

# 相対論的流体揺らぎ とその応用

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## arXiv:

1602.00794  
1601.02260  
1407.8357  
1304.3243

## 参考文献：

村瀬功一、東京大学大学院  
理学系研究科博士論文 (2015)  
平野哲文、「流体力学と方  
程式」数理科学623, 32 (2015)

# はじめに

力学  $\Leftrightarrow$  運動学  $\Leftrightarrow$  流体力学  $\Leftrightarrow$  熱力学

小  $\leftarrow$  ----- 時間・空間スケール、粗視化 -----  $\rightarrow$  大

揺らぎ流体  
熱揺らぎ  
入り  
流体

因果的流体  
散逸に  
緩和を含む  
流体

粘性流体  
局所平衡  
散逸有り  
流体

完全流体  
局所平衡  
散逸無し  
流体

相対論的な  
流体で

# 全体の流れ

- 相対論的流体力学関連の歴史的なこと
- 相対論的流体力学の定式化
- 因果的な流体揺らぎ
  - 揺動散逸関係
  - 1次元膨張系の例
  - 3次元揺らぎ流体のシミュレーション
- まとめ

# 相対論的流体力学関連の 歴史的なこと

# Eckartによる流体力学の相対論化

NOVEMBER 15, 1940

PHYSICAL REVIEW

VOLUME 58

## The Thermodynamics of Irreversible Processes

### III. Relativistic Theory of the Simple Fluid

CARL ECKART

*Ryerson Physical Laboratory, University of Chicago, Chicago, Illinois*

(Received September 26, 1940)

The considerations of the first paper of this series are modified so as to be consistent with the special theory of relativity. It is shown that the inertia of energy does not obviate the necessity for assuming the conservation of matter. *Matter* is to be interpreted as number of molecules, therefore, and not as inertia. Its velocity vector serves to define local proper-time axes, and the energy momentum tensor is resolved into proper-time and -space components. It is shown that the first law of thermodynamics is a scalar equation, and not the fourth component of the energy-momentum principle. Temperature and entropy also prove to be scalars. Simple relativistic generalizations of Fourier's law of heat conduction, and of the laws of viscosity are obtained from the requirements of the second law. The same considerations lead directly to the accepted relativistic form of Ohm's law.

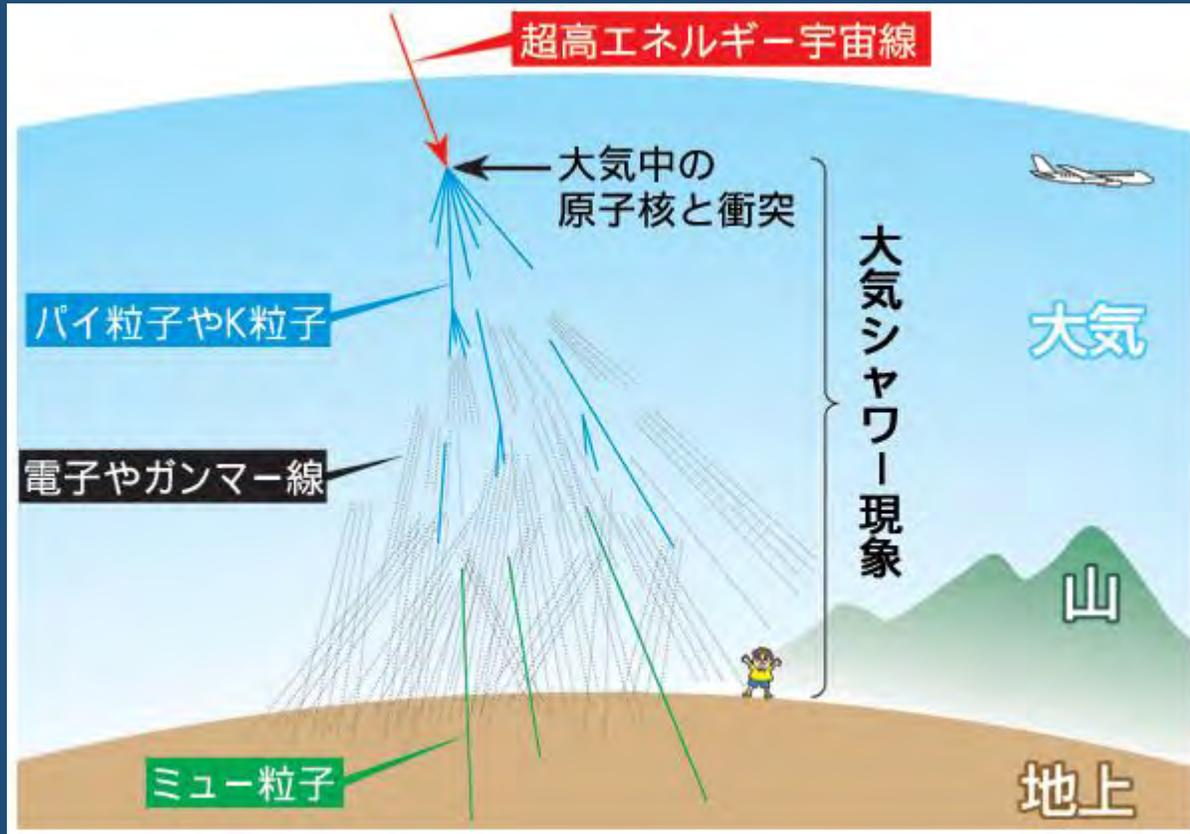
#### INTRODUCTION

IN the second paper of this series,<sup>1</sup> the theory of  $\epsilon$ -substitutions was outlined, and it was shown that this device can be used to simplify the derivation of some thermodynamic formulae. However, the author was reluctant to use it in the derivation of any fundamental formulae because the  $\epsilon$ -substitution depends on the fact

rather than inertia. The principles (b) and (c) combine into a single tensor equation, as is well known. This is somewhat disconcerting, for the first law of thermodynamics is a scalar equation; its relation to the energy-momentum principle must be discovered. Moreover, the correct form of the energy-momentum tensor is still a matter of discussion, and some assumption must be made before the theory can be com-

C. Eckart (1940)

# Landauによる多重発生への適用(1953)



<http://www2.kek.jp/ja/newskek/2008/mayjun/TAlinac.html>

## 火の玉の生成(Fermi)と膨張

### CHAPTER XV

## RELATIVISTIC FLUID DYNAMICS

### §125. The energy-momentum tensor

THE establishment of the relativistic equations of fluid motion is of fundamental importance. The necessity of allowing for relativistic effects may be due not only to a large velocity of the macroscopic motion (comparable with that of light), but also, as we shall see, to a large velocity of the microscopic motion of the fluid particles.

We must first of all determine the form of the energy-momentum 4-tensor  $T_{ik}$  for a fluid in motion.† The momentum flux through an element  $df$  of the surface of a body is just the force on that element. Hence  $T_{\alpha\beta} df^\beta$  is the  $\alpha$ -component of the force on a surface element.‡ Let us consider some element of volume in the fluid, and use a frame of reference in which this element is at rest (the “proper” frame). In such a frame Pascal’s law holds: the pressure exerted by a given portion of fluid is the same in all directions and perpendicular to the area on which it acts. We can therefore write  $T_{\alpha\beta} df^\beta = p df_\alpha$ , whence††  $T_{\alpha\beta} = p g_{\alpha\beta}$ .

