量子少数系における普遍性と（スーパー）エフィモフ効果

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第４回 統計物理学懇談会

2016年3月7-8日 @ 学習院大学

Plan of this talk

1. Universality in physics

2. What is the Efimov effect ?

   Keywords: universality
   scale invariance
   quantum anomaly
   RG limit cycle

3. Beyond cold atoms: Quantum magnets

4. New progress: Super Efimov effect

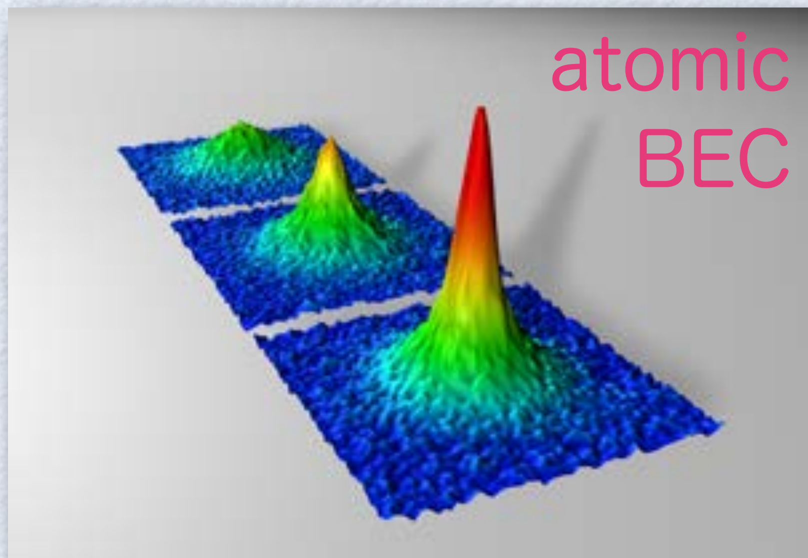
Introduction

1. **Universality in physics**
2. What is the Efimov effect?
3. Beyond cold atoms: Quantum magnets
4. New progress: Super Efimov effect

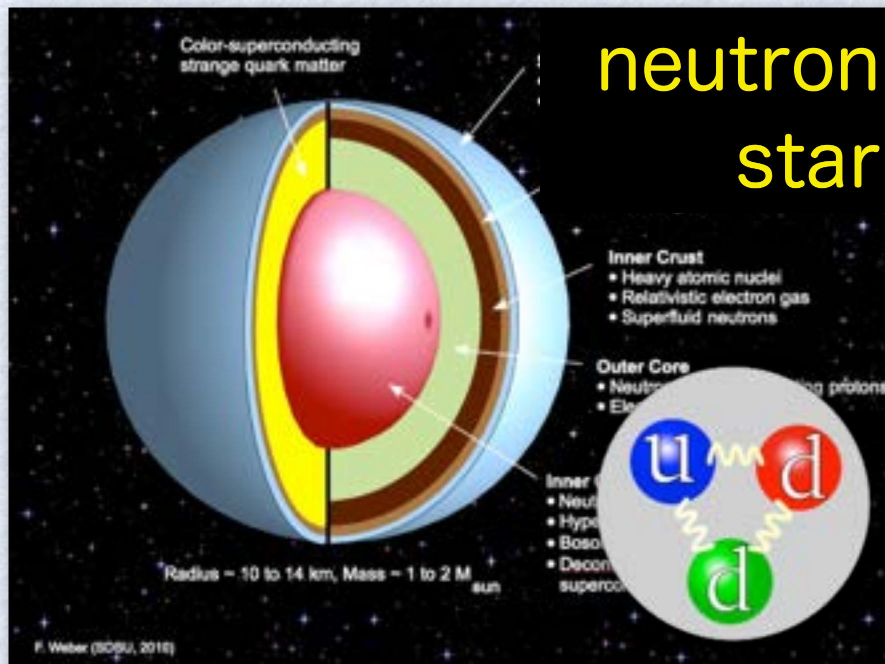
(ultimate) Goal of research

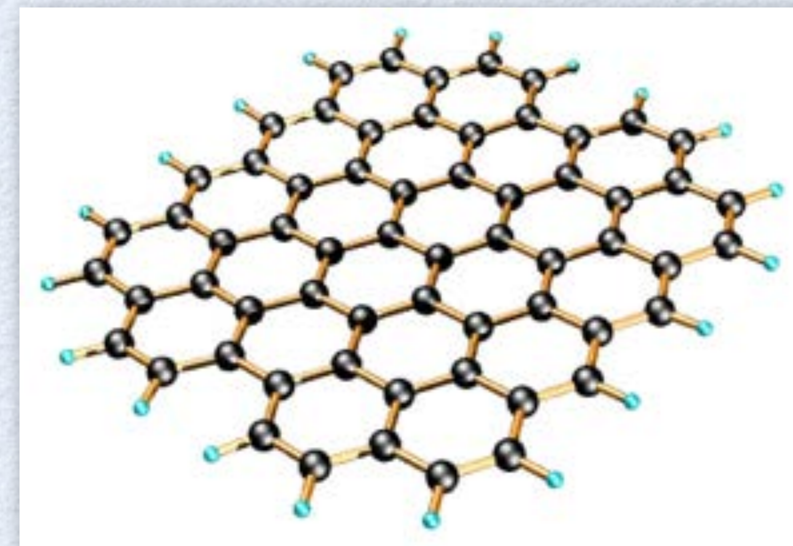
Understand physics of few and many particles governed by quantum mechanics

