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On a Scheme of Driver's License

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1. Introduction

Not a few papers in theoretical economics are concerned with the relationships between market equilibrium and efficient allocation. Concerning papers on externality, one can find one of the examples in discussions of the relationships between efficiency and solutions in tax-subsidy scheme or artificial market. It is already a well known fact that the Pigovian tax-subsidy solution is identical to the one of artificial markets, the only difference resulting in income distribution if lump-sum transfer is feasible. It must be noted, however, that the usual market mechanism or tax-subsidy scheme can work well only if the relevant commodity is measurable and the quantity of the commodity is recognized by the participants of the trade. Since externalities are rather difficult to measure compared to usual commodities, it may be reasonable to consider the other policies for externality than the usual market mechanism or tax-subsidy scheme.

Let us discuss this problem in case of road congestion as a kind of externality. As William Vickrey ([8] and [9]) pointed out, use of pricing as a means of obtaining improved utilization of transportation facilities within metropolitan areas has hardly been an outstanding success in the past. In this paper, we are not concerned with the question whether the electric pricing introduced by him is practical or not, but we are concerned with implications of a scheme in which the measurement of each driver's contribution to road congestion is not implimented, therefore, it becomes impossible to make drivers pay toll proportional to their contributions to the road congestion, and full Pareto efficiency cannot be obtained.[1]

It will be shown that we are going to investigate the second best policy for road congestion. By the second best we mean a constrained optimum under available policy rules. In this spirit of the second best, the subject of this paper deals with

1) Our license scheme is a kind of indirect charging and is close formally to the dayly license in the Smeed Report [5]. The report as well as W. Vickrey encourages most strongly direct charging for road use with meter systems.

the same problem as the theory of commodity taxation or of optimal departure from marginal cost pricing.[2] When lump-sum transfers are not feasible, commodity taxation or departure of price from marginal cost may be available for income redistribution and for public budgetary requirement. The limitations to the set of policy rules available may come from administrative, political, or social constraints. Even if political or social constraints can be ignored, under considerations of administration, a full Pareto efficient solution does not always mean it is the first best. More precisely, if administrative costs in each economic policy are taken into account, it is not clear whether a full Pareto efficient solution entailing a high administrative cost is the first best one or not.[3]

The purpose of this paper is to examine a scheme of license as a policy tool which is assumed to be chosen from the administrative considerations. In exploring the implications of a policy tool available to the government, the paper has the same motivation as in the theory of commodity taxation or of optimal departure from marginal cost pricing.

The purpose of this scheme that we are going to propose is to reduce the traffic in a community to the optimum. In this paper we make the following simplifying assumptions:

(1) The analysis is limited to a short-term problem how to use most efficiently the roads that already exist.

(2) The analysis is concerned only with traffic derived from consumers enjoying their driving. (The congestion arising from physical distribution is ignored).

(3) The congestion arising from bus service is ignored.

The license scheme will be defined in the following. The public authority issues license, and consumers can get the license by paying some price. A consumer with the license can enjoy driving in the community with no limit of time or distance. The authority can control the amount of the issued license by deciding the price of the license (the license fee). Revenue from the sale of the license is, after subtracting the administrative cost of the license scheme, divided equally between all the non-drivers.

The role of the license is to decrease the aggregate traffic volume to the Pareto efficiency with rather low administrative cost. The license scheme is not an institution to give the full Pareto efficiency because of the discrimination between drivers and non-drivers. To get the full efficiency, all the consumers must have the right to drive, paying money proportional to degree of their contributions to the congestion on roads as in the congestion toll scheme. The administrative cost in the license

2) See, for example, [2], [3] and [4].

3) Arrow [1] pointed out that transaction costs as costs of running the economic system should be an important factor in the choice of mode of resource allocation.

scheme seems to be rather low compared with one in the congestion toll scheme, because the public authority has only to inspect whether each driving person has the license or not, and has neither to know how much he has driven nor to make him pay money proportional to the time or the distance of driving.[4]