The components  $T_{0\alpha}$  which give the momentum density are zero for a given volume element in its proper frame. The component  $T_{00}$  is the proper internal energy density of the fluid, which we shall denote in this chapter by  $e$ .

Thus the energy-momentum tensor for a given portion of fluid is, in the

# 磯、森、並木による場の理論的基礎付け

403

Progress of Theoretical Physics, Vol. 22, No. 3, September 1959

## Applicability Conditions of the Hydrodynamical Model of Multiple Production of Particles from the Point of View of Quantum Field Theory

Chikashi ISO,\* Kenju MORI\*\* and Mikio NAMIKI\*\*\*

\**Research Institute for Fundamental Physics, Kyoto University, Kyoto*

\*\**Department of Physics, Tokyo University of Education, Tokyo*

\*\*\**Department of Applied Physics, Waseda University, Tokyo*

(Received June 10, 1959)

In the framework of quantum field theory, it is attempted to investigate whether the hydrodynamical description is applicable to the meson cloud produced in extremely high energy collision of nucleons as considered in Landau's theory of the multiple production of particles. The applicability conditions of the hydrodynamical model consist of local equilibrium and conditions for the possibilities of defining the local system in the meson cloud, which are prepared by the methods based on quantum statistical mechanics of irreversible processes.

Iso, Mori, Namiki (1959)

See also Namiki, Iso (1957)

線形応答(1957)直後に相対論的場の理論に適用

# 最近（といっても10年前）の話題

asahi.com

社会 asahi.comトップ > 社会 > その他・話題

## 宇宙の始まりはしずく？ 「クォークは液体」と発表

2005年04月18日 23時34分

宇宙誕生の大爆発「ビッグバン」直後に相当する超高温・高密度の状態を再現する実験をしてきた日米などの国際チームは18日、物質を形づくる究極の基本粒子クォークは超高温でバラバラになるが、気体のように自由に跳び回るのでなく、しずくのような液体状態にあったと考えられる、と発表した。理論的に予想外の発見で、宇宙や物質のなりたちを説明するシナリオに影響を与える可能性がある。

The Washington Post

## Universe May Have Begun as Liquid, Not Gas

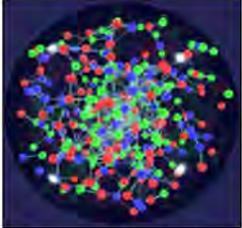
Associated Press  
Tuesday, April 19, 2005; Page A05

New results from a particle collider suggest that the universe behaved like a liquid in its earliest moments, not the fiery gas that was thought to have pervaded the first microseconds of existence.

BBC NEWS

## Early Universe was 'liquid-like'

Physicists say they have created a new state of hot, dense matter by crashing together the nuclei of gold atoms.



The high-energy collisions prised open the nuclei to reveal their most basic particles, known as quarks and gluons.

The researchers, at the US Brookhaven National Laboratory, say these particles were seen to behave as an almost perfect "liquid".

The impression is of matter that is more strongly interacting than predicted.

The work is expected to help scientists explain the conditions that existed just milliseconds after the Big Bang.

素粒子極限物質 (>~2兆度)  
クォーク・グルーオン・プラズマ  
の完全流体のような振る舞い  
← 相対論的流体力学が重要な役割



日本評論社(2015)

# ギネス世界記録

Browser: Highest man-made x  
Address: www.guinnessworldrecords.com/world-records/highest-man-made-temperature

OFFICIALLY  AMAZING SEARCH 

EXPLORE RECORDS SET A RECORD

## Highest man-made temperature

Share   

			Who
			What
			Where
			SWITZERLAND
			When
			13 AUGUST 2012

On 13 August 2012 scientists at CERN's Large Hadron Collider, Geneva, Switzerland, announced that they had achieved temperatures of over 5 trillion K and perhaps as high as 5.5 trillion K. The team had been using the ALICE experiment to smash together lead ions at 99% of the speed of light to create a quark gluon plasma – an exotic state of matter believed to have filled the universe just after the Big Bang.



# 相対論的流体力学の定式化

平野哲文, 「流体力学と方程式」

数理科学 623, 32 (2015)

# エネルギー・運動量保存則

$$\partial_{\mu} T^{\mu\nu} = 0$$

エネルギー・運動量テンソル

以下では

$$\partial_{\mu} = \frac{\partial}{\partial x^{\mu}}$$

ギリシャ文字の添え字は時空間

( $\mu = 0, 1, 2, 3 = t, x, y, z$ )

$c = \hbar = k_B = 1$ の単位系

4本の連続の方程式

→ 4つの保存量

$\nu = 0$ : エネルギー保存則

$\nu = 1, 2, 3$ : 運動量保存則

# エネルギー・運動量テンソル

$$T^{\mu\nu} = e u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

エネルギー密度

4元流速

静水圧

体積粘性

応力テンソル

静止系(Local Rest Frame)

$$u_{\text{LRF}}^\mu = (1, 0, 0, 0)$$

$$T_{\text{LRF}}^{\mu\nu} = \begin{pmatrix} e & 0 & 0 & 0 \\ 0 & p + \Pi + \pi^{11} & \pi^{12} & \pi^{13} \\ 0 & \pi^{21} & p + \Pi + \pi^{22} & \pi^{23} \\ 0 & \pi^{31} & \pi^{32} & p + \Pi + \pi^{33} \end{pmatrix}$$

# 慣性系取り方

何が流れているのか？

$$T_{\text{LRF}}^{\mu\nu} = \begin{pmatrix} e & 0 & 0 & 0 \\ 0 & p + \Pi + \pi^{11} & \pi^{12} & \pi^{13} \\ 0 & \pi^{21} & p + \Pi + \pi^{22} & \pi^{23} \\ 0 & \pi^{31} & \pi^{32} & p + \Pi + \pi^{33} \end{pmatrix}$$

- エネルギー・運動量テンソルの  $0i$  成分
  - 流体の場合：熱流
  - 電磁場の場合：Poyntingベクトル

何故これらは0か？

# 慣性系の取り方（つづき）

$$T^{\mu}_{\nu} u^{\nu} = e u^{\nu}$$

- エネルギー流に乗った慣性系で流速を定義（Landau系）
  - エネルギー・運動量テンソルの固有値問題
  - 相対論では質量も熱もエネルギーの一形態
  - 質量は非保存 ex.)  $e^+ + e^- \leftrightarrow 2\gamma, q + \bar{q} \leftrightarrow 2g$
  - 粒子数も非保存

## 慣性系の取り方（つづき）

$$\partial_{\mu} N^{\mu} = 0, \quad u^{\mu} = \frac{N_{\mu} N^{\mu}}{\sqrt{N_{\mu} N^{\mu}}}$$

