Addenda and Errata to “Physics and Mathematics of Quantum Many-Body Systems”

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Addendum to Section 2.1

In Problem 2.1.g (page 20), I asked the reader to derive explicit matrix forms of the $\pi$-rotation operators about the three axes. Since I was not able to devise any compact direct derivation, I suggested to reduce the problem to that of a collection of $S = 1/2$ spins. After the publication of the book, however, I learned the following elegant derivation from Keisuke Muota. I believe this one is much better than the solution I gave in the book.

A better solution to Problem 2.1.g

We shall show below that
\[ \langle y^s_j \hat{u}_1 y^t \rangle = (-i)^{2S} \delta_{s,t}; \] (AE.1)

for $\alpha = 2, 3$. Since
\[ \hat{S}^{(3)} \hat{u}_1 \psi^\sigma = -\hat{u}_1 \hat{S}^{(3)} \psi^\sigma = -\sigma \hat{u}_1 \psi^\sigma, \] (AE.2)

we find that
\[ \hat{u}_1 = \sum_{\sigma} \lambda_\sigma \langle \psi^{\sigma} \rangle \langle \psi^\sigma \rangle, \] (AE.3)

with some $\lambda_\sigma$.

Note that (AE.1) also implies $\hat{S}^+ \hat{u}_1 = \hat{u}_1 \hat{S}^-$. By using (2.1.3) and (AE.3), we find
\[ \hat{S}^+ \hat{u}_1 \psi^\sigma = \lambda_\sigma \hat{S}^+ \psi^{-\sigma} = \lambda_\sigma \sqrt{S(S+1) - \sigma(\sigma+1)} \psi^{-\sigma+1}, \] (AE.4)

and
\[ \hat{u}_1 \hat{S}^- \psi^\sigma = \sqrt{S(S+1) - \sigma(\sigma-1)} \hat{u}_1 \psi^{\sigma-1} = \lambda_{\sigma-1} \sqrt{S(S+1) - \sigma(\sigma-1)} \psi^{-\sigma+1}. \] (AE.5)

Since these two must be identical, we find $\lambda_\sigma = \lambda_{\sigma-1}$, and hence
\[ \hat{u}_1 = \lambda \hat{R}, \] (AE.6)

for some $\lambda \in \mathbb{C}$, where
\[ \hat{R} = \sum_\sigma |\psi^{-\sigma}\rangle\langle \psi^\sigma|. \quad \text{(AE.7)} \]

We only need to determine \( \lambda \). Let \( |\xi^S\rangle \) be the unique state such that \( \hat{S}^{(1)}|\xi^S\rangle = S^2|\xi^S\rangle \), which obviously satisfies \( \hat{u}_1|\xi^S\rangle = e^{-i S^3} |\xi^S\rangle \). We claim that \( \hat{R}|\xi^S\rangle = |\xi^S\rangle \), which shows the desired result \( \lambda = e^{-i S^3} = (-i)^{\Delta S} \).

To see the claim we first note that \( \hat{R}|\xi^S\rangle = \pm |\xi^S\rangle \) because \([\hat{R}, \hat{u}_1] = 0\) and \(\hat{R}^2 = \hat{1} \). Note that \( |\xi^S\rangle \) is obtained by maximizing \( \langle \xi | \hat{S}^{(1)} | \xi \rangle = \langle \xi | (\hat{S}^+ + \hat{S}^-) | \xi \rangle / 2 \) over all normalized \( |\xi\rangle = \sum_\sigma c_\sigma |\psi^\sigma\rangle \). It is obvious that the maximum is attained when \( c_\sigma \geq 0 \) for all \( \sigma \), and hence it is only possible that \( \hat{R}|\xi^S\rangle = |\xi^S\rangle \). (This is essentially the same as using the Perron-Frobenius theorem.)

**Addendum to Section 8.3.5**

In footnote 50 (page 278), I mentioned a generalization of Theorem 8.6 that is parallel to Theorem 8.7. Recently we found a new proof of such a generalized theorem. Interestingly the proof makes an essential use of the index introduced in the study of symmetry protected topological phase.

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General Lieb-Schultz-Mattis type theorems for quantum spin chains

**Addendum to Section 11.3.2**

Theorem 11.13, which is based on [34], is not correct as it is. When \( (\tilde{A}, \tilde{\mathcal{B}}) \) is not bipartite, the condition of biconnectedness does not guarantee the existence of ferromagnetism. See footnote in page 4338 of [35].
Errata

- p. vii, 6th line from the bottom:
  detal → detail
- p. 30, 3rd line below (2.3.28):
  identiccal → identical
- p. 37, 4th line:
  in such way → in such a way
- p. 40, 3rd and 4th lines in the Proof of Theorem 2.2:
  “that the whole spectrum of $\hat{H}$ is contained in the subspace $\mathcal{H}_0$” should better be “that, for any eigenvalue of $\hat{H}$, there is at least one corresponding eigenstate in $\mathcal{H}_0$.”
- p. 54, below (3.2.9):
  $\sum_{x \in L} \langle \sigma_0, \sigma_x \rangle_{\beta, \infty}$ should be $\sum_{x \in \mathbb{Z}^d} \langle \sigma_0, \sigma_x \rangle_{\beta, \infty}$, i.e., change $L$ to $\mathbb{Z}^d$.
- p. 212, 8th line:
  proposal → proposed
- p. 212, 15th line:
  the the → the
- p. 437, 2 lines above (11.4.70):
  The right-hand sides of the inequality should be modified. (I will work out the detail soon.)
- p. 437, (11.4.74):
  The right-hand side of the inequality should be modified. (I will work out the detail soon.)
- p. 497, (S.19), 1st and 2nd lines:
  $e^{-iM\phi/2}$ → $e^{-i\phi}$