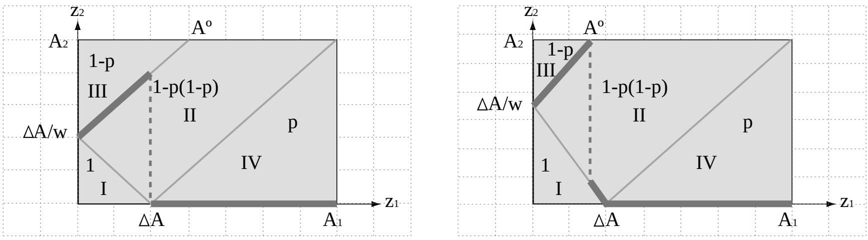
$$If\ p>1/2 \quad z_2^*(z_1)=\begin{cases} 0, & if\ z_1\in[\max(A^0,\Delta A),A_1] \\ \dfrac{\Delta A-z_1}{w}+\alpha, & if\ z_1\in[A^0,\Delta A] \\ \dfrac{\Delta A}{w}+z_1+\alpha, & if\ z_1\in[0,A^0) \end{cases}$$

Figure 4a. Rich player's best-response (θ<0)



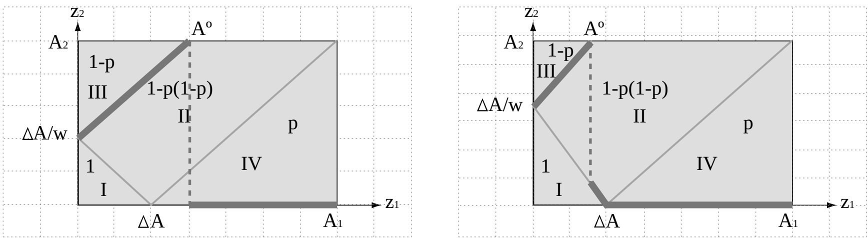
ΔA < A°

Figure 4b. Poor player's best-response (θ<0, p<1/2)



ΔA < A° ΔA ≥ A°

Figure 4c. Poor player's best-response (θ<0, p>1/2)



ΔA < A° ΔA ≥ A°

3.3.2 Positive expected return case

Although he is risk-neutral and is facing a positive-expected-return investment opportunity, the possibility of losing his investment and thereby losing relative position in the wealth distribution limits how much the rich player invests. Specifically, if the poor invests less than $\Delta A/w$, then the rich can guaran-

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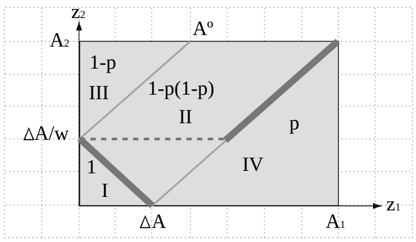
tee keeping top wealth rank (Case I) by investing no more that $\Delta A - z_2$. If the poor invests more than $\Delta A/w$, then the rich cannot obtain Case I and must settle for the next-best Case II. The rich player's best-response function is (see Figure 5a)

$$z_1^*(z_2) = \begin{cases} \Delta A - wz_2, \text{ } if \text{ } z_2 \in [0, \frac{\Delta A}{w}] \\ z_2 + \Delta A, \quad if \text{ } z_2 \in (\frac{\Delta A}{w}, A_2] \end{cases}$$

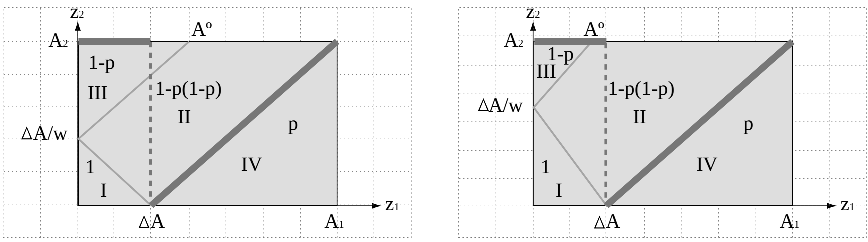
The poor player may also under-invest in a positive-expected-return opportunity in order to maximize probability of achieving top wealth rank. Specifically, if the rich invests an amount large enough that losing it would change relative wealth standing, the poor prefers to limit his investment. But if the rich invests an amount so small that losing it will not change relative wealth standing, then the poor player's