atomic BEC



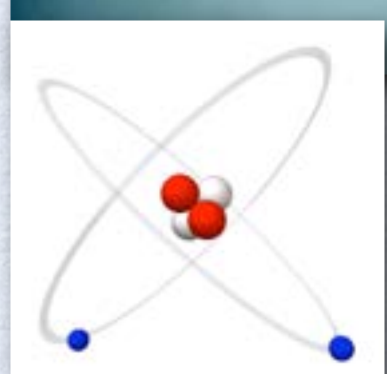
neutron star

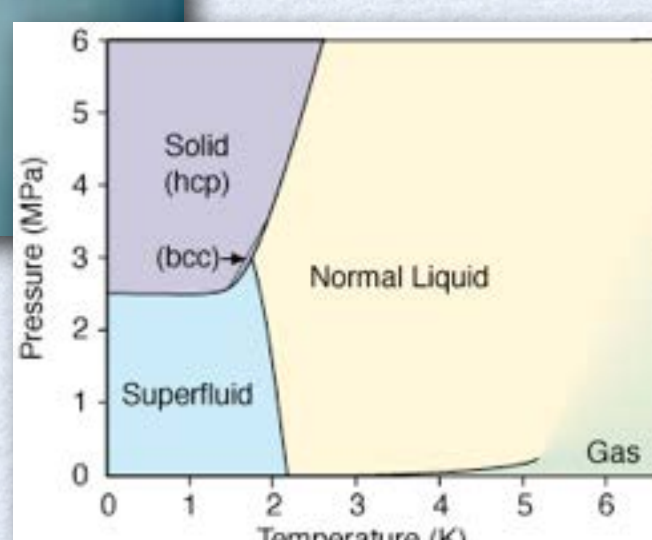




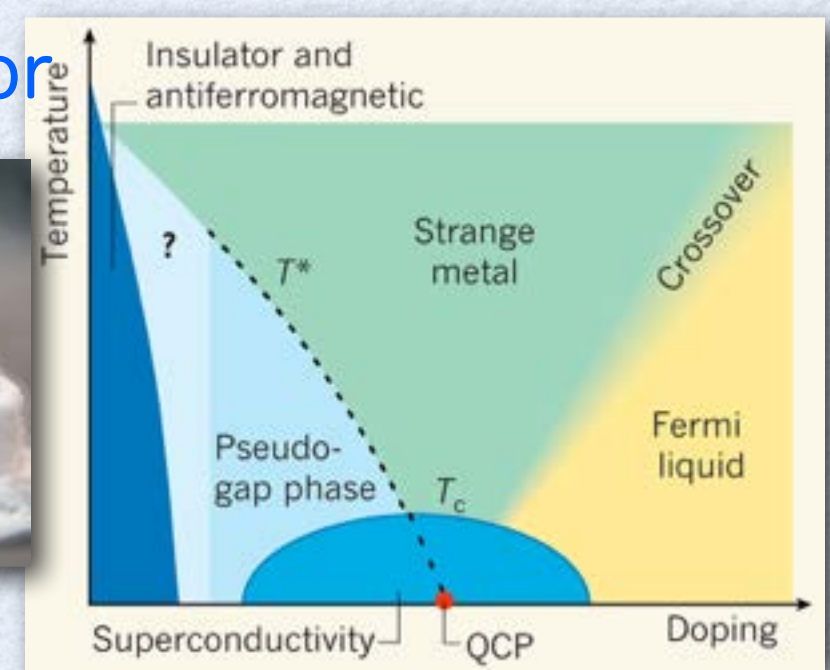
graphene

liquid helium





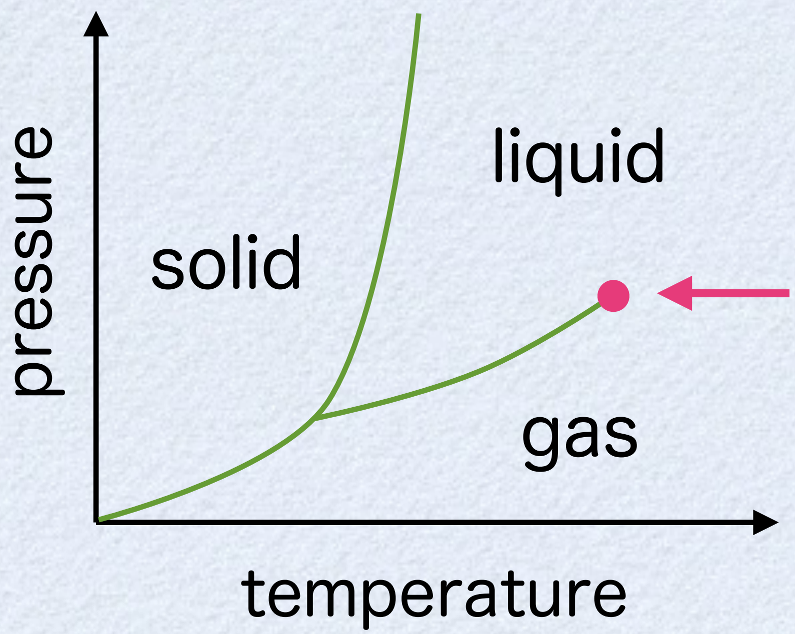
superconductor

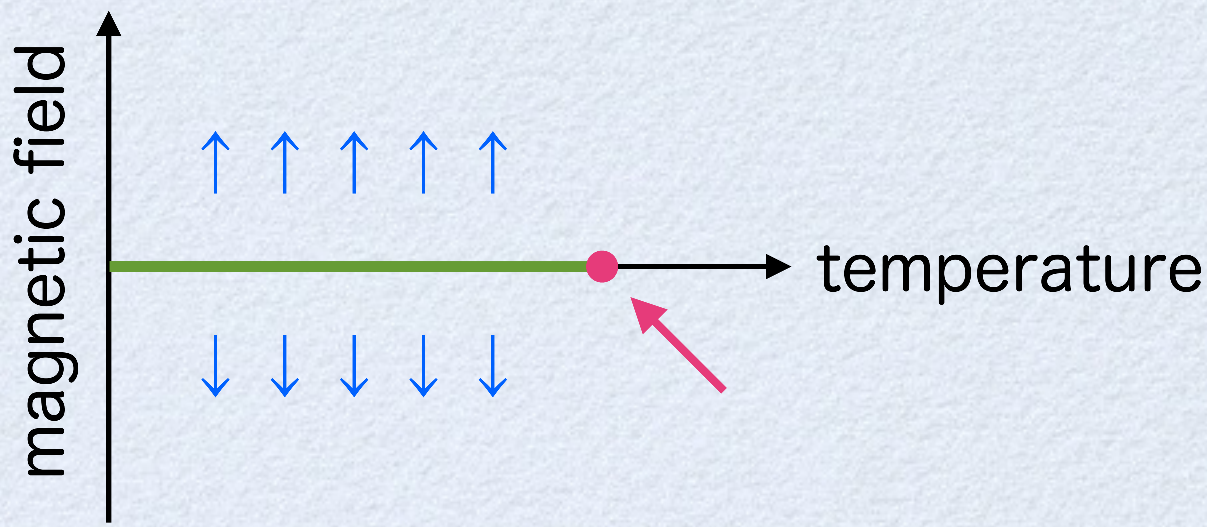


When physics is universal ?

A1. Continuous phase transitions $\Leftrightarrow$ $\xi / r_0 \to \infty$

