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Spontaneous symmetry breaking: some history and some variations on the theme

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WHAT IS SPONTANEOUS SYMMETRY BREAKING ?

Spontaneous (dynamical) symmetry breaking

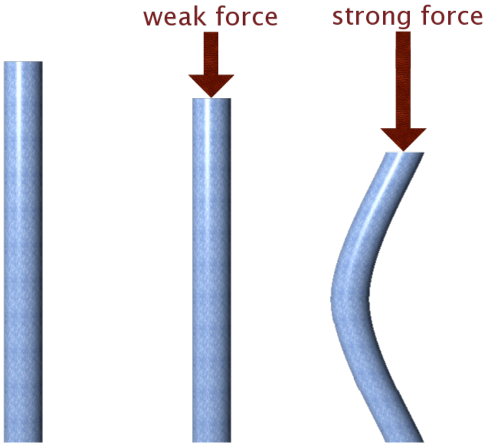

Figure: Elastic rod compressed by a force of increasing strength

Other examples

physical system	broken symmetry
ferromagnets	rotational invariance
crystals	translational invariance
superconductors	local gauge invariance which implies local conservation of the electric charge

When spontaneous symmetry breaking takes place, the ground state of the system is degenerate

Spontaneous breakdown of symmetry is a concept that is applicable only to systems with infinitely many degrees of freedom. Although it pervaded the physics of condensed matter for a very long time, magnetism is a prominent example, its formalization and the recognition of its importance has been an achievement of the second half of the $XXth$ century. Strangely enough the name was adopted only after its introduction in particle physics: it is due to Baker and Glashow. Very often concepts acquire a proper name only when they attain their full maturity.

Spontaneous symmetry breaking was successfully introduced in elementary particle physics through an analogy with condensed matter. The transfer of ideas from one domain of science to another is a complicated process which involves that ill-defined concept that we call intuition. This is something very subjective and uses entirely different paths according to the cultural background and inclinations of each scientist. In theoretical physics an intuition may have as a starting point some imperfect or incomplete parallelism of physical concepts, but sometimes it is a mathematical analogy which is at the origin of a new development and physical concepts are shaped along the way.

Heisenberg, Z. Naturforsch. **14**, 441 (1959), Proceedings of the 1960 Rochester Conference p 851, was the first to consider SBS as a possibly relevant concept in particle physics. To appreciate the innovative character of this concept in particle physics one should consider the strict dogmas which constituted the foundation of relativistic quantum field theory at the time. One of the dogmas stated that in the lowest energy state, the vacuum, nothing physically observable can happen and all the symmetries of the theory, implemented by unitary operators, must leave the vacuum invariant. While the first of these requirements may appear basic for the interpretation of the theory, the second one is far less obvious: in fact even if SBS is not directly observable in the vacuum, its consequences may affect the structure of observables like the particle spectrum. The theory of superconductivity of Bardeen, Cooper and Schrieffer which appeared in 1957 provided the key paradigm for the introduction of SBS in relativistic quantum field theory on the basis of an analogy proposed by Nambu, Phys. Rev. Lett. **4**, 380 (1960).

Nambu's background

Y. Nambu, Nobel Lecture, Rev. Mod. Phys. **81**, 1015 (2009)

I will begin by a short story about my background. I studied physics at the University of Tokyo. I was attracted to particle physics because of the three famous names, Nishina, Tomonaga and Yukawa, who were the founders of particle physics in Japan. But these people were at different institutions than mine. On the other hand, condensed matter physics was pretty good at Tokyo. I got into particle physics only when I came back to Tokyo after the war. Tomonaga had just started to develop the renormalization theory at his university nearby, and some of my room mates who were working with him initiated me into particle physics. The remarkable discoveries of the Lamb shift and the pion-muon decay chain soon occurred, vindicating Tomonaga and Yukawa. In hindsight, though, I must say that my early exposure to condensed matter physics has been quite beneficial to me.