Figure 5a. Rich player's best-response (θ>0)



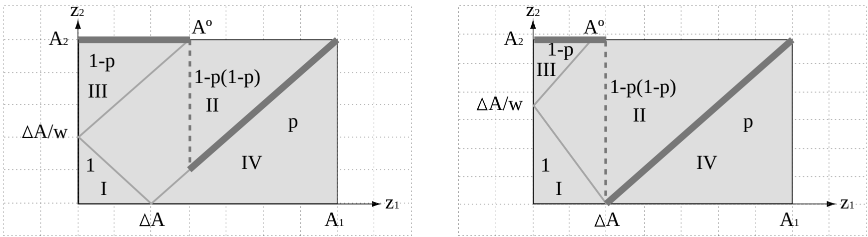
ΔA < A°

Figure 5b. Poor player's best-response (θ>0, p<1/2)



ΔA < A°　　ΔA ≥ A°

Figure 5c. Poor player's best-response (θ>0, p>1/2)



ΔA < A°　　ΔA ≥ A°

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only hope of leapfrogging the rich is through investing himself, and then he might as well invest his entire endowment. Specifically, the poor player's best response function is (see Figure 5b-c)

$$If\ p<1/2,\quad z_2^*(z_1)=\begin{cases} A_2,\ if\ z_1\in[0,\triangle A] \\ z_1-\triangle A-\alpha,\ if\ z_1\in(\triangle A, A_1] \end{cases}$$

$$If\ p>1/2,\quad z_2^*(z_1)=\begin{cases} A_2,\ if\ z_1\in[0,\max(A^0,\triangle A)] \\ z_1-\triangle A-\alpha,\ if\ z_1\in(\max(A^0,\triangle A), A_1] \end{cases}$$

4. Equilibria

The reaction curves derived in the previous section do not intersect. Therefore the game in which players place bets simultaneously has no Nash equilibrium in pure strategies. In this section, we examine equilibria of games in which players place bets sequentially.

4.1 Exogenous order of betting

In the poor-first game, player 2 chooses z_2, player 1 observes it and then chooses z_1, and finally the two lottery draws are made, winnings (if any) paid out, and the prize awarded. In the rich-first game, the order of betting is reversed. Each game has a unique subgame perfect equilibrium. As shown in Figure 6a, when expected return is negative, the poor-first game has one equilibrium and the rich-first game has one of three possible equilibria, depending on exogenous parameter values. Analogously, Figure 6b shows the four equilibria in the case of positive expected return.

In every equilibrium, there is a positive probability that the poor leapfrogs the rich and thereby obtains the prize. Equilibria in the poor-first game make intuitive sense. When expected return is negative, the poor player gambles a substantial portion of his endowment but the rich only a nominal amount. When expected return is positive, both players invest their entire endowments. Equilibria in the rich-first game, however, are counter-intuitive under some parameter values. Specifically, the rich may gamble more than the poor on a negative expected return lottery and invest less than the poor in a positive expected return lottery. We next show that endogenizing the betting order eliminates the rich-first equilibria and retains only the poor-first equilibria.

4.2. Endogenous order of betting

Endogenizing betting order is problematic for two reasons. First, comparing equilibrium outcomes in poor-first and rich-first games reveals that each player prefers to be the follower. This is because, given any parameter values, in the equilibrium of the game in which he is the follower, as compared to the equilibrium of the game in which he is the leader, a player either obtains a higher success probability or the same success probability but with higher expected absolute wealth (due to less gambling when $\theta<0$ or more investing when $\theta>0$.[4] For this reason we cannot follow Baik and Shogren's (1992) approach to endogenizing the sequence of effort choices in a contest by letting each player pre-commit to a time at

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4 However, when $\theta>0$ and $\triangle A>A^0$, player 2 is indifferent between being the leader or the follower.