- 保存流に乗った慣性系でも流速定義可能（Eckart系）
  - $T^{0i}$ は有限（熱流は定義可）
  - しかしクォーク・グルーオン・プラズマの場合  
粒子数 $\approx$ 反粒子数 →  $N_{\mu} N^{\mu} \sim 0$

# 構成方程式

※通常は熱力学第2法則から決定

線形応答

$$\Gamma(x) = \int_{t > t'} d^4 x' G(x, x') F(x')$$

散逸流

記憶関数

熱学的な力

	散逸流 $\Gamma$	熱学的な力 $F$
ずり粘性	$\pi^{\mu\nu}$	$\nabla \langle \mu u^\nu \rangle$
体積粘性	$\Pi$	$-\nabla_\mu u^\mu$

$\nabla^\mu (= \Delta^{\mu\nu} \partial_\nu)$ : ラグランジュ空間微分 (エネルギーの慣性系)

# 記憶関数 1

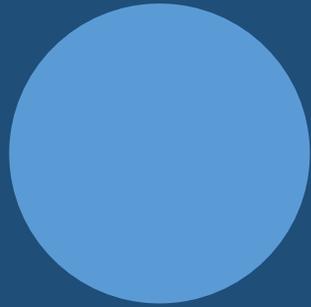
例 1) 瞬間的応答

$$G(x, x') = \kappa \delta(t - t') \delta^{(3)}(\mathbf{x} - \mathbf{x}')$$

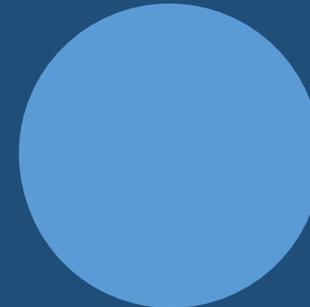
$$\kappa = \begin{cases} \eta & \text{ずり粘性係数} \\ \zeta & \text{体積粘性係数} \end{cases} \quad \longrightarrow \quad \begin{aligned} \pi^{\mu\nu} &= 2\eta \nabla^{\langle\mu} u^{\nu\rangle} \\ \Pi &= -\zeta \nabla_{\mu} u^{\mu} \end{aligned}$$

相对論的Navier-Stokes方程式

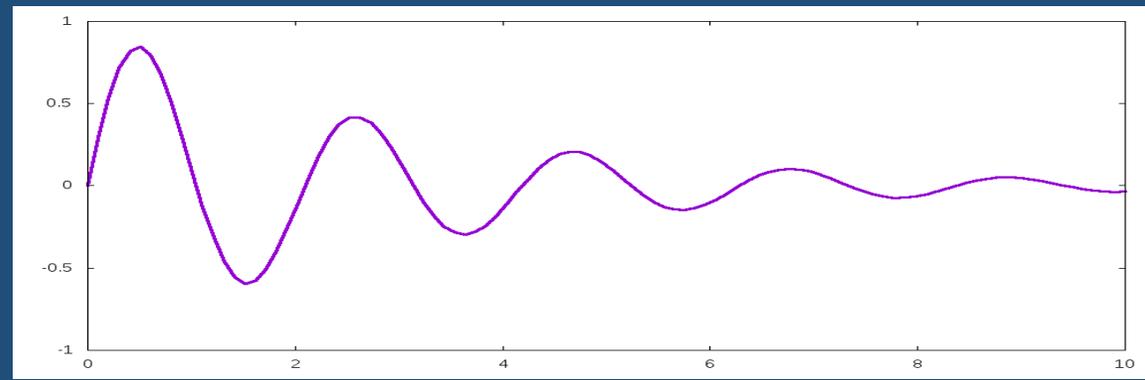
# 減衰モード



体積粘性  
→体積変化の妨げ



ずり粘性  
→変形の妨げ



# 因果律の破れ

相対論的Navier-Stokes方程式

- 拡散型方程式（放物型方程式）
- 時間 1 階微分、空間 2 階微分
- 情報伝達速度無限大
- 因果律の破れ、不安定解

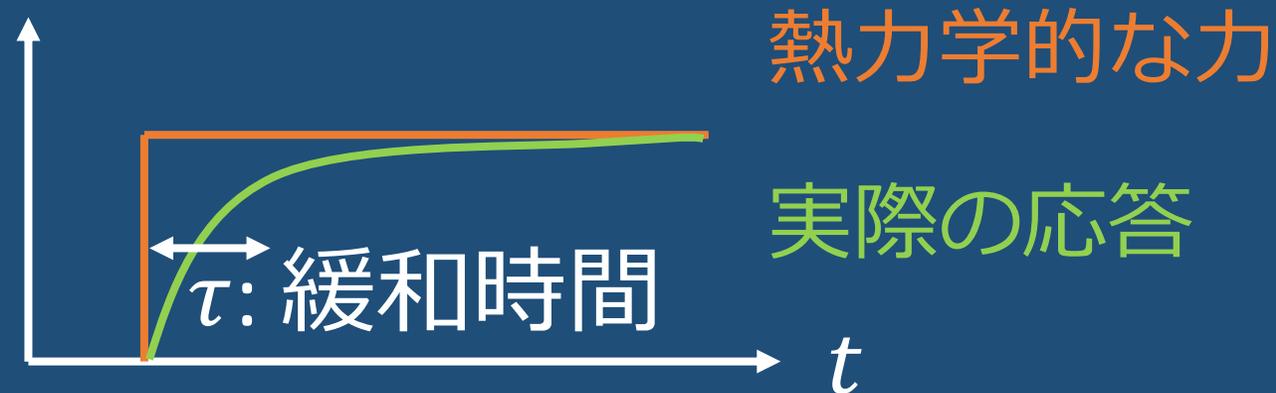
Hiscock-Lindblom (1985)