protohistory

The BCS theory of superconductivity was reformulated and developed by various authors including Bogolubov, Valatin, Anderson, Ricayzen and Nambu. The paper of Nambu, Phys. Rev. **117**, 648 (1960), used a language akin to quantum field theory, that is Green's functions formalism. Two facts emerged clearly

1. The elementary fermionic excitations (quasi-particles) are not eigenstates of the charge as they appear as a superposition of an electron and a hole.
2. In order to restore charge conservation these excitations must be the source of bosonic excitations described by a long range (zero mass) field. In this way the original gauge invariance of the theory is restored.

The analogies in particle physics were built on these two facts.

Quasi-particles in superconductivity

Electrons near the Fermi surface are described by the following equation

$$
\begin{aligned}
E\psi_{p,+} &= \epsilon_p \psi_{p,+} + \phi \psi^{\dagger}_{-p,-} \\
E\psi^{\dagger}_{-p,-} &= -\epsilon_p \psi^{\dagger}_{-p,-} + \phi \psi_{p,+}
\end{aligned}
$$

with eigenvalues

$$E = \pm\sqrt{\epsilon_p^2 + \phi^2}$$

Here, $\psi_{p,+}$ and $\psi^{\dagger}_{-p,-}$ are the wavefunctions for an electron and a hole of momentum p and spin $+$

Analogy with the Dirac equation

In the Weyl representation, the Dirac equations reads

$$\begin{aligned} E\psi_1 &= \boldsymbol{\sigma} \cdot \boldsymbol{p}\psi_1 + m\psi_2 \\ E\psi_2 &= -\boldsymbol{\sigma} \cdot \boldsymbol{p}\psi_2 + m\psi_1 \end{aligned}$$

with eigenvalues

$$E = \pm\sqrt{p^2 + m^2}$$

Here, ψ_1 and ψ_2 are the eigenstates of the chirality operator γ_5

Recovering gauge invariance in superconductivity

Approximate expressions for the charge density and the current associated to a quasi-particle in a BCS superconductor

$$\begin{aligned} \rho(x,t) &\simeq \rho_0 + \frac{1}{\alpha^2}\partial_t f \\ \boldsymbol{j}(x,t) &\simeq \boldsymbol{j}_0 - \boldsymbol{\nabla} f \end{aligned}$$

where $\rho_0 = e\Psi^\dagger\sigma_3 Z\Psi$ and $\boldsymbol{j}_0 = e\Psi^\dagger(\boldsymbol{p}/m)Y\Psi$ with Y, Z and α constants and f satisfies the wave equation

$$\left(\nabla^2 - \frac{1}{\alpha^2}{\partial_t}^2\right) f \simeq -2e\Psi^\dagger\sigma_2\phi\Psi$$

Here, $\Psi^\dagger = (\psi_1^\dagger, \psi_2)$ and $\partial_t\rho + \boldsymbol{\nabla}\cdot\boldsymbol{j} \simeq 0$

The axial vector current

Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960)

Electromagnetic current		Axial current
$\bar{\psi}\gamma_\mu\psi$	$\Longleftrightarrow$	$\bar{\psi}\gamma_5\gamma_\mu\psi$

The axial current is the analog of the electromagnetic current in BCS theory. In the hypothesis of exact conservation, the matrix elements of the axial current between nucleon states of four-momentum p and p' have the form

$$\Gamma_\mu^A(p',p) = \left(i\gamma_5\gamma_\mu - 2m\gamma_5 q_\mu/q^2\right) F(q^2) \qquad q = p' - p$$

Conservation is compatible with a finite nucleon mass m provided there exists a massless pseudoscalar particle.