E.g. Water  vs. Magnet 



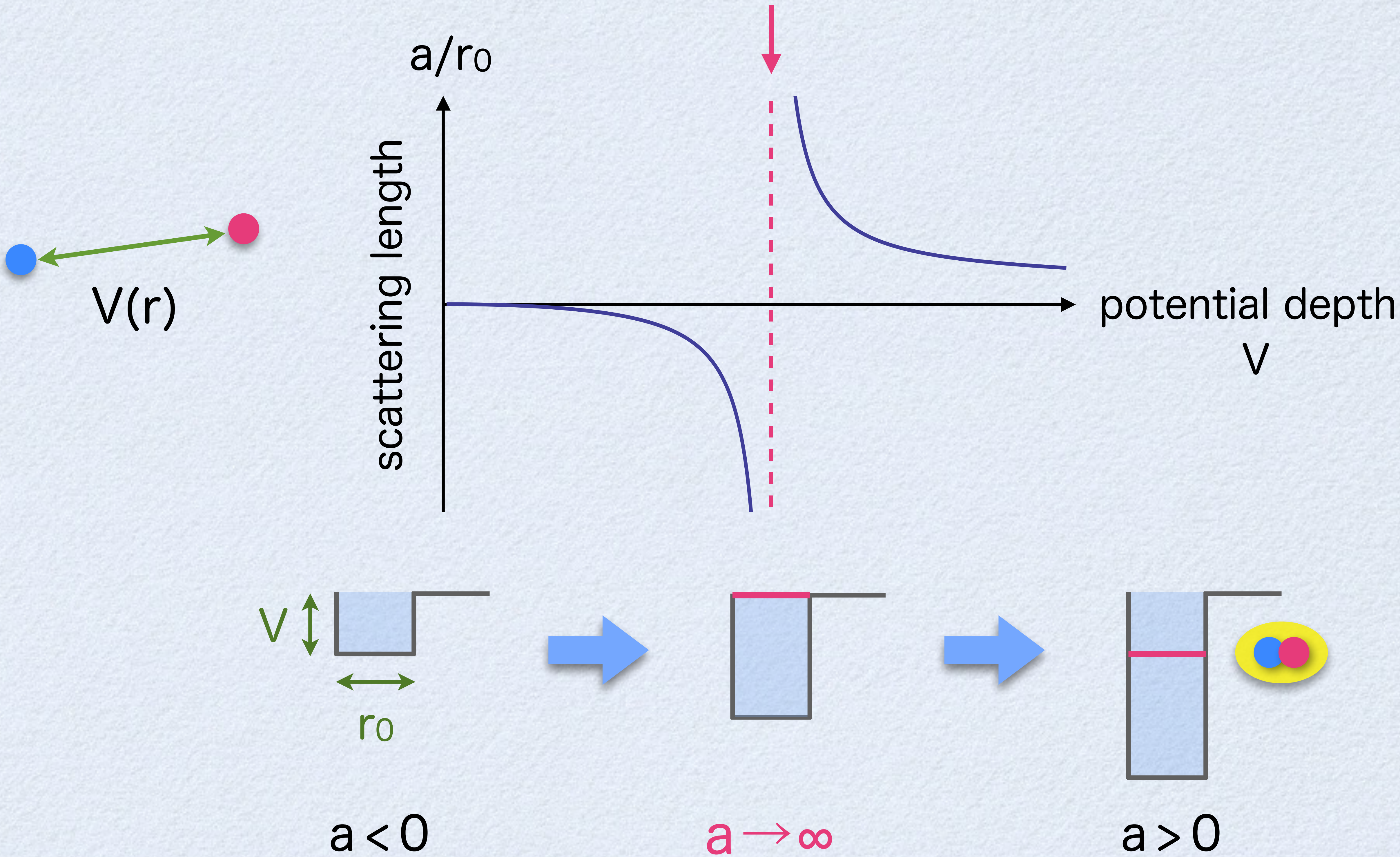


Water and magnet have the same exponent $\beta \approx 0.325$

$$\rho_{\rm liq} - \rho_{\rm gas} \sim (T_{\rm c} - T)^{\beta} \qquad M_{\uparrow} - M_{\downarrow} \sim (T_{\rm c} - T)^{\beta}$$

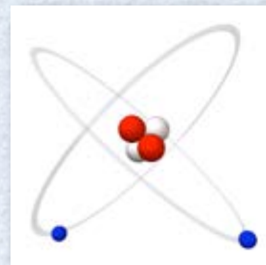
When physics is universal ?