緩和過程、遅延効果の必要性

# 記憶関数 2

例 2) 遅延応答の一例

$$G(t - t') = \frac{\kappa}{\tau} \exp\left(-\frac{t - t'}{\tau}\right)$$



# 緩和過程を含む構成方程式

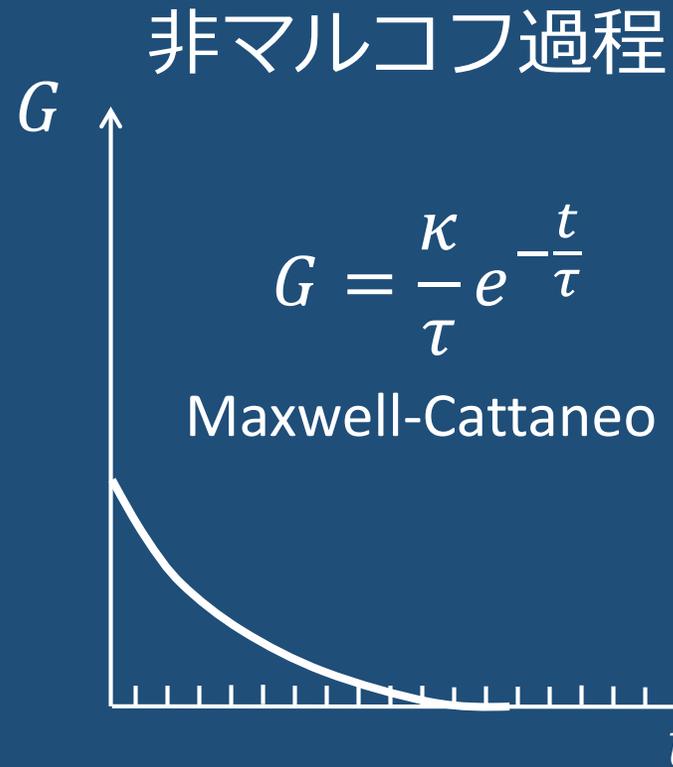
$$\text{積分形} \left\{ \begin{array}{l} \pi^{\mu\nu} = \int d^4x' G_\eta(x-x')^{\mu\nu}_{\alpha\beta} \nabla^{\langle\alpha} u^{\beta\rangle}(x') \\ \Pi = -\int d^4x' G_\zeta(x-x') \nabla_\mu u^\mu(x') \end{array} \right.$$

$$\text{微分形} \left\{ \begin{array}{l} (1 + \tau_\eta D) \pi^{\langle\mu\nu\rangle} = 2\eta \nabla^{\langle\mu} u^{\nu\rangle} \\ (1 + \tau_\zeta D) \Pi = -\zeta \nabla_\mu u^\mu \end{array} \right.$$

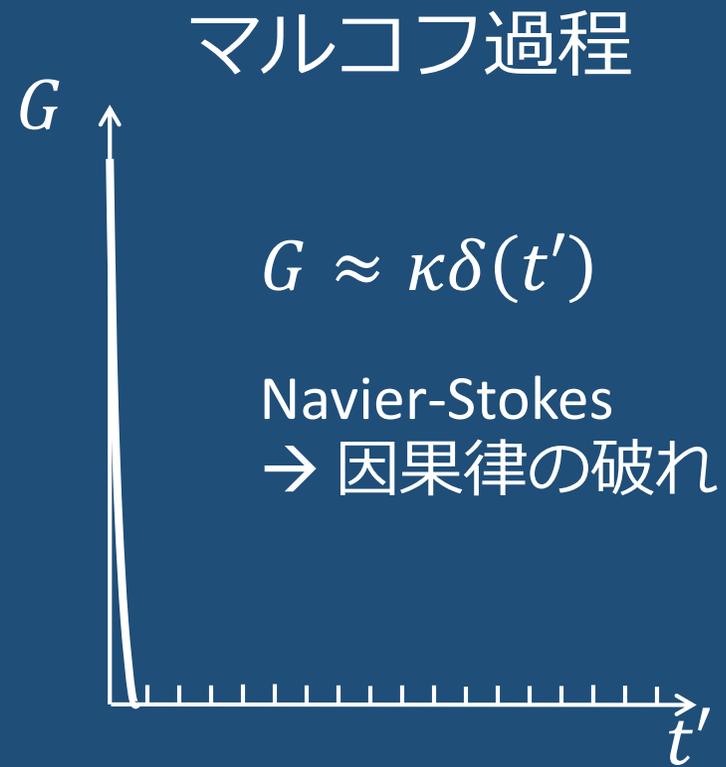
$D(= u_\mu \partial^\mu)$ : ラグランジュ時間微分

散逸流が独立な自由度  $\rightarrow$  拡張された不可逆過程の熱力学

# 時間の粗視化



時間の粗視化



相対論的理論における粗視化の制限（限界！？）

# 因果的な流体揺らぎ

K. Murase, T. Hirano, “Relativistic fluctuating hydrodynamics with memory functions and colored noises”, arXiv:1304.3243

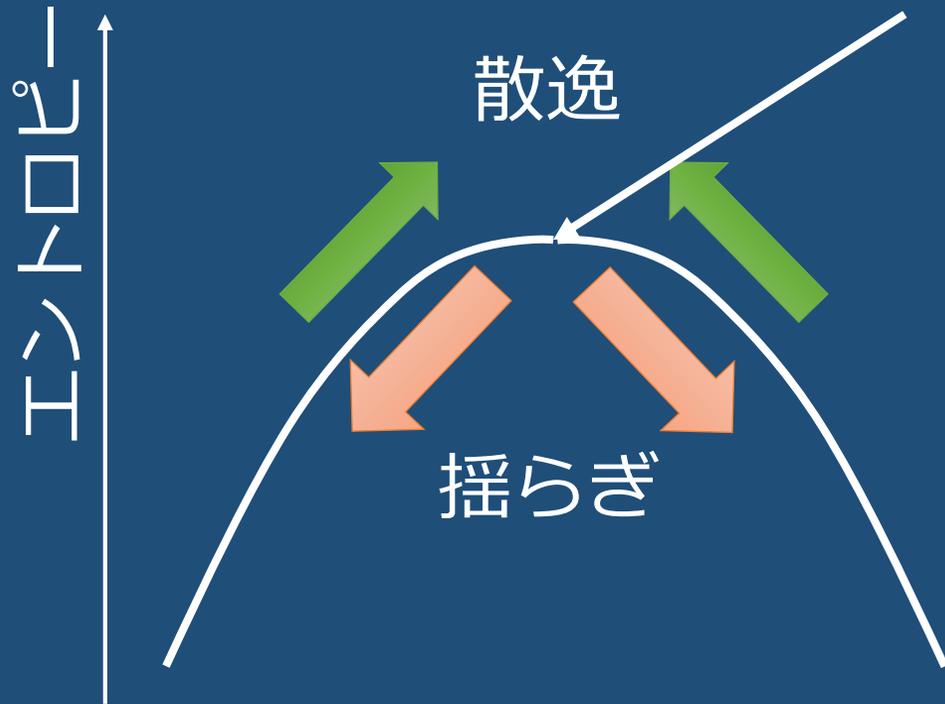
# 構成方程式のLangevin化

$$\Gamma(x) = \int d^4x' G(x, x') F(x') \\ + \delta\Gamma(x) : \text{流体揺らぎ}$$

揺動散逸関係

$$\langle \delta\Gamma(x) \delta\Gamma(x') \rangle \\ = T [G(x, x') + G(x', x)] \\ \langle \delta\Gamma(x) \rangle = 0$$

# 揺動散逸関係



マクロな視点  
熱平衡状態 = 最大エントロピー状態

ミクロな視点  
揺らぎと散逸のせめぎ合い  
による熱平衡状態の安定性

# フーリエ空間

緩和過程の場合

$$G(\boldsymbol{x} - \boldsymbol{x}') = \frac{\kappa}{\tau} \exp\left(-\frac{t - t'}{\tau}\right) \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}')$$



$$\langle \delta\Gamma_{\omega, \boldsymbol{k}}^* \delta\Gamma_{\omega', \boldsymbol{k}'} \rangle = \frac{\kappa T}{1 + \omega^2 \tau^2} (2\pi)^4 \delta(\omega - \omega') \delta^{(3)}(\boldsymbol{k} - \boldsymbol{k}')$$