2. The Model

Let there be s consumers in the community concerned, each having the composite good including his leisure I_j, $j=1, 2, \dots s$. A consumer's utility is assumed to be a function of the composite good disposable C_j, driving distance and the road congestion. In the laissez-faire economy, a consumer j maximizes

$$U^j(C_j, d_j, \sum_{\neq j} d_i)$$

$$\equiv U^j(I_j - u_j d_j, d_j, \sum_{\neq j} d_i)$$

under $d_j \geqq 0$, where d_j denotes his driving distance and u_j denotes the composite good used up per unit of driving and $\sum_{\neq j} d_i$ denotes the road congestion effective for j. If $d_j > 0$, he enjoys driving. If $d_j = 0$, he does not enjoy driving at all.[5]

Let m, n and z be the per capita compensation to non-drivers, the license fee, and the administrative cost of the license scheme, respectively. If r consumers are drivers out of the s consumers, the budgetary equation of the public authority is:

$$rn = (s-r)m + z.$$

When we define $e = m + n$, we get the rule (R) for determining m and n:

$$m = \frac{er - z}{s}$$
$$n = \frac{(s-r)e + z}{s}. \qquad \text{(R)}$$

e turns out to be degree of discriminating between a driver and a non-driver with respect to the composite good. Since e is non-negative and z is positive, n is positive

4) Some license schemes for efficient pollution control were analytically explored by [6] and [7]. Our scheme differs from them in that with a license an individual can enjoy driving with no limit of driving distance while in their model individual economic unit can discharge pollutants according to the quantity of licenses it buys. Since our type of license has not been explored yet and it is presumably rather practical one in the case of road congestion, we tried to investigate the scheme in some detail.

5) It is very restrictive that u_j is independent of the road congestion. If the consumption of the gasoline is increased by the delay of cars owing to the road congestion, then u_j will be dependent on the road congestion; $u_j = u_j\,(\sum_{\neq j} d_i)$. Under our assumption, the congestion is considered to be purely psychological factor, not to be costly in terms of the consumption of fuel.

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while m's sign is indeterminate.

If $\max\limits_{d_j \geqq 0} U^j(I_j - n - u_j d_j, d_j, \sum\limits_{\neq j} d_i)$ is greater than $U_j(I_j + m, 0, \sum\limits_{\neq j} d_i)$, he would buy the license. If less, he would not buy the license.

When $e = e^0$, an equilibrium in this scheme is defined:

$$(e^0, r^0, d_1^0, d_2^0, \dots, d_s^0)$$

such that only r^0 consumers buy the license under road congestion (as defined by the sum of driving distances of the others) $\sum\limits_{\neq j} d_i^0$, and m^0 and n^0 determined by e^0 and r^0 according to (R). There would be a sequence of equilibria corresponding to the changing e.

In the following, we are going to investigate the stability of an adjustment process when the public authority increases the control variable e and properties of the sequence of the equilibria.

3. An Adjustment Process and the Sequence of Equilibria

When the public authority increases the control variable e up to e^* from an equilibrium $(e^0, r^0, d_1^0, d_2^0, \dots, d_s^0)$, will the system $(e^*, r, d_1, d_2, \dots, d_s)$ converge to new equilibrium? The adjustment process of the license scheme is defined as follows:[6]

$$\begin{cases} r = r^0 \\ e = e^* \end{cases} \xrightarrow{\text{(R)}} \begin{cases} m^1 \\ n^1 \end{cases} \xrightarrow{(d_1^0, d_2^0, \dots, d_s^0)\downarrow} \begin{cases} r^1 \\ (d_1^1, d_2^1, \dots, d_s^1) \end{cases}$$

$$\begin{cases} r = r^i \\ e = e^* \end{cases} \xrightarrow{\text{(R)}} \begin{cases} m^{i+1} \\ n^{i+1} \end{cases} \xrightarrow{(d_1^i, d_2^i, \dots, d_s^i)\downarrow} \begin{cases} r^{i+1} \\ (d_1^{i+1}, d_2^{i+1}, \dots, d_s^{i+1}). \end{cases}$$

$$(i = 1, 2, \dots,)$$

Assumptions

(A, 1) Utility function is separable, that is, $U(C, d) + V(\sum\limits_{\neq j} d_i)$, or $V(\sum\limits_{\neq j} d_i) U(C, d)$.

(A, 2) As for the signs of derivatives, $U_C > 0$, $U_d > 0$, $V' < 0$, $U_{CC} < 0$ and $U_{Cd} > 0$. Strictly diminishing marginal rate of substitution in C and d.