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Figure 6a. Summary of equilibria in the negative returns case (θ<0). Each box represents an equilibrium and corresponds to one of cases I through IV in Table 1, as indicated on the top line within the box. The bottom two lines within each box indicate equibrium bets z_1 and z_2

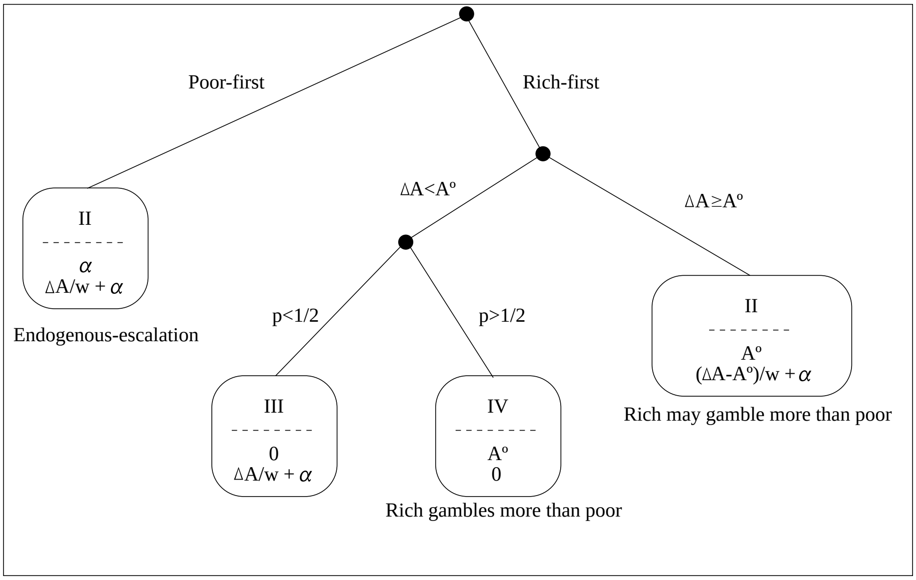
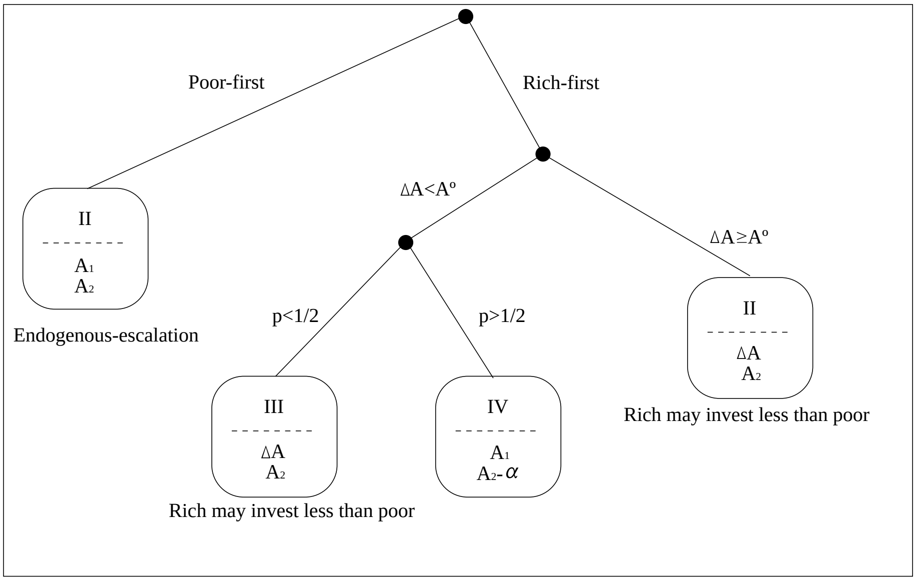


Figure 6b. Summary of equilibria in the positive returns case (θ>0).



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which he will announce his bet. The second difficutly is that if players are allowed to respond to each others' bets an unlimited number of times they will cycle endlessly raising and lowering their bets.

A straightforward way endogenize betting order in spite of these difficulties is the following. Players take turns observing the other's latest bet and adjusting one's own bet. A player may increase his bet or keep it unchanged; decreasing a bet is disallowed. Thus, on the r^{th} round of bet-setting, player i observes z_j^{r-1}, the bet announced by player $j \neq i$ on the previous round, and announces his bet $z_i^r \in [z_i^{r-1}, A_i]$. Initially all bets are zero: $z_1^0 = z_2^0 = 0$. The first time a round is completed in which neither player increases his bet, i.e. $z_1^r = z_1^{r-1}$ and $z_2^r = z_2^{r-1}$, the bets are considered final and the game proceeds to the lottery drawings.