パワースペクトルの振動数依存性 → 色付き雑音  
← 相対論特有の性質！？

# 微分形の構成方程式

記憶関数は数値計算に不向き→微分方程式に落とし込み

$$\left\{ \begin{array}{l} \mathcal{L}\Gamma(x) = \mathcal{L} \int d^4x' G(x, x') F(x') + \mathcal{L}\delta\Gamma(x) \\ \mathcal{L} = 1 + \tau D \end{array} \right.$$

※より一般の線形演算子と応答関数の場合→村瀬D論


$$\left\{ \begin{array}{l} (1 + \tau D)\Gamma = \kappa F + \xi \\ \xi = \mathcal{L}\delta\Gamma \end{array} \right.$$

揺動散逸関係は？

# 微分形における揺動散逸関係

$$\langle \delta\Gamma(x)\delta\Gamma(x') \rangle = T[G(x, x') + G(x', x)]$$



$$\begin{aligned} \langle \xi(x)\xi(x') \rangle &= \mathcal{L}_x \mathcal{L}_{x'} \langle \delta\Gamma(x)\delta\Gamma(x') \rangle \\ &= 2[\kappa T + \tau D(\kappa T)] \delta^{(4)}(x - x') \end{aligned}$$

- 様な背景場の場合落とせる項
- 非線形流体の場合？
- 揺らぎの定理では必要！？

# 1次元膨張系の例

# 1次元ハッブル膨張モデル J.D.Bjorken (1983)

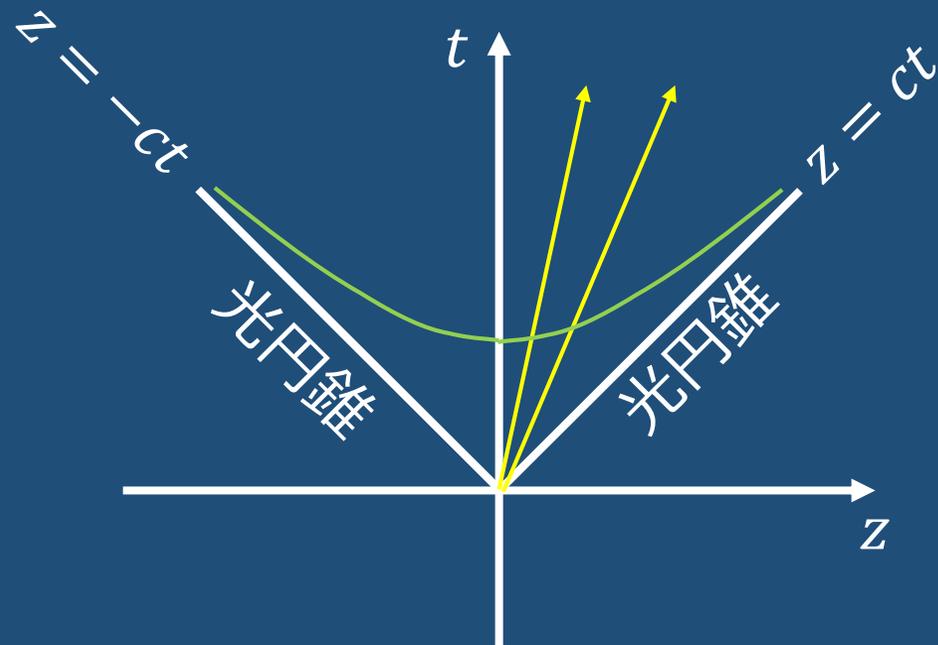
## 4元流速のAnsatz

$$\begin{aligned} u^\mu &= \gamma(1, v_x, v_y, v_z) \\ &= \frac{t}{\tau} \left( 1, 0, 0, \frac{z}{t} \right) \end{aligned}$$

変数変換  $(t, z) \rightarrow (\tau, \eta_s)$

$$\begin{cases} \tau = \sqrt{t^2 - z^2} \\ \eta_s = (1/2) \ln(t + z)/(t - z) \end{cases}$$

(3+1)次元問題  $\rightarrow$  (0+1)次元問題



# バランス方程式

エネルギー保存則

$$\frac{de}{d\tau} = -\frac{e+p}{\tau} \left( 1 - \frac{\pi}{sT} + \frac{\Pi}{sT} \right)$$

$$\pi = \pi^{tt} - \pi^{zz}$$

エントロピー密度

温度

# 構成方程式 1

## 応力テンソル

$$\tau_\eta \frac{d\pi}{d\tau} + \pi = \frac{4\eta}{3\tau} + \xi_\pi$$

$$\langle \xi_\pi(x) \xi_\pi(x') \rangle = \frac{8\eta T}{3} \delta^{(4)}(x - x')$$