A2. Scattering resonances $\Leftrightarrow$ $a/r_0 \to \infty$



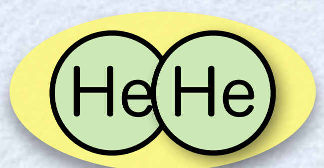
When physics is universal ?

A2. Scattering resonances ⇔ $a/r_0 \to \infty$

E.g. ^{4}He atoms  vs. proton/neutron 

van der Waals force:

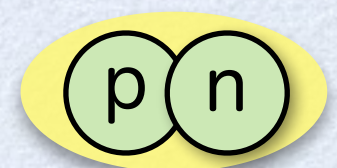
$a \approx 1\times10^{-8}$ m $\approx 20\ r_0$



$E_{binding} \approx 1.3\times10^{-3}$ K

nuclear force:

$a \approx 5\times10^{-15}$ m $\approx 4\ r_0$



$E_{binding} \approx 2.6\times10^{10}$ K

Atoms and nucleons have the same form of binding energy

$$E_{\text{binding}} \to -\frac{\hbar^2}{m\,a^2} \qquad (a/r_0 \to \infty)$$

➡ Physics only depends on the scattering length “a”

Efimov effect

1. Universality in physics
2. **What is the Efimov effect?**
3. Beyond cold atoms: Quantum magnets
4. New progress: Super Efimov effect

Efimov effect

Efimov (1970)

Volume 33B, number 8 PHYSICS LETTERS 21 Decembe

ENERGY LEVELS ARISING FROM RESONANT TWO-BODY FORCES IN A THREE-BODY SYSTEM

V. EFIMOV
A.F.Ioffe Physico-Technical Institute, Leningrad, USSR

Received 20 October 1970

Resonant two-body forces are shown to give rise to a series of levels in three-particle systems. The number of such levels may be very large. Possibility of the existence of such levels in systems of three α-particles (^{12}C nucleus) and three nucleons (^{3}H) is discussed.

The range of nucleon-nucleon forces r_0 is known to be considerably smaller than the scattering lengts a. This fact is a consequence of the resonant character of nucleon-nucleon forces. Apart from this, many other forces in nuclear physics are resonant. The aim of this letter is to expose an interesting effect of resonant forces in a three-body system. Namely, for $a \gg r_0$ a series of bound levels appears. In a certain case, the number of levels may become infinite.

Let us explicitly formulate this result in the simplest case. Consider three spinless neutral

ticle bound states emerge one after the other. At $g = g_0$ (infinite scattering length) their number is infinite. As g grows on beyond g_0, levels leave into continuum one after the other (see fig. 1).

The number of levels is given by the equation

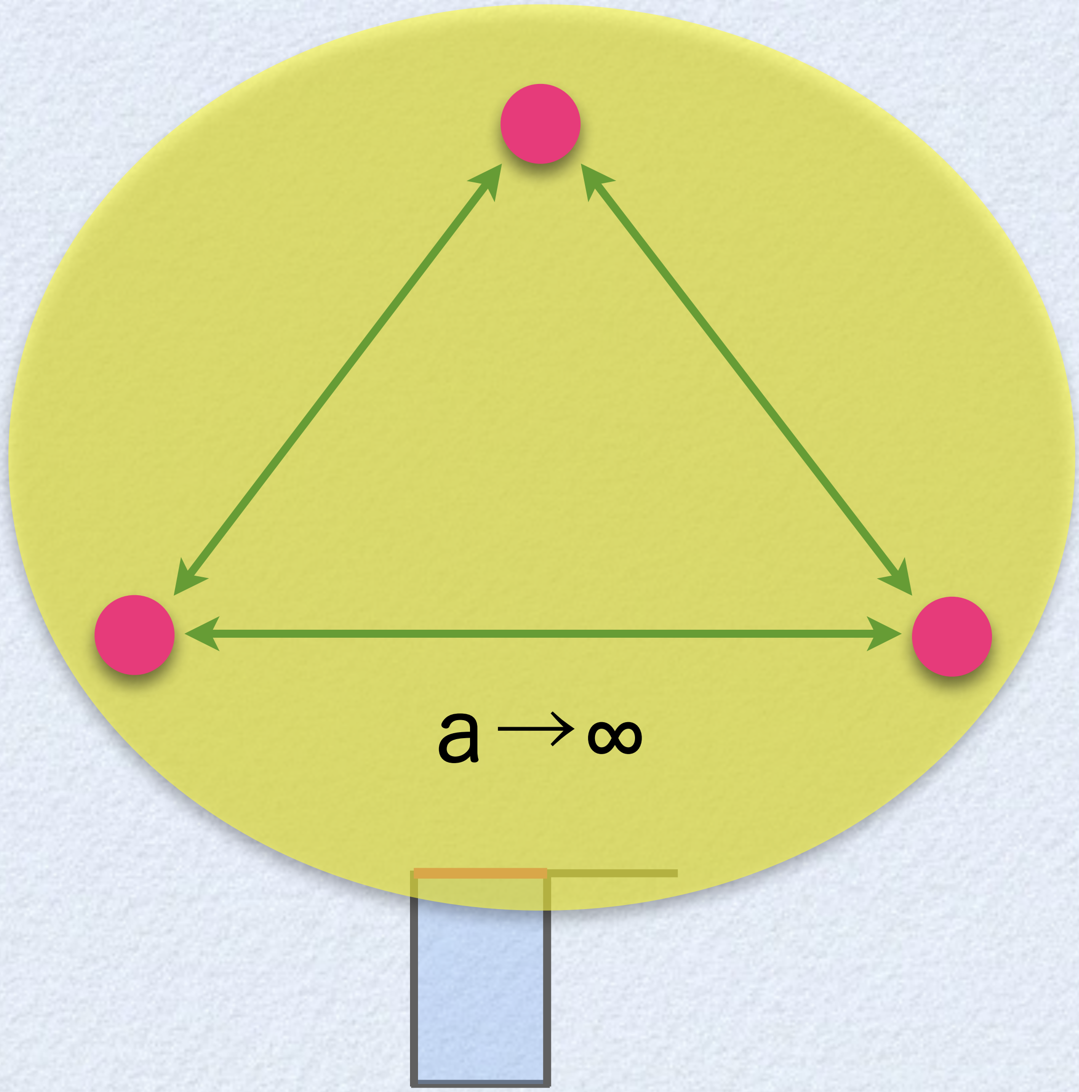
$$N \approx \frac{1}{\pi} \ln(|a|/r_0) \qquad (1)$$