First consider the case of negative expected return. On the first round, the rich player does not bet because he is satisfied with the status quo and has no reason to bet preemptively since another round of betting is guaranteed if the poor player bets something. The poor player follows his best-response and bets $z_2^1 = \frac{\Delta A}{w} + \alpha$. On the second round, the rich player follows his best response by betting $z_1^2 = \alpha$, thereby establishing Case II. Player 2 then considers following his best response function to raise his bet to $z_2^2 = \frac{\Delta A}{w} + 2\alpha$ and thereby establish Case III. However, he knows that player 1 would then respond according by betting $z_1^3 = \alpha$ and thereby re-establishing Case II. Such escalation could continue until $z_2 = A_2$ and $z_1 = A_2 - \Delta A / w$, at which point the poor player would reach his budget constraint and the rich player would succeed in re-establishing Case II. Thus, by escalating beyond $z_2 = \Delta A / w + \alpha$, the poor player gains no increase in success probability but shoulders more negative-return risk. By backward induction, the poor player anticipates the inevitability of Case II and does not escalate. By virtue of having a larger endowment, the rich player has the last move in the endogenous-move game and the game has the same equilibrium as the poor-first game.

In the case of positive expected return, the rich player starts out betting to establish his most-preferred Case I. The poor player responds by betting to establish Case II or Case III. The players continue to escalate until both have fully invested their endowments and Case II obtains. This equilibrium is identical to the equilibrium in the poor-first game.

5. Negative expected return case with many players and prizes

It is interesting to examine the case of negative expected return with $N \geq 2$ players. We assume that no two players have the same endowment and index the players such that $A_i > A_{i+1} \forall i \in \{1, ..., N-1\}$. Let the number of prizes be K, an integer such that $1 \leq K < N$. All prizes are identical. Each player who gets a prize obtains additional utility $V > (w+1) A_1$. The "rich" players are players $i = 1, ..., K$; each would obtain a prize if the lottery were not available and prizes were allocated based on endowed wealth rank. The "poor" are players $i = K+1, ..., N$.[5] Assume $(w+1) A_N > A_1$, which implies that the poorest player can afford a bet that gives him a chance of surpassing the wealth of the richest. V is sufficiently large in the sense of Section 3.2, which allows us to reduce each player's expected utility maximization problem to the problem of betting the least amount to obtain the largest contest success probability. Bets

5 Separate from the absolute rich/poor distinction, we will refer to a player i as "richer" (or "poorer") than player j if $A_i > A_j$ (or $A_i < A_j$).

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are placed according to the same routine as described in Section 4.2. Specifically, each player has his turn to maintain or increase his bet according to some particular order such as $1, 2, \ldots, N$. Bet-setting rounds are repeated until a full round is made without a single player increasing his bet. Then all bets are considered final and an independent random draw of the lottery is held for each player.

After the lottery, prizes are allocated according to rank in the achieved wealth distribution, one prize per player until all K prizes are awarded. Specifically, allocation begins with the player(s) who have the most post-lottery wealth and proceeds down one rank at a time. If, due to ties, the lowest rank for which there is at least one prize still available includes more players than prizes remaining, the remaining prizes are allocated among these players according to their rank in the endowment distribution.

The simple case of $N=3$ and $K=2$ illustrates the logic of the many-player many-prize contest. One strategy the single poor player 3 may consider is to aim to surpass the poorest rich player $i=2$ by betting $z_3=(A_2-A_3)/w+\alpha$. The threatened player 2 will then respond with $z_2=\alpha$, giving the poor player a success probability of $\pi_3=p(1-p)$ (Case II). Player 1 will not be threatened and will not gamble. However, this minimal gambling is not the best strategy for the poor player. He can obtain more success probability by aiming to surpass the richest player by betting $z_3=(A_1-A_3)/w+\alpha$. Then the rich players will respond with $z_1=\alpha$ and $z_2=(A_1-A_2)/w+\alpha$. The poor player stands to obtain a prize if he wins the lottery and either one or both of the rich players lose in the lottery. This gives the poor player a success probability of $\pi_3=p(1-p^2)>p(1-p)$. None of the players wants to escalate betting further since each can deduce that the escalation will be reciprocated, potentially all the way up to the budget constraint, while the distribution of success probability would not change.