Theorem

Under (A, 1) *and* (A,2), *the system* $(e^*, r, d_1, d_2, \dots, d_s)$ *converges to an equilibrium, and* r *cannot increase.*

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6) The same kind of discussion applies when the authority decreases e.

Proof. From (A, 1), it depends only on the amount of $U(C,d)$ whether a consumer buys the license or not.

Since $e^*>e^0$ means $m^1>m^0$ and $n^1>n^0$ according to the rule (R), $r^0 \geqq r^1$. The number of drivers remains r^1 even when $\sum_{\neq j} d_i^0$ is reduced to $\sum_{\neq j} d_i^1$.

$$m^1-m^2=n^2-n^1=\frac{r^0-r^1}{s}e^* \geqq 0.$$

Change in utility of a consumer as a non-driver is:

$$U_C(I+\alpha,0)(m^2-m^1),\quad m^1 \geqq \alpha \geqq m^2,$$

where $U_C(I+\alpha,0)$ denotes value of partial derivative of U with respect to C at $(I+\alpha,0)$.

Change in utility of a consumer as a driver is:

$$\{U_C(I-\beta-ud,d)+(-uU_C+U_d)d'\}(n^1-n^2)=U_C(I-\beta-ud,d)(n^1-n^2),$$
$$n^2 \geqq \beta \geqq n^1,$$

where d' denotes value of derivative of d with respect to $I-n$, at $I-\beta$.

Then, by (A, 2) and since $\alpha+\beta>n^1+m^2>0$,

$$U_C(I-\beta-ud,d)-U_C(I+\alpha,0)=U_{CC}(a,b)(-\beta-ud-\alpha)+U_{Cd}(a,b)d>0,$$

where $a=I+\alpha+\theta(-\beta-ud-\alpha)$, $b=\theta d$, $1>\theta>0$.

Therefore, we get $0 \geqq$ change in utility as a non-driver $\geqq$ change in utility as a driver. Thus, $r^1 \geqq r^2$. When the process is repeated, we obtain

$$0 \leqq \dots r^{i+1} \leqq r^i \dots r^3 \leqq r^2 \leqq r^1 \leqq r^0.$$

Therefore, after some q, $r^q=r^{q+1}=\dots$, $m^{q+1}=m^{q+2}=\dots$, and $n^{q+1}=n^{q+2}=\dots$. Then, $(e^*, r^{q+1}, d_1^{q+1}, d_2^{q+1}, \dots, d_s^{q+1})$ is an equilibrium. (*q. e. d.*)

It is easy to see that under the separable utility functions for $e=0$ $(n=-m=\frac{z}{s})$ all the consumers buy the license and an equilibrium exists. Therefore, existence of an equilibrium is not a problem for any $e(\geqq 0)$.

As the process of proof shows, equilibrium corresponding to each e may be not unique. Different equilibria are possible according as a consumer remains a driver or becomes a non-driver when he is indifferent at some m and n.

As a corollary of Theorem, we can say that the number of drivers does not increase in the sequence of the equilibria corresponding to the increasing e under the assumption (A, 1) and (A, 2). Then m and n behave as in Figure 1 and Figure 2, respectively, corresponding to the increasing e. At each discontinuous point in positive e, a driver becomes a non-driver.

How about the changes in utilities of drivers or non-drivers, corresponding to the increasing e? When e increases, the road congestion $\sum_{\neq j} d_i$ monotonically decreases