All the levels are of the 0^+ kind; corresponding wave functions are symmetric; the energies $E_N \ll 1/r_0^2$ (we use $\hbar = m = 1$); the range of these bound states is much larger than r_0.

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

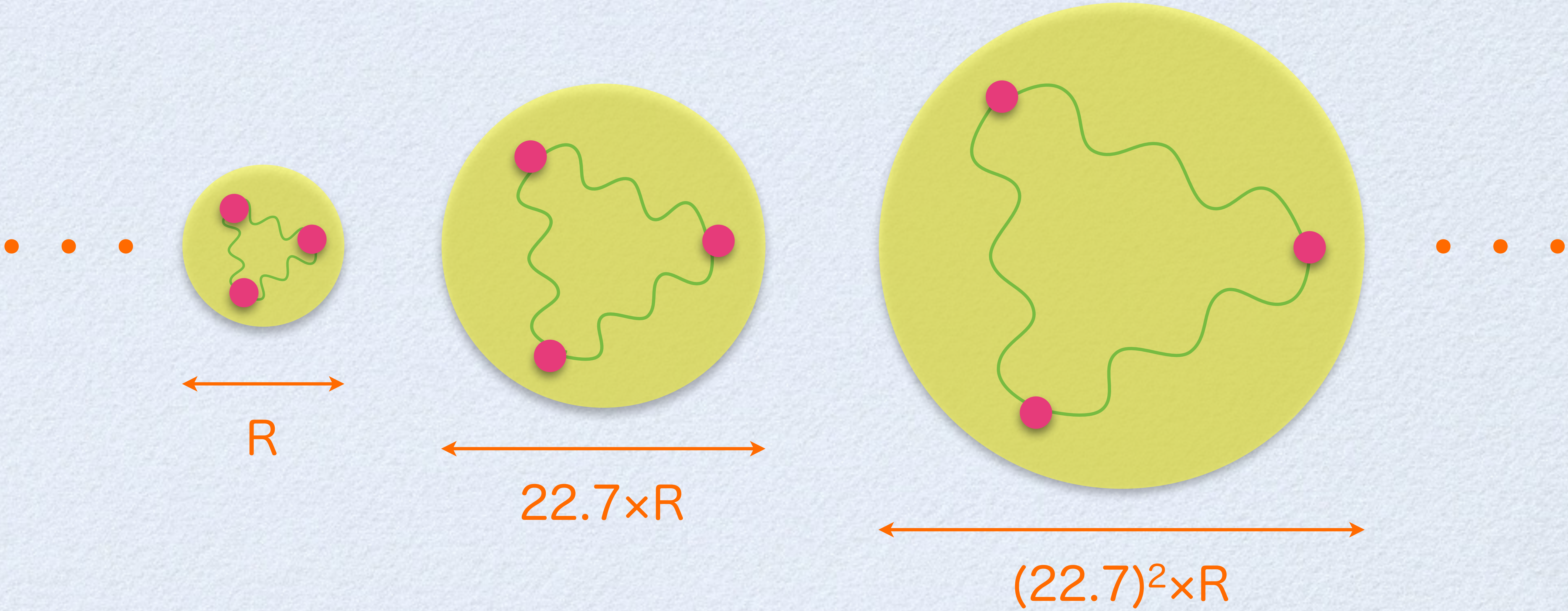
Efimov (1970)



Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons **always** form **a series of bound states**