This logic generalizes to any number of players when there is only one poor player, i.e., $K=N-1$. The poor player will gamble $z_N=(A_1-A_N)/w+\alpha$, threatening the richest player and thereby inducing every rich player $i=1,\ldots,K$ to gamble defensively $z_i=(A_1-A_i)/w+\alpha$, each seeking to avoid being the one who gives up his prize to the poor player in case the poor player wins the lottery. By inducing such defensive gambling among the rich, the poor player gets a prize if he wins the lottery and at least one of the rich players loses in the lottery, which implies his success probability is the highest achievable: $\pi_N=p(1-p^N)$.

Adding an arbitrary number of poor players to the previous case brings us to the general case of $1\le K<N$. By analogous logic it can be shown that in equilibrium each player rich and poor bets $z_i=(A_1-A_i)/w+\alpha$. Each player who wins the lottery joins the "winner's circle" with $A_1+w\alpha$ in wealth. Let k be a random variable representing the number of lottery winners. There are four mutually exclusive, collectively exhaustive cases:

1. If $k=0$, then each rich player gets a prize and each poor player goes prizeless.
2. If $k=K$, then all winners get prizes and all losers go prizeless, regardless of who was rich and who was poor by endowment.
3. If $1\le k<K$, then all winners and the richest $K-k$ losers get prizes; all other losers go prizeless.
4. If $K<k\le N$, then the richest K winners get prizes; the remaining $k-K$ winners and all losers go prizeless.

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For a poor player, it is necessary but not sufficient to win the lottery to get a prize. This is because of case 4, wherein a poor player may win the lottery but nevertheless go prizeless if K or more richer players also win the lottery. Specifically, for a poor player i,

$$Pr\,(i\ gets\ prize) = Pr\,(i\ wins\ lottery) \times Pr\,(fewer\ than\ K\ lottery\ winners\ are\ richer\ than\ i)$$

which can be expressed using the binomial distribution as follows:

$$\pi_i = p\sum_{k=0}^{K-1}\binom{i-1}{k}p^k(1-p)^{i-1-k}, \quad K < i \leq N$$

Remarkably, the probability of a poor player getting a prize is independent of the total number of players N. All that matters to a poor player i is the number of players richer than him and the number of prizes; the number of players poorer than him is irrelevant. In other words, how he ranks in terms of his endowment matters to a poor player only by looking up the endowment hierarchy.

For a rich player, it is sufficient but not necessary to win the lottery to get a prize. This is because of case 3, wherein a rich player i may lose in the lottery but nevertheless get a prize if $k<K$ and fewer than $K-k$ losers are richer than i. Specifically, for a rich player i,

$$\begin{aligned}Pr\,(i\ gets\ prize) = {} & Pr\,(i\ wins\ lottery) + [Pr\,(i\ loses\ lottery) \times \\ & Pr\,(k<K\ and\ number\ of\ lottery\ losers\ richer\ than\ i\ is\ less\ than\ K-k)]\end{aligned}$$

which can be expressed using binomial distributions as follows:

$$\pi_i = p + (1-p)\sum_{y=0}^{i-1}\binom{i-1}{y}(1-p)^y p^{i-1-y}\sum_{k=0}^{K-i}\binom{N-i}{k}p^k(1-p)^{N-i-k},\ 1 \leq i \leq K$$

Although a poor player’s success probability is unaffected if additional players poorer than him join the contest, a rich player’s success probability decreases with increasing number of poor players. As Table 2 and Figure 7 demonstrate, for a given number of prizes, having more poor players corresponds to lower success probability for every rich player. Each additional poor player joining the contest does not affect other poor players’ chances but redistributes some of the rich players’ success probability to himself. In the limit as the number of poor players grows large, the success probability of each rich player approaches p.