※しばらく揺動散逸関係に現れた  $\tau D(\kappa T)$  の項を無視

# 構成方程式 2

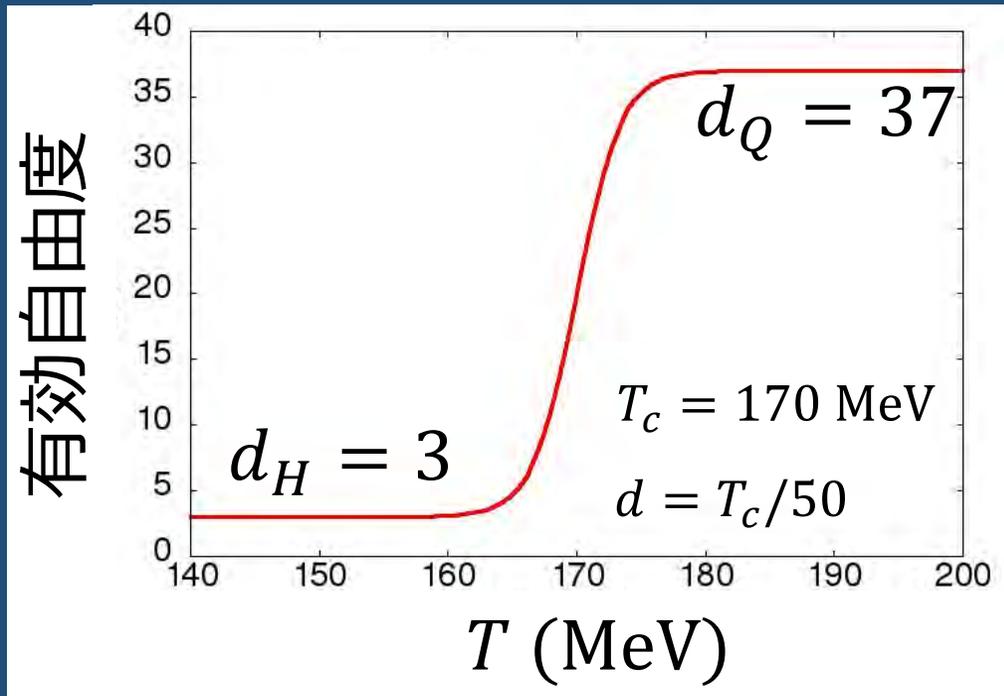
体積粘性圧力

$$\tau_\zeta \frac{d\Pi}{d\tau} + \Pi = -\frac{\zeta}{\tau} + \xi_\Pi$$

$$\langle \xi_\Pi(x) \xi_\Pi(x') \rangle = 2\zeta T \delta^{(4)}(x - x')$$

構成方程式のみが確率微分方程式

# 状態方程式のモデル



$$s(T) = d_{\text{eff}} \frac{4\pi^2}{90} T^4$$

$$p(T) = \int_0^T s(T') dT'$$

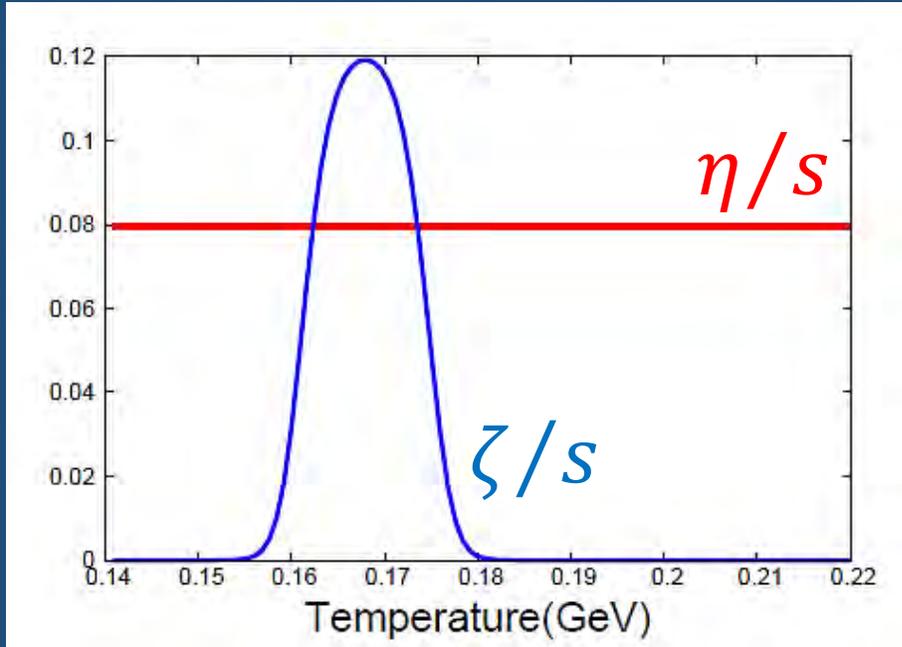
$$e(T) = Ts - p$$

ハドロン  
ガス

クロスオーバー

クォーク  
グルーオン  
プラズマ

# 輸送係数のモデル



ずり粘性係数:

$$\eta/s = 1/4\pi$$

P.Kovtun, D.T.Son, A.O.Starinets (2005)

体積粘性係数:

$$\zeta/s \propto 1/3 - c_s^2$$

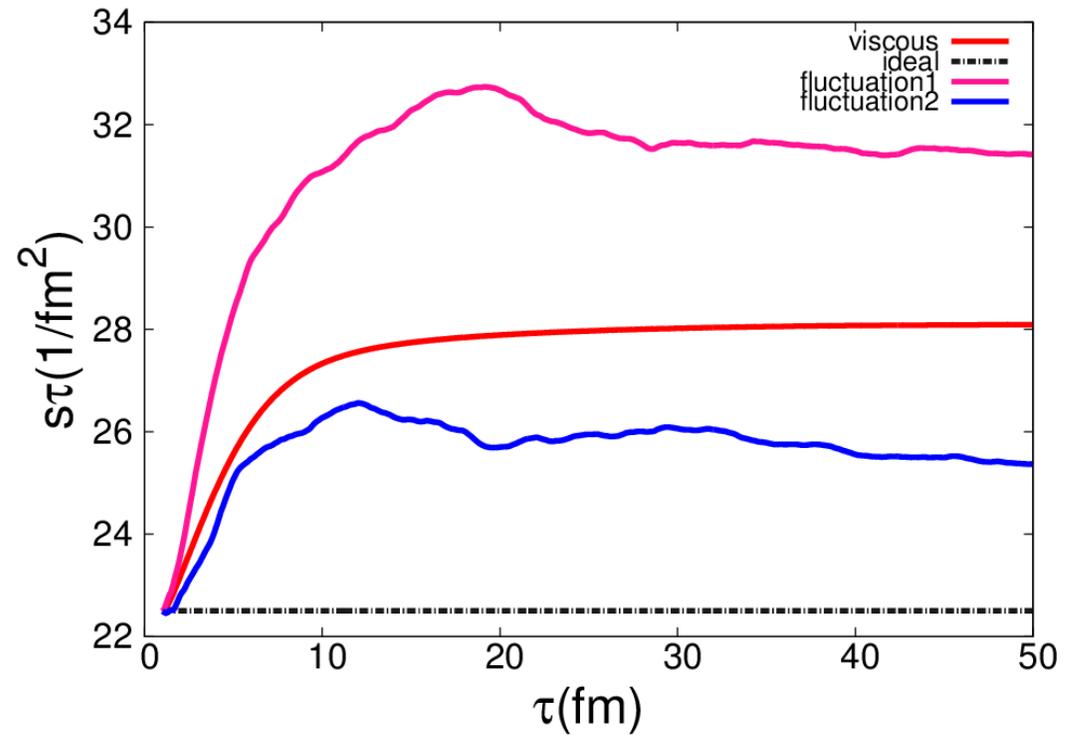
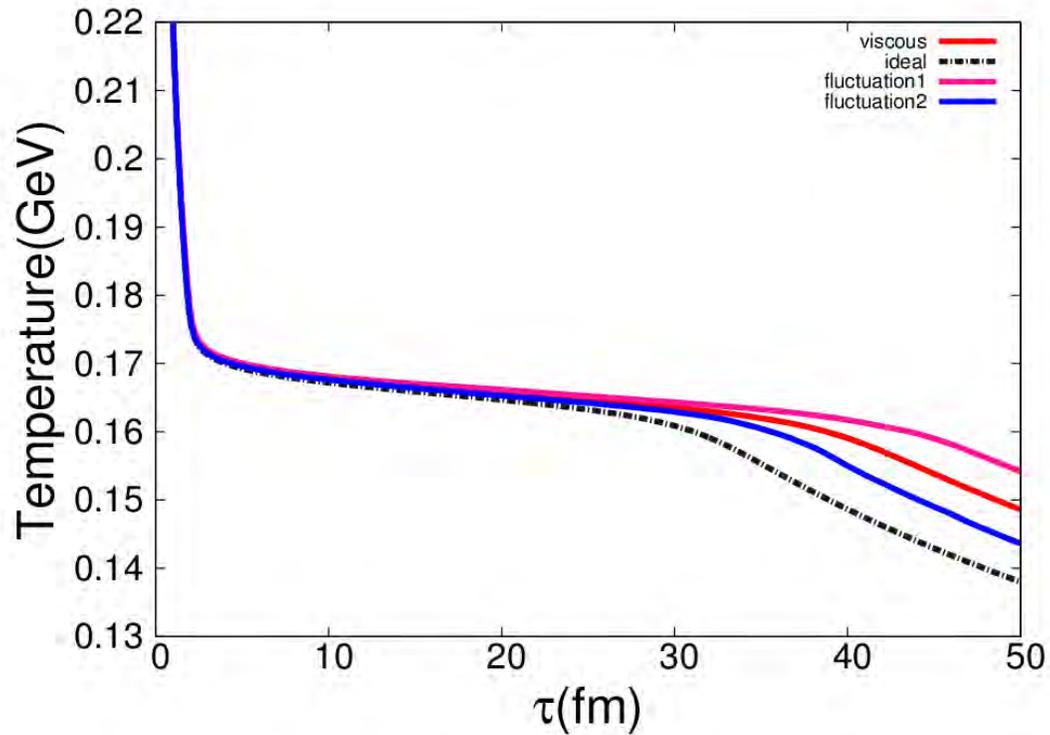
S.Weinberg (1971)