The Goldstone theorem

J. Goldstone, Nuovo Cimento **19**, 154 (1961)

Whenever the original Lagrangian has a continuous symmetry group, which does not leave the ground state invariant, massless bosons appear in the spectrum of the theory.

physical system	broken symmetry	massless bosons
ferromagnets	rotational invariance	spin waves
crystals	translational invariance	phonons

In Nature, the axial current is only approximately conserved. Nambu's hypothesis was that the small violation of axial current conservation gives a mass to the massless boson, which is then identified with the π meson. Under this hypothesis, one can write

$$\Gamma_\mu^A(p', p) \simeq \left(i\gamma_5\gamma_\mu - \frac{2m\gamma_5 q_\mu}{q^2 + m_\pi^2} \right) F(q^2) \qquad q = p' - p$$

This expression implies a relationship between the pion nucleon coupling constant G_π, the pion decay coupling g_π and the axial current β-decay constant g_A

$$2mg_A \simeq \sqrt{2} G_\pi g_\pi$$

This is the Goldberger–Treiman relation

An encouraging calculation

Y. Nambu, G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961), Appendix

It was experimentally known that the ratio between the axial vector and vector β-decay constants $R = g_A/g_V$ was slightly greater than 1 and about 1.25. The following two hypotheses were then natural:

1. under strict axial current conservation there is no renormalization of g_A;
2. the violation of the conservation gives rise to the finite pion mass as well as to the ratio $R > 1$ so that there is some relation between these quantities.

Under these assumptions a perturbative calculation gave a value of R close to the experimental one. More important, the renormalization effect due to a positive pion mass went in the right direction.

One had to show that the dynamical mechanism leading to superconductivity has a counterpart in quantum field theory. In a superconductor a basic fact is the attractive nature of electron-electron interaction, due to phonon exchange, near the Fermi surface. In a picture of the vacuum of a massless Dirac field as a sea of occupied negative energy states, an attractive force between particle and antiparticle should have the effect of producing a finite mass, the counterpart of the gap. At this point the choice of the model became important. In a relativistic theory interactions are usually associated with the exchange of bosons but due to the novelty of the approach a choice was far from obvious.

At that time Heisenberg and his collaborators had developed a comprehensive theory of elementary particles based on a non linear spinor interaction: the physical principle was that spin $\frac{1}{2}$ fermions could provide the building blocks of all known elementary particles. Heisenberg was however very ambitious and wanted at the same time to solve in a consistent way the dynamical problem of a non renormalizable theory. This made their approach very complicated and not transparent but it contained for the first time the idea of SBS in a field theoretic context. Nambu considered Heisenberg theory very formal but the four spinor interaction was attractive due to its simplicity and analogy with the many-body case. I had a more enthousiastic attitude. Shortly after my graduation in Rome, in the two years before going to Chicago, I had been exposed several times to the nonlinear spinor theory, first in a meeting in Venice where a very interesting discussion between Heisenberg and Pauli took place, then in Rome that Heisenberg visited just to explain his theory. At that time Bruno Touschek in Rome was deeply interested in these ideas and I was impressed by comprehensive "fundamental" theories.

1960 Midwest Conference in Theoretical Physics, Purdue University

A 'SUPERCONDUCTOR' MODEL OF ELEMENTARY PARTICLES AND ITS CONSEQUENCES by Y. Nambu (University of Chicago)†

(In absence of the author the paper was presented by G. Jona-Lasinio.)

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In recent years it has become fashionable to apply field-theoretical techniques to the many-body problems one encounters in solid state physics and nuclear physics. This is not surprising because in a quantized field theory there is always the possibility of pair creation (real or virtual), which is essentially a many-body problem. We are familiar with a number of close analogies between ideas and problems in elementary particle theory and the corresponding ones in solid state physics. For example, the Fermi sea of electrons in a metal is analogous to the Dirac sea of electrons in the vacuum, and we speak about electrons and holes in both cases. Some people must have thought of the meson field as something like the shielded Coulomb field. Of course, in elementary particles we have more symmetries and invariance properties