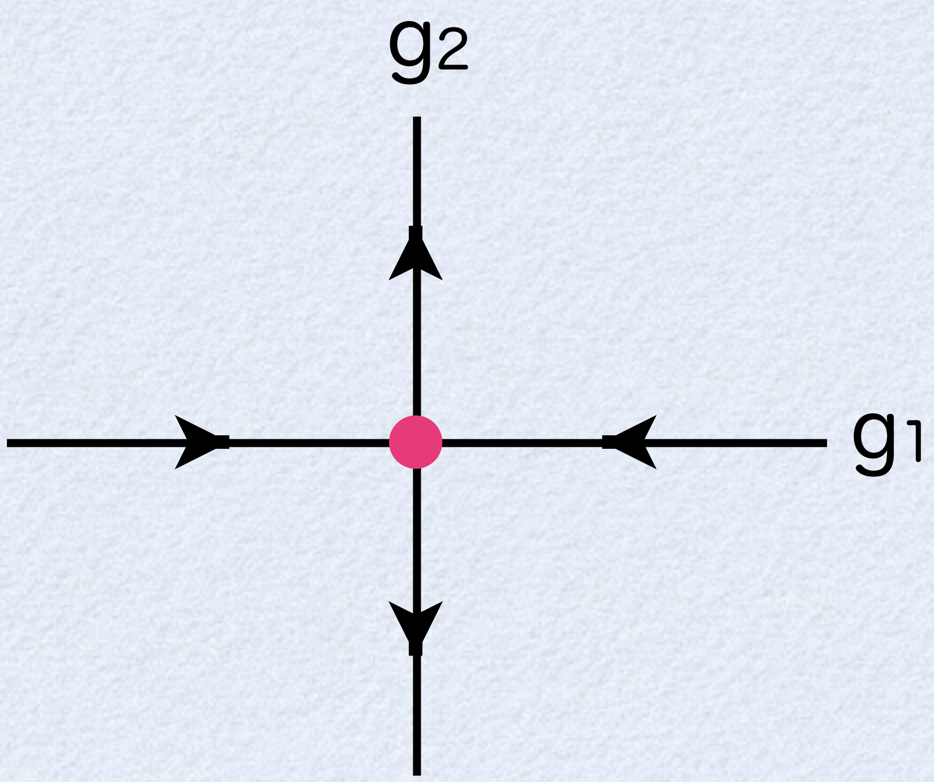
Efimov (1970)

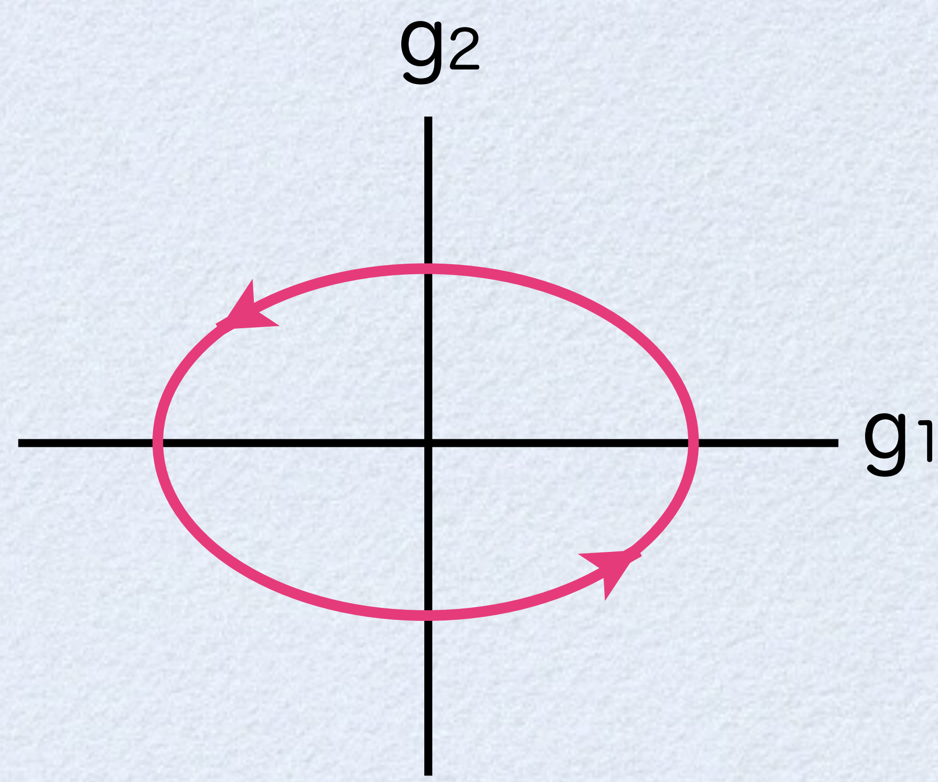


Discrete scaling symmetry

Renormalization group limit cycle

Renormalization group flow diagram in coupling space





RG fixed point

⇒ Scale invariance

E.g. critical phenomena

RG limit cycle

⇒ Discrete scale invariance

E.g. E????v effect

Renormalization group limit cycle

K. Wilson (1971) considered for strong interactions

L REVIEW D VOLUME 3, NUMBER 8 15 APRIL 1971

Renormalization Group and Strong Interactions*

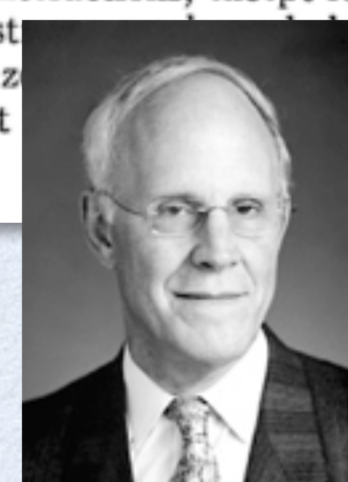
KENNETH G. WILSON
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
and
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850†

(Received 30 November 1970)

The renormalization-group method of Gell-Mann and Low is applied to field theories of strong interactions. It is assumed that renormalization-group equations exist for strong interactions which involve one or several momentum-dependent coupling constants. The further assumption that these coupling constants approach fixed values as the momentum goes to infinity is discussed in detail. However, an alternative is suggested, namely, that these coupling constants approach a limit cycle in the limit of large momenta. Some results of this paper are: (1) The e^+-e^- annihilation experiments above 1-GeV energy may distinguish a fixed point from a limit cycle or other asymptotic behavior. (2) If electrodynamics or weak interactions become strong above some large momentum Λ, then the renormalization group can be used (in principle) to determine the renormalized coupling constants of strong interactions, except for $U(3)\times U(3)$ symmetry-breaking parameters. (3) Mass terms in the Lagrangian of st[...] must break a symmetry of the combined interactions with z[...] weak interactions can be understood assuming only that [...] interactions.