緩和時間

$$\tau_\eta = \tau_\zeta = 3\eta/2p$$

W.Israel (1976)

# 温度とエントロピーの時間発展の例

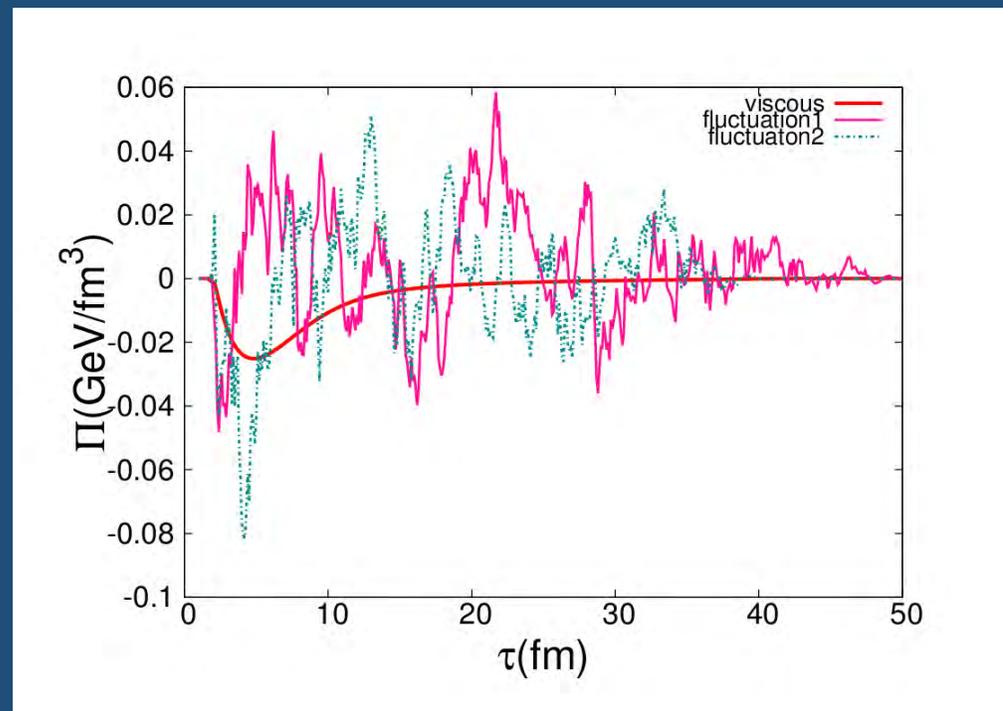
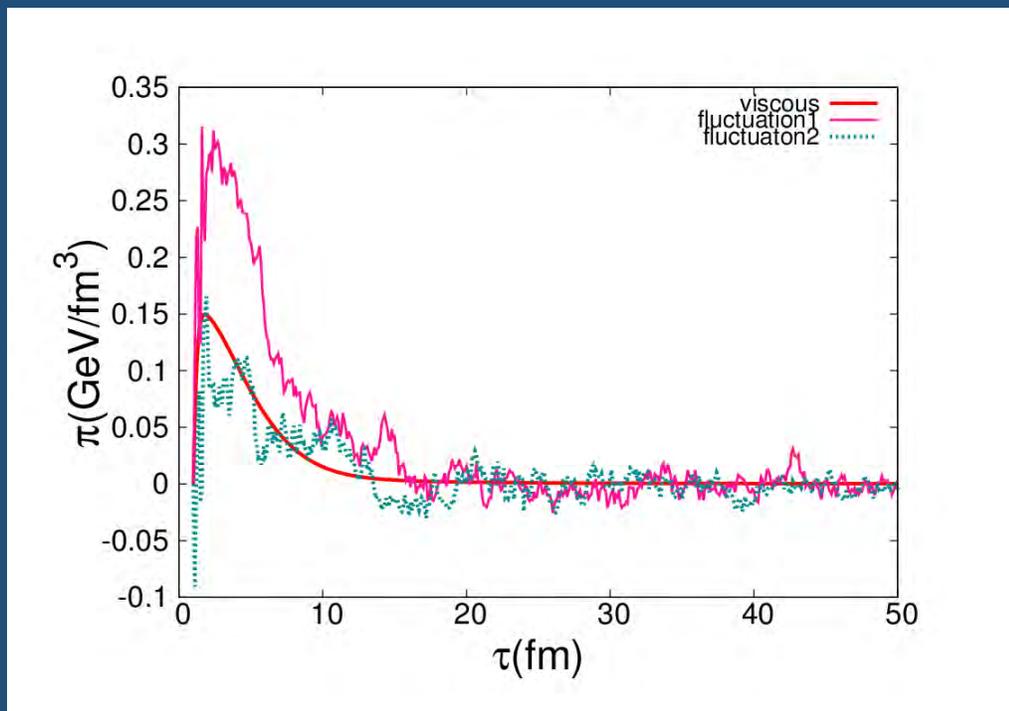


完全流体  $\rightarrow$  エントロピー保存、粘性流体  $\rightarrow$  単調増加  
揺らぎ流体  $\rightarrow$  エントロピーの揺らぎ

# エントロピー生成率

単位時間あたり

$$\sigma = \frac{dS}{d\tau} = \frac{\pi - \Pi}{T} dx dy d\eta_s$$



流体揺らぎ → 正負のエントロピー生成率

# 1次元膨張系における 揺らぎの定理

# “定常的”揺らぎの定理

$$\frac{P(\bar{\sigma} = \alpha)}{P(\bar{\sigma} = -\alpha)} = e^{\alpha t}$$

エントロピー生成率の確率分布

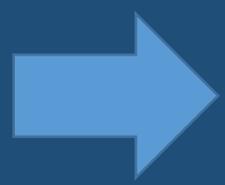
ガウス分布の場合

$$P(\bar{\sigma})d\bar{\sigma} = \frac{1}{\sqrt{2\pi a^2}} \exp\left[-\frac{(\bar{\sigma} - \langle\bar{\sigma}\rangle)^2}{2a^2}\right] d\bar{\sigma} \quad \longrightarrow \quad \frac{2\langle\bar{\sigma}\rangle}{a^2} = t$$