The Nambu–Jona-Lasinio (NJL) model

Y. Nambu, G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)

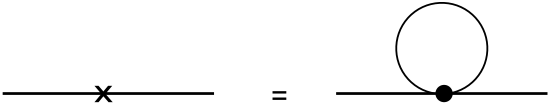
The Lagrangian of the model is

$$L = -\bar{\psi}\gamma_\mu \partial_\mu \psi + g\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right]$$

It is invariant under ordinary and γ_5 gauge transformations

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$$
$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}$$

Mean field approximation



$$m = -\frac{g_0 m i}{2\pi^4} \int \frac{d^4 p}{p^2 - m^2 - i\varepsilon} \, F(p, \Lambda)$$

The spectrum of the NJL model

Mass equation

$$\frac{2\pi^2}{g\Lambda^2} = 1 - \frac{m^2}{\Lambda^2} \ln\left(1 + \frac{\Lambda^2}{m^2}\right)$$

where Λ is the invariant cut-off

Spectrum of bound states

nucleon number	mass μ	spin-parity	spectroscopic notation
0	0	0^-	${}^1S_0$
0	$2m$	0^+	${}^3P_0$
0	$\mu^2 > \frac{8}{3}m^2$	1^-	${}^3P_1$
± 2	$\mu^2 > 2m^2$	0^+	${}^1S_0$

The self consistent field approximation in our model was not very satisfactory. A characteristic feature of statistical mechanics, both classical and quantum, is the existence of variational principles determining the stable states of a system. A quantum field and a many-body system have both infinitely many degrees of freedom: it is therefore natural to look for analogies at a very general level. Variational principles in quantum statistical mechanics have been introduced by Lee and Yang followed by Balian, Bloch and De Dominicis. The variables appearing in these principles are typically average occupation numbers and the corresponding stationary functionals.The key ingredient to derive variational principles is the functional Legendre transform with respect to space-time dependent potentials. At this point the formal analogy with statistical mechanics is obvious it becomes natural to introduce the effective action, a c-number action functional for quantum field theory whose arguments are the vacuum expectation values of the fields.

The effective action

G. Jona-Lasinio, Nuovo Cimento **34**, 1790 (1964)

Define the *partition function*

$$Z[J] = \langle 0|T \exp i[\int dx(L_I + \sum J_i\Phi_i)]|0\rangle$$

where the fields Φ_i transform, e.g., according to the fundamental representation of the orthogonal group. Then

$$G[J] = -i \log Z[J]$$

is the generator of the time ordered vacuum expectation values (in statistical mechanics G is the free energy in the presence of an external field J)

$$\frac{\delta G}{\delta J} = \langle \Phi \rangle = \phi$$

The effective action is the dual functional $\Gamma[\phi]$ defined by the Legendre transformation

$$\frac{\delta \Gamma}{\delta \phi} = -J$$

The vacuum of the theory is defined by the variational principle

$$\frac{\delta \Gamma}{\delta \phi} = 0$$

$\Gamma[\phi]$ is the generator of the vertex functions and can be constructed by simple diagrammatic rules. Its general form is

$$\Gamma[\phi] = L_{\mathrm{cl}}[\phi] + \hbar Q[\phi]$$

Electroweak unification

S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967)

After the appearance of the Nambu-Jona-Lasinio model, spontaneous symmetry breaking became popular and was a key idea for the unification of weak and electromagnetic interactions. This marked its transition from an interesting possibility to an actual basic concept of particle physics.

The form of spontaneous symmetry breaking involved in the electroweak unification is different from the breaking of chiral symmetry and is known as the Brout-Englert-Higgs mechanism (1964).