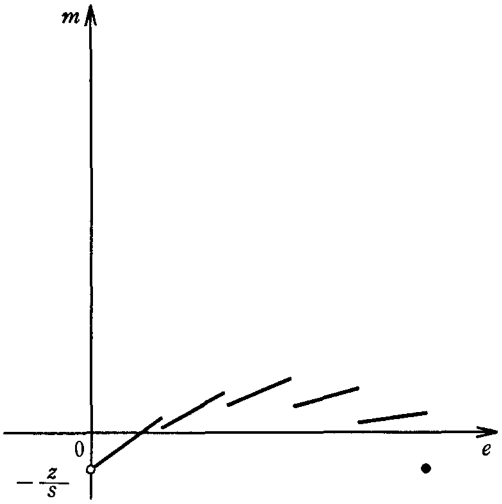

Figure 1

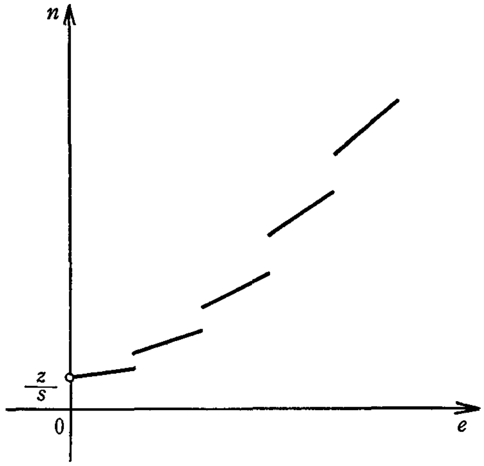

Figure 2

since the number of drivers decreases and, also, each driver's driving time decreases (if driving is assumed to be a normal good).

If m increases, then a non-driver becomes better off. The decrease in m counteracts the increase in utility brought about by the reduced road congestion. Since the increase in utility by the reducing road congestion becomes more and more less, meanwhile, non-driver's utility has a tendency to decrease.

A driver's utility increases with the increasing e in a smaller e because the increase in utility by the reducing road congestion overweighs the decrease in utility by the increase in n. When e becomes greater, the decrease in utility by the increase in n overweighs the increase in utility by the reducing road congestion. Therefore, the driver's utility is a decreasing function of e in the greater e.

It is easy to understand that, if each consumer is not so much different from each other in income distribution and in sensitivity to the road congestion, then non-driver gets his maximum utility, when e is increased, after driver gets his maximum utility. Therefore, the choice of e will be made not only to get some Pareto superior solution but also to give a solution in a conflict of interests between drivers and non-drivers.

4. More General Case of Non-separable Utility Function

In this section, we try to extend the results derived in Section 3 to the case of more general non-separable utility function.

Just as in Section 3, we will compare the change in utility of an individual as a driver with that in utility of him as a non-driver as the step of the adjustment process proceeds from 1 to 2.

Change in utility of a consumer (j) as a non-driver

$$\Delta U^n = U_C(I+\alpha, 0, \hat{D})(m^2-m^1) + U_D(I+\alpha, 0, \hat{D})(D^1-D^0)$$

where $D \equiv \sum_{i \neq j} d_i$ (the road congestion effective for j),

$$\alpha = \theta m^2 + (1-\theta) m^1,$$

and $\hat{D} = \theta D^1 + (1-\theta) D^0$, for $1 >$ some $\theta > 0$.

Change in utility of a consumer as a driver

$$\Delta U^d = U_C(I-\beta-u\hat{\hat{d}}, \hat{\hat{d}}, \hat{\hat{D}})(n^1-n^2) + U_D(I-\beta-u\hat{\hat{d}}, \hat{\hat{d}}, \hat{\hat{D}})(D^1-D^0)$$

where $\beta = \tau n^2 + (1-\tau) n^1$

$\hat{\hat{D}} = \tau D^1 + (1-\tau) D^0$, for $1 >$ some $\tau > 0$.

$\hat{\hat{d}}$ is driving distance under income $I-\beta$ and the road congestion $\hat{\hat{D}}$.

Then, from the mean value theorem,

$$\begin{aligned} &\Delta U^n - \Delta U^d \\ &= \{U_C(I+\alpha, 0, \hat{D}) - U_C(I-\beta-u\hat{\hat{d}}, \hat{\hat{d}}, \hat{\hat{D}})\}\Delta I + \{U_D(I+\alpha, 0, \hat{D}) - U_D(I-\beta-u\hat{\hat{d}}, \hat{\hat{d}}, \hat{\hat{D}})\}\Delta D \\ &= \{(\alpha+\beta+u\hat{\hat{d}})U_{CC} - \hat{\hat{d}}U_{Cd} + (\hat{D}-\hat{\hat{D}})U_{CD}\}\Delta I + \{(\alpha+\beta+u\hat{\hat{d}})U_{DC} - \hat{\hat{d}}U_{Dd} + (\hat{D}-\hat{\hat{D}})U_{DD}\}\Delta D, \end{aligned}$$

where $\Delta I = m^2 - m^1 = n^1 - n^2 = \frac{r^1 - r^0}{s} e^* \leqq 0$ and $\Delta D = D^1 - D^0 < 0$ since the number of drivers decreases and driving distance of each individual decreases as the process proceeds.