# 積分形

$$\langle \bar{\sigma} \rangle \approx \frac{dx dy d\eta_s}{\tau - \tau_i} \int_{\tau_i}^{\tau} \frac{d\tau_1}{T_0(\tau_1)} \int_{\tau_i}^{\tau_1} d\tau_2 G(\tau_1, \tau_2) \left( \frac{4\eta}{3\tau_2} + \frac{\zeta}{\tau_2} \right)$$

$$\langle \bar{\sigma}^2 \rangle - \langle \bar{\sigma} \rangle^2 \approx \frac{2dx dy d\eta_s}{(\tau - \tau_i)^2} \int_{\tau_i}^{\tau} \frac{d\tau_1}{T_0(\tau_1)} \int_{\tau_i}^{\tau_1} d\tau_2 G(\tau_1, \tau_2) \left( \frac{4\eta}{3\tau_2} + \frac{\zeta}{\tau_2} \right)$$


$$\frac{2\langle \bar{\sigma} \rangle}{\langle (\bar{\sigma} - \langle \bar{\sigma} \rangle)^2 \rangle} = \tau - \tau_i \quad \text{が成立}$$

※背景温度場の揺らぎは高次

# 微分形

$$\langle \delta\Gamma(x)\delta\Gamma(x') \rangle = T[G(x, x') + G(x', x)]$$



$$\begin{aligned}\langle \xi(x)\xi(x') \rangle &= \mathcal{L}_x \mathcal{L}_{x'} \langle \delta\Gamma(x)\delta\Gamma(x') \rangle \\ &= 2[\kappa T + \tau D(\kappa T)] \delta^{(4)}(x - x')\end{aligned}$$

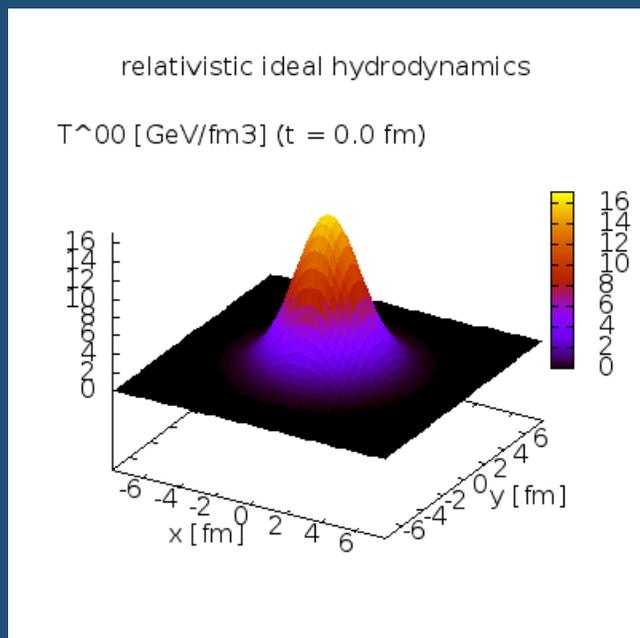
数值計算 → Work in progress

# 3次元揺らぎ流体の シミュレーション

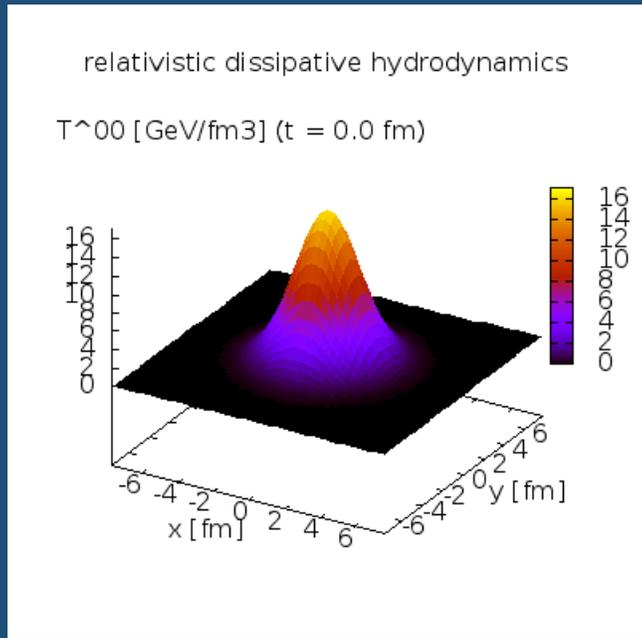
# エネルギー密度の時間発展

シミュレーション by 村瀬

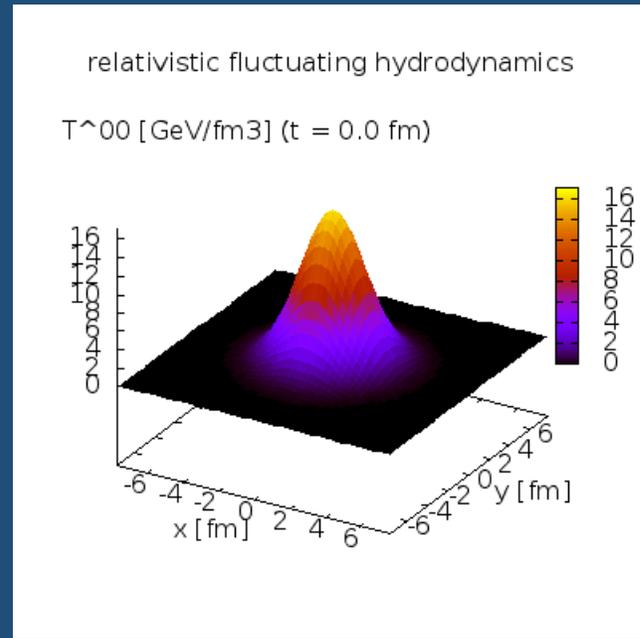
## 周期境界条件を課した2次元の箱



第1世代  
完全流体



第2世代  
粘性流体



第3世代  
揺らぎ流体

※ $\tau D(\kappa T)$ の項は無視

# まとめ

- 相対論的流体力学と因果律を守る流体揺らぎの導入
  - 緩和時間が重要な役割 → 色付き雑音の現れ
  - 1次元膨張系への適用
  - クォーク・グルーオン・プラズマの物理への適用
  - 揺らぎの定理周辺は今後の課題

# Green-Kubo Formula

$$\eta = \lim_{\omega \rightarrow 0} \lim_{q \rightarrow 0} \frac{1}{2\omega} \int dt dx e^{i(\omega t - qx)} \\ \times \langle [T_{xy}(t, x), T_{xy}(0, 0)] \rangle$$

Slow dynamics  $\rightarrow$  How slow?

Macroscopic time scale  $\sim 1/\omega \leftarrow t_{\text{macro}}$

Microscopic time scale  $\sim \tau$

cf.) Long tail problem (liquid in 2D, glassy system, super-cooling, etc. )