QCD is asymptotic free

(2004 Nobel prize)



Renormalization group limit cycle

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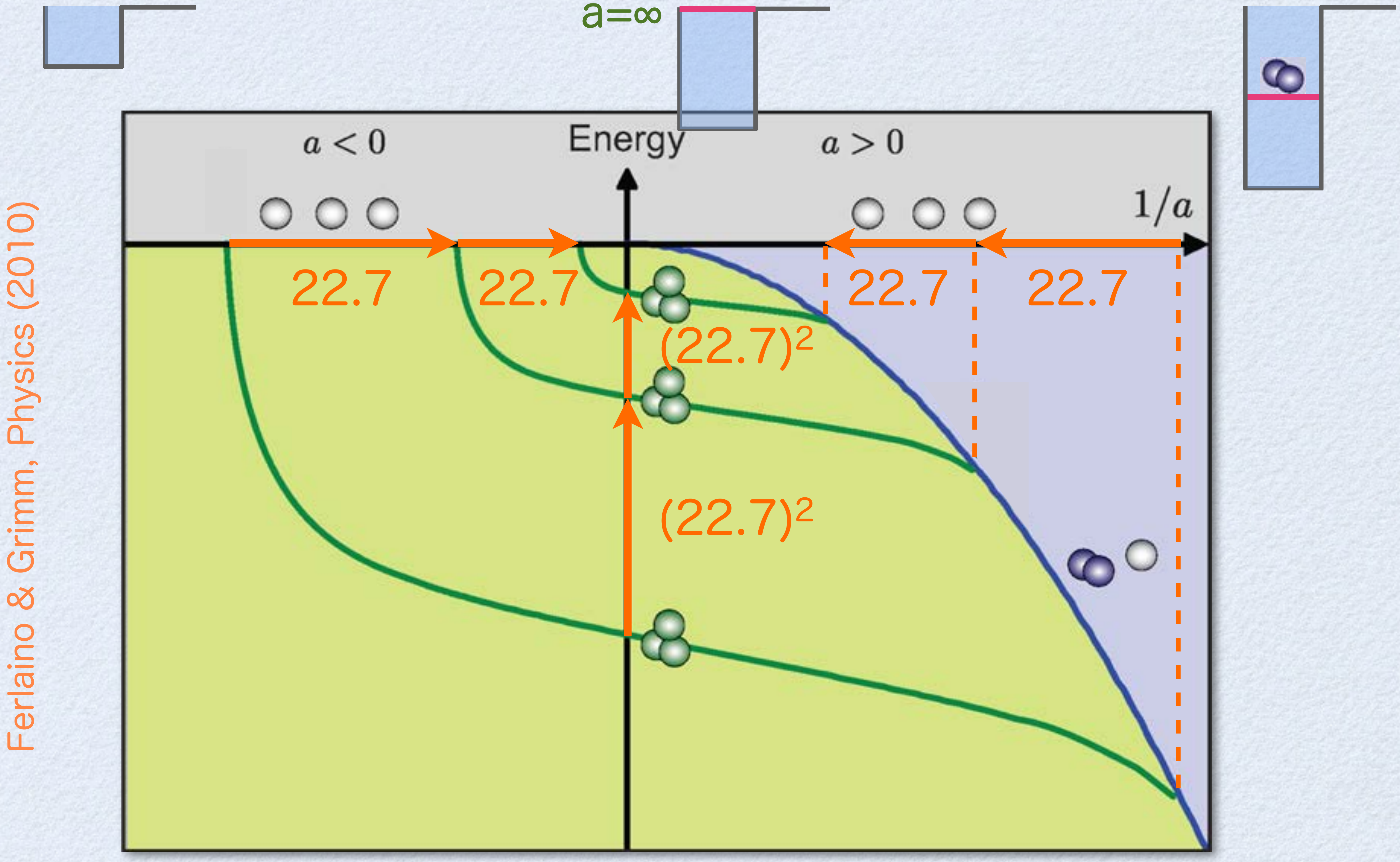
Efimov effect (1970) is its rare manifestation!

Efimov effect at a≠∞



Discrete scaling symmetry

Why 22.7 ?

Just a numerical number given by

22.694382595366695192860217 1369...

ln(22.694382595366695192860217 1369...)

= 3.1222117431104219680730917 32438...

= π / 1.006237825102781489064066 81234...

= π / s_0

$$\frac{2\pi}{s_0}\frac{\sinh(\frac{\pi}{6}s_0)}{\cosh(\frac{\pi}{2}s_0)}=\frac{\sqrt{3}\pi}{4}$$

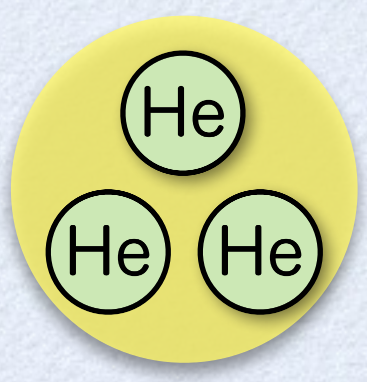
22.7 = exp(π / 1.006...)