The NJL model as a low-energy effective theory of QCD

T. Hatsuda, T. Kunihiro, Phys. Rep. **247**, 221 (1994)

The NJL model has been reinterpreted in terms of quark variables. One is interested in the low energy degrees of freedom on a scale smaller than some cut-off $\Lambda \sim 1$ Gev. The short distance dynamics above Λ is dictated by perturbative QCD and is treated as a small perturbation. Confinement is also treated as a small perturbation. The total Lagrangian is then

$$L_{\mathrm{QCD}} \simeq L_{\mathrm{NJL}} + L_{\mathrm{KMT}} + \varepsilon \left(L_{\mathrm{conf}} + L_{\mathrm{OGE}} \right)$$

where the Kobayashi–Maskawa–'t Hooft term

$$L_{\mathrm{KMT}} = g_D \det_{i,j} \left[\bar{q}_i (1 - \gamma_5) q_j + \mathrm{h.c.} \right]$$

mimics the axial anomaly and L_{OGE} is the one gluon exchange potential.

Variation I: Pyramidal molecules

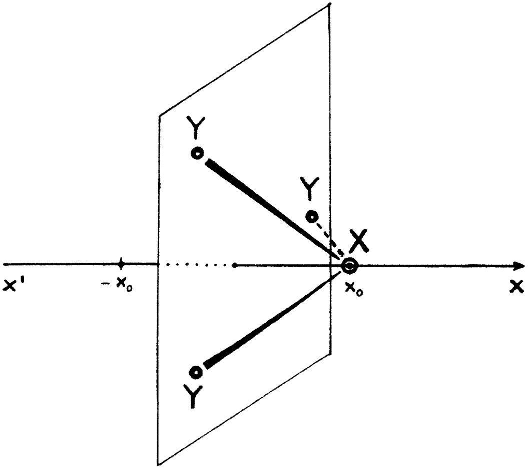


FIG. 1. A pyramidal molecule XY_3 with its inversion axis $x'x$ and the two nuclear equilibrium configurations $-x_0$ and x_0.

Chiral molecules

Examples of pyramidal molecules are NH_3 ammonia, PH_3 phosphine, AsH_3 arsine. Their chemical symbol is of the form XY_3 .

Suppose that we replace two of the hydrogens with different atoms like deuterium and tritium: we obtain a molecule of the form $XYWZ$. This is called an enantiomer, that is a molecule whose mirror image cannot be superimposed to the original one. These molecules are optically active as they rotate the polarization plane of light. For this property they are called chiral.

Hund's paradox 1927

According to quantum mechanics chiral molecules cannot exist as stationary states.

In fact the corresponding Hamiltonian is invariant under parity and its ground state is necessarily a superposition with equal weights of the two enantiomers. It is therefore delocalized and its dipole moment is zero.

A way out: we never observe isolated molecules

P. Claverie, G. Jona-Lasinio Phys. Rev. A **33**, 2245 (1986)

If we deal with a set of molecules (e.g. in the gaseous state), once localization happens for a molecule there appears a cooperative effect which tends to stabilize this localization. The mechanism is called the reaction field mechanism.

Let μ the dipole moment of the localized molecule; this moment polarizes the environment which in turn creates the *reaction field* $\mathcal{E}$ which is collinear with μ and the interaction energy $V = -\mu \cdot \mathcal{E}$ is negative.

If $|V| >> \Delta E$, where ΔE is the doublet splitting due to tunneling in the isolated symmetric state, the molecules of the gas are localized. As a consequence the doublet should disappear when $|V|$ increases for example by incresing the pressure.

