Addenda and Errata to “Physics and Mathematics of Quantum Many-Body Systems”

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Addendum to Section 2.1

In Problem 2.1.g (page 20), I asked the reader to derive explicit matrix forms of the \(\pi\)-rotation operators about the three axes. Since I was not able to devise any compact direct derivation, I suggested to reduce the problem to that of a collection of \(S = 1/2\) spins. After the publication of the book, however, I learned the following elegant derivation from Keisuke Muota. I believe this one is much better than the solution I gave in the book.

A better solution to Problem 2.1.g

We shall show below that

\[
\langle y_s \hat{u}_1^* y_t \rangle = (-i)^{2S} \delta_{\sigma,-\tau}.
\]

The relation

\[
\langle y_s \hat{u}_3^* y_t \rangle = e^{-i\pi \delta_{\sigma,\tau}}
\]

is obvious. Then the relation

\[
\langle y_s \hat{u}_2^* y_t \rangle = \delta_{\sigma,+}\delta_{\sigma,-}\tau
\]

follows from \(\hat{u}_2 = \hat{u}_3\hat{u}_1\).

From (2.1.21), we see that

\[
\hat{S}^\alpha \hat{u}_1 = -\hat{u}_1 \hat{S}^\alpha
\]

for \(\alpha = 2,3\). Since

\[
\hat{S}^{(3)} \hat{u}_1 |\psi\rangle = -\hat{u}_1 \hat{S}^{(3)} |\psi\rangle = -\sigma \hat{u}_1 |\psi\rangle,
\]

we find that

\[
\hat{u}_1 = \sum_\sigma \lambda_\sigma |\psi^-\rangle \langle \psi\sigma|,
\]

with some \(\lambda_\sigma\).

Note that (AE.1) also implies \(\hat{S}^+ \hat{u}_1 = \hat{u}_1 \hat{S}^-\). By using (2.1.3) and (AE.3), we find

\[
\hat{S}^+ \hat{u}_1 |\psi\sigma\rangle = \lambda_\sigma \hat{S}^+ |\psi-\sigma\rangle = \lambda_\sigma \sqrt{S(S+1) + \sigma(\sigma+1)} |\psi^-\sigma+1\rangle,
\]

and

\[
\hat{u}_1 \hat{S}^- |\psi\sigma\rangle = \sqrt{S(S+1) - \sigma(\sigma-1)} \hat{u}_1 |\psi^-\sigma-1\rangle
\]

\[
= \lambda_{\sigma-1} \sqrt{S(S+1) - \sigma(\sigma-1)} |\psi^-\sigma+1\rangle.
\]

Since these two must be identical, we find \(\lambda_\sigma = \lambda_{\sigma-1}\), and hence

\[
\hat{u}_1 = \lambda \hat{R},
\]

for some \(\lambda \in \mathbb{C}\), where
\[ \hat{R} = \sum_{\sigma} |\psi^{-\sigma}\rangle\langle\psi^{\sigma}|. \]  

(AE.7)

We only need to determine \( \lambda \). Let \( |\xi^S\rangle \) be the unique state such that \( \hat{S}^{(1)}|\xi^S\rangle = S|\xi^S\rangle \), which obviously satisfies \( \hat{u}_1|\xi^S\rangle = e^{-i\pi S}|\xi^S\rangle \). We claim that \( \hat{R}|\xi^S\rangle = |\xi^S\rangle \), which shows the desired result \( \lambda = e^{-i\pi S} = (-i)^{2S} \).