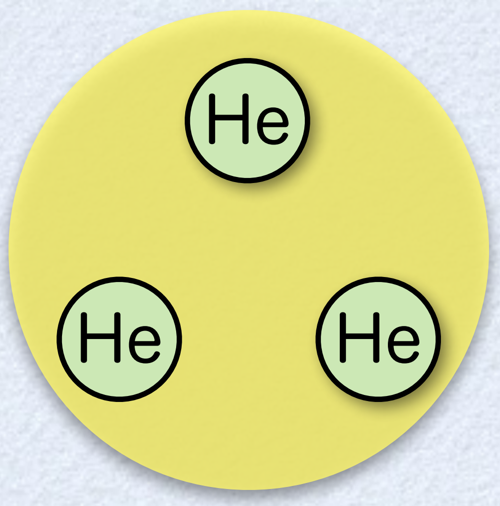
Where Efimov effect appears ?

× Originally, Efimov considered
^3H nucleus (≈3n) and ^{12}C nucleus (≈3α)

△ ^4He atoms ($a \approx 1\times10^{-8}$ m $\approx 20 r_0$) ?

2 trimer states were predicted
and observed in 1994 and 2015



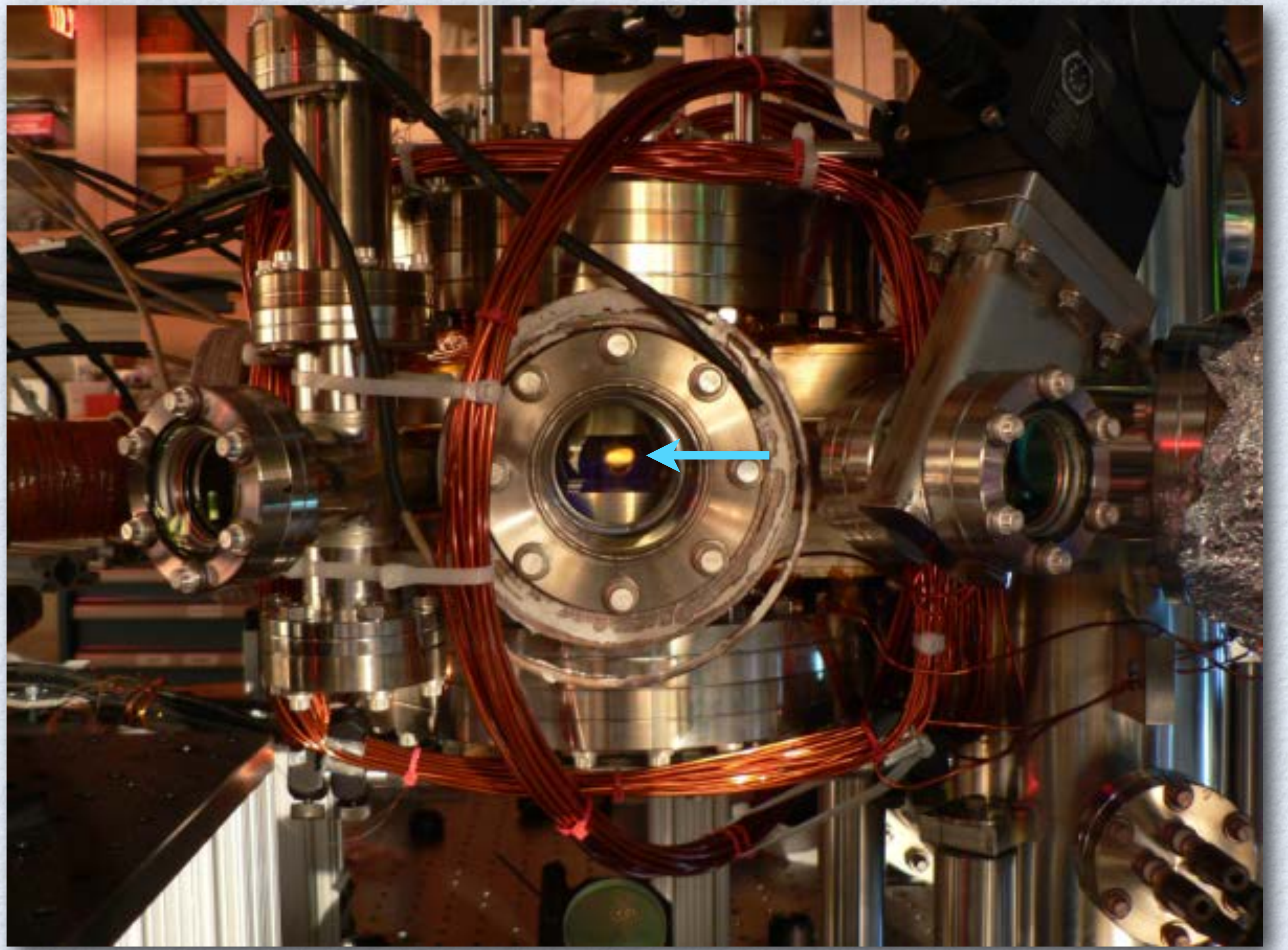


E_b = 125.8 mK E_b = 2.28 mK

→ Ultracold atoms !

Ultracold atom experiments

Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

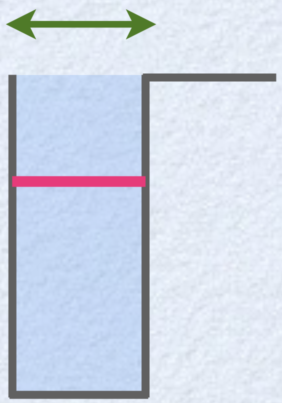