Variation II: Spontaneous symmetry breaking in nonequilibrium

M. R. Evans, D. P. Foster, C. Godreche, D. Mukamel, Phys. Rev. Lett. **74**, 208 (1995)

C. Godreche, J. M. Luck, M. R. Evans, D. Mukamel, S. Sandow, E. R. Speer, J. Phys. A **28**, 6039 (1995)

V. Popkov, M. R. Evans, D. Mukamel, J. Phys. A **41**, 432002 (2008)

S. Gupta, D. Mukamel, G. M. Schuetz, arXiv:0908.25 [cond-mat]

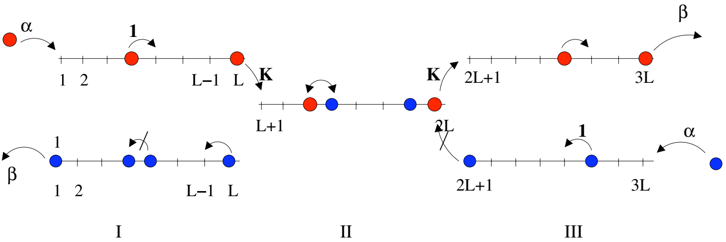
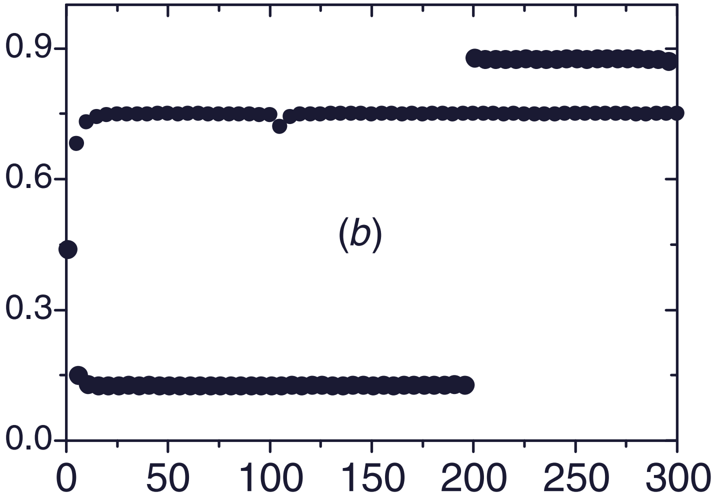
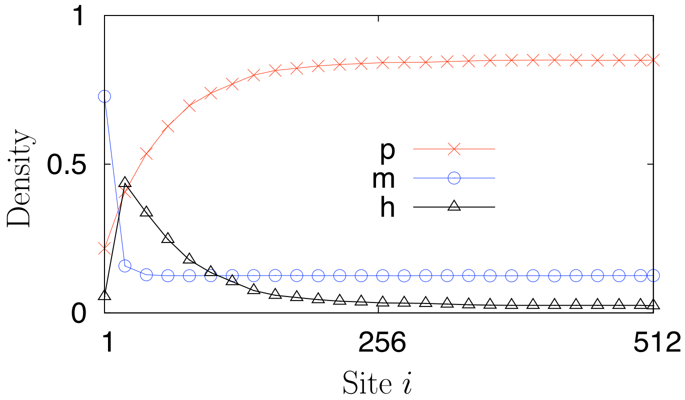


Figure 1. The bridge model with two junctions. Positively (negatively) charged particles hop to the right (left). The model is invariant with respect to left–right reflection and charge inversion. Section II is the bridge. It contains positive and negative particles and holes. Sections I and III comprise parallel segments each containing pluses and holes or minuses and holes.



The segments $[0, 100]$ and $[200, 300]$ correspond to sections I and III. The middle points represent the positive charges.


1
0.5
0
Density
1
256
512
Site i
p
m
h

The existence of two SSB steady states can be easily established in mean field approximation for appropriate values of the rates $\alpha > \beta$. The SSB states are connected by the CP operation.

When the size L is finite the system flips between the two states. The flipping time τ_{flip} can be estimated

$$\tau_{flip} \simeq \exp \kappa L \quad \kappa = 2 \log \frac{\alpha(1-\alpha)}{\beta(1-\beta)}$$

A conclusion and some open problems

Spontaneous symmetry breaking pervades many sectors of physics and can be relevant also in situations of our social life (e.g. traffic problems!)

Noticeable asymmetries still awaiting a convincing explanation in which spontaneous symmetry breaking may play a role:

1. matter dominance over antimatter in the near universe
2. dominance of left-handed molecules over right-handed in living matter