6. Concluding remarks

If the lottery were to be disabled in our model, the player with the larger endowment would obtain the prize with probability one. Since in every equilibrium both players have positive probability of obtaining the prize, the betting is not mutually-offsetting and the lottery serves to narrow the opportunity gap between the players regardless of whether it offers positive or negative expected return. If the expected

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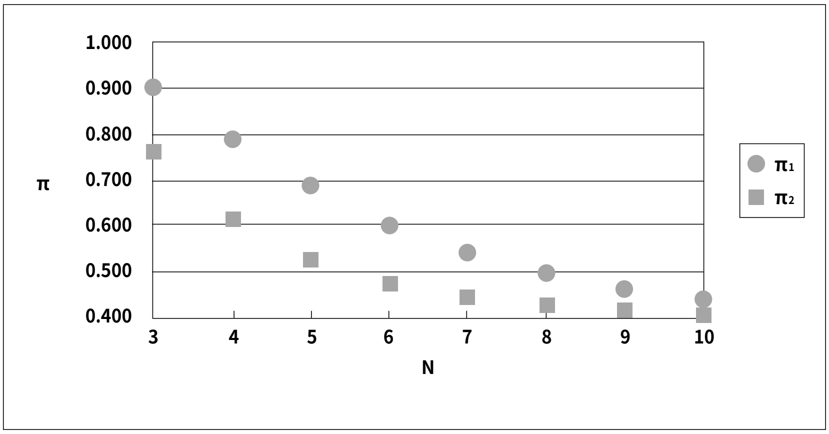
Table 2. Contest success probabilities in a contest with endogenous order of bets, K = 2 prizes and N = 3 to 10 players. Probability of winning the negative expected return lottery is p = 0.4. Keeping the number of prizes constant and adding a poor player at the bottom of the endowment hierarchy redistributes contest success probabilities away from the two rich players to the newly added poor player, without affecting the success probability of the richer poor players.

Players Prizes	N K	3 2	4 2	5 2	6 2	7 2	8 2	9 2	10 2
Rich	1	0.904	0.789	0.685	0.602	0.540	0.495	0.464	0.442
	2	0.760	0.616	0.530	0.478	0.447	0.428	0.417	0.410
Poor	3	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336
	4		0.259	0.259	0.259	0.259	0.259	0.259	0.259
	5			0.190	0.190	0.190	0.190	0.190	0.190
	6				0.135	0.135	0.135	0.135	0.135
	7					0.093	0.093	0.093	0.093
	8						0.063	0.063	0.063
	9							0.043	0.043
	10								0.028

Figure 7. Equilibrium success probabilities of the two rich players in Table 2 as a function of number of players. (K=2, N = 3, ..., 10, p = 0.4)



return is negative, expected absolute wealth is sacrificed in all equilibria; if it is positive, expected absolute wealth is enhanced, but, in some equilibria, not to the full extent possible.

If players are solely concerned with maximizing absolute wealth per utility function (1), the sign of the expected return constitutes necessary and sufficient information that each player must know about the lottery to determine his optimal bet. If players place first priority on relative wealth and second priority on absolute wealth, as in utility function (2), players' best response depends not only on $sign(\theta)$ but also on $sign(p-1/2)$ and on w. In this case, the rate of return is neither necessary nor sufficient in-

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formation for selecting the optimal bet.

Another way to see how the logic of the risk-bearing decision differs under absolute and relative wealth maximization is to consider players' preferences among lotteries. If offered a choice of two lotteries, a player who maximizes absolute wealth per (1) would prefer the lottery with the highest positive expected rate of return or be indifferent if both lotteries offer negative expected return. Rate of return is necessary and sufficient information for an absolute wealth maximizer to decide which lottery he prefers. For a relative wealth maximizer represented by (2), however, the rate of return is neither necessary nor sufficient to decide which lottery he prefers be used in the contest.[6] We have shown that with poor-first or endogenous sequence of bets, the equilibrium allocation of success probability is $\pi_1 = 1 - p(1-p)$ and $\pi_2 = p(1-p)$. Since $p(1-p)$ is the variance of the lottery and has a unique maximum at $p = 1/2$, the rich player prefers the lower-variance lottery with p furthest from $1/2$ whereas the poor prefers the higher-variance lottery with p closest to $p = 1/2$. Unless both lotteries have the same variance, the two players will prefer different lotteries. This implies that even if one lottery first-order stochastically dominates the other, one of the players will nevertheless prefer that it be chosen for use in the contest.

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6 We are continuing to assume that a single lottery is to be used for all players in the contest.

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