Let us assume that $U_D < 0$, $U_{CC} < 0$, $U_{Cd} > 0$, $U_{DC} < 0$, $U_{Dd} > 0$ and $U_{DD} < 0$. Then if it is assured that $\hat{D} - \hat{\hat{D}} > 0$, one can obtain that $\Delta U^n - \Delta U^d > 0$. Therefore it is not possible that the number of drivers increases as the process moves from 1 to 2.

This property obtains through every step as long as the number of drivers and the income for each individual decrease and the road congestion becomes improved, and both effects make it more uncomfortable for each consumer to be a driver than a non-driver. However, if at some step $\hat{D} - \hat{\hat{D}}$ become sufficiently negative so that some non-drivers return to drivers and moreover, the number of them overweighs the number of individuals who quit drivers, then at the next step income for each individual increases and the road congestion presumably becomes worse, and those are again conducive to increasing the number of drivers. When the process attains the vicinity of an equilibrium corresponding to e^*, since the number of individuals who quit drivers is very small, even a small number of individuals who return again to drivers are very likely to be decisive of determining whether the total number of drivers is increasing or not. Even a small number of individuals can reverse the direction of change in the total number of drivers. This implies that the process is very unstable just in the vicinity of an equilibrium.

Unfortunately we could not derive any plausible condition for $\hat{D}-\hat{\hat{D}}>0$. However we can have a comment on the condition for $\hat{D}-\hat{\hat{D}}>0$.

If one assumes, taking into account difference of places to be involved in the congestion, that drivers feel less unpleasant with the congestion than non-driver so that the effective congestion for non-driver is D while that for driver is δD $(0<\delta<1)$, then necessarily $\hat{D}-\hat{\hat{D}}>0$ if $\delta D^0<D^1$. It is easy to see that this condition is the more likely to be satisfied if δ is the smaller, that is, driver is in the more favorable position concerning the road congestion.

5. Implication of Presence of Administrative Cost

So far we have not investigated whether the license scheme is more desirable than the laissez-faire economy or not. If the capacity of the roads is enough not to raise any serious congestion problem, the laissez-faire economy is more desirable for some drivers than the license economy.

Let us precisely prove this conjecture in case of additively separable utility function in assumption (A, 1). In the laissez-faire economy, a driver's utility is

$$U(I-ud^*, d^*)+V\left(\frac{\sum_{\neq j} d_i^*}{K}\right)\equiv U(*)+V(*)$$

where K denotes the capacity of the roads.

In the license economy, the driver's utility is

$$U(I-n-ud^{**}, d^{**})+V\left(\frac{\sum_{\neq j} d_i^{**}}{K}\right)\equiv U(**)+V(**).$$

Since $U(*)-U(**)$ is a positive constant because of the positive n, when K increases, there is some $\tilde{K}$ such that $U(*)+V(*)>U(**)+V(**)$ for $K>\tilde{K}$ under plausible assumptions.

It plays a very important role in the choice of the license scheme how high the administrative cost of the license scheme is. If z is 0, the schedule of n starts from the origin in Figure 2. As z increases, n shifts upwards at a very small e. Then, $U(*)-U(**)$ increases at the e, which means that the minimum $\tilde{K}$ decreases in which the laissez-faire economy becomes more desirable than the license scheme. The increase in the administrative cost enlarges the case for the laissez-faire economy.

6. Conclusion

We have analyzed the license scheme by presenting a very simplified model. We also have examined how an adjustment process works which is adopted by the public

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authortiy when it increases or decreases the degree of discriminating between a driver and a non-driver. We have found out that the adjustment process is at least likely not to be monotonically convergent to an equilibrium. Since the monotonical convergence has been proved in the case of a separable utility function, it turns out that the process becomes unstable since individuals change their mind on whether he buys a license or not owing to worsening or improving road congestion.

Concerning the stability of the process and the equilibrium to be attained, an important role was played by the decision of each individual of whether he should be a driver or a non-driver when he was indifferent. The decisions become more effective and can bring about cumulative process in the vicinity of an equilibrium. The instability of the process is a result of interactions of individuals by way of the externality of road congestion. Although we pointed out the possibility of cumulative instability near the equilibrium, our intention was not to refute the license scheme for the reason of the possible instability but to show some difficulties in enforcing a practical adjustment process. Our main result will be mentioned as one of difficulties relating to the second best policy.

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