To see the claim we first note that \( \hat{R}|\xi^S\rangle = \pm|\xi^S\rangle \) because \( [\hat{R}, \hat{u}_1] = 0 \) and \( \hat{R}^2 = \hat{1} \). Note that \( |\xi^S\rangle \) is obtained by maximizing \( \langle \xi | \hat{S}^{(1)} | \xi \rangle = \langle \xi | (\hat{S}^+ + \hat{S}^-) | \xi \rangle / 2 \) over all normalized \( |\xi\rangle = \sum_{\sigma} c_{\sigma} |\psi^{\sigma}\rangle \). It is obvious that the maximum is attained when \( c_{\sigma} \geq 0 \) for all \( \sigma \), and hence it is only possible that \( \hat{R}|\xi^S\rangle = |\xi^S\rangle \). (This is essentially the same as using the Perron-Frobenius theorem.)

Addendum to Section 8.3.5

In footnote 50 (page 278), I mentioned a generalization of Theorem 8.6 that is parallel to Theorem 8.7. Recently we found a new proof of such a generalized theorem. Interestingly the proof makes an essential use of the index introduced in the study of symmetry protected topological phase.


Addendum to Section 11.3.2

Theorem 11.13, which is based on [34], is not correct as it is. When \( (\hat{A}, \hat{B}) \) is not bipartite, the condition of biconnectedness does not guarantee the existence of ferromagnetism. See footnote in page 4338 of [35].
Errata

- p. vii, 6th line from the bottom:
  detal → detail
- p. 30, 3rd line below (2.3.28):
  identical → identical
- p. 37, 4th line:
  in such way → in such a way
- p. 40, 3rd and 4th lines in the Proof of Theorem 2.2:
  "that the whole spectrum of $\hat{H}$ is contained in the subspace $\mathcal{H}_0$" should better be
  "that, for any eigenvalue of $\hat{H}$, there is at least one corresponding eigenstate in $\mathcal{H}_0$"
- p. 54, below (3.2.9):
  $\sum_{x \in A} \langle \sigma_x, \sigma_x \rangle_{\beta,\alpha}$ should be $\sum_{x \in \mathbb{Z}^d} \langle \sigma_x, \sigma_x \rangle_{\beta,\alpha}$, i.e., change $A_L$ to $\mathbb{Z}^d$.
- p. 193, (7.2.7):
  the minus sign on the right-hand side should be removed.
- p. 212, 8th line:
  proposal → proposed
- p. 243, 2nd line of (8.2.16):
  $\sum_{x=1}^{L} \rightarrow \sum_{x=1}^{L-1}$
- p. 257, 12th line:
  the map $x \rightarrow L - x \rightarrow$ the map $x \rightarrow L + 1 - x$
- p. 263, 18th and 19th lines:
  $x \rightarrow 1 - x \rightarrow x \rightarrow -1 - x$
- p. 265, footnote 37:
  This is not an erratum, but it is better to add the following sentence to the foot-
  note. "Recalling that $A^\sigma, \tilde{A}^\sigma$ and $B^\sigma$ are independent of $L$, we see that $\eta_L/L$ is
  independent of $L$.”
- p. 295, the first paragraph:
  The assertion about the uniqueness of the infinite volume ground state is incor-
  rect. In fact, it is proved in [14] that the toric code model on $\mathbb{Z}^2$ has exactly
  four ground states. One of them is the frustration free ground state obtained as
  the unique infinite volume limit of the ground states of finite systems. The re-
  maining three ground states are not frustration free, and are characterized by the
  presence of an anyon. See [14] for details.
- p. 212, 15th line:
  the the → the
- p. 265:
  The logic on this page is incomplete. Theorem 7.6, as is stated, does not
- p. 316, the line below (9.2.37):
  know → known
- p. 437, 2 lines above (11.4.70):
The inequality $\hat{P}_0 \hat{h}_{\text{eff}} \hat{P}_0 \geq -2s' = -2sv^4/(1 + v^2)$ should be understood as the inequality in a suitable space.

- p. 437, (11.4.74):
The inequality should be understood as inequality in a suitable space.

- p. 497, (S.19), 1st and 2nd lines:
  $e^{-iM\phi/2} \rightarrow e^{iM\phi}$

- p. 508, (S.77):
The product should be from $x = 1$ to $L - 1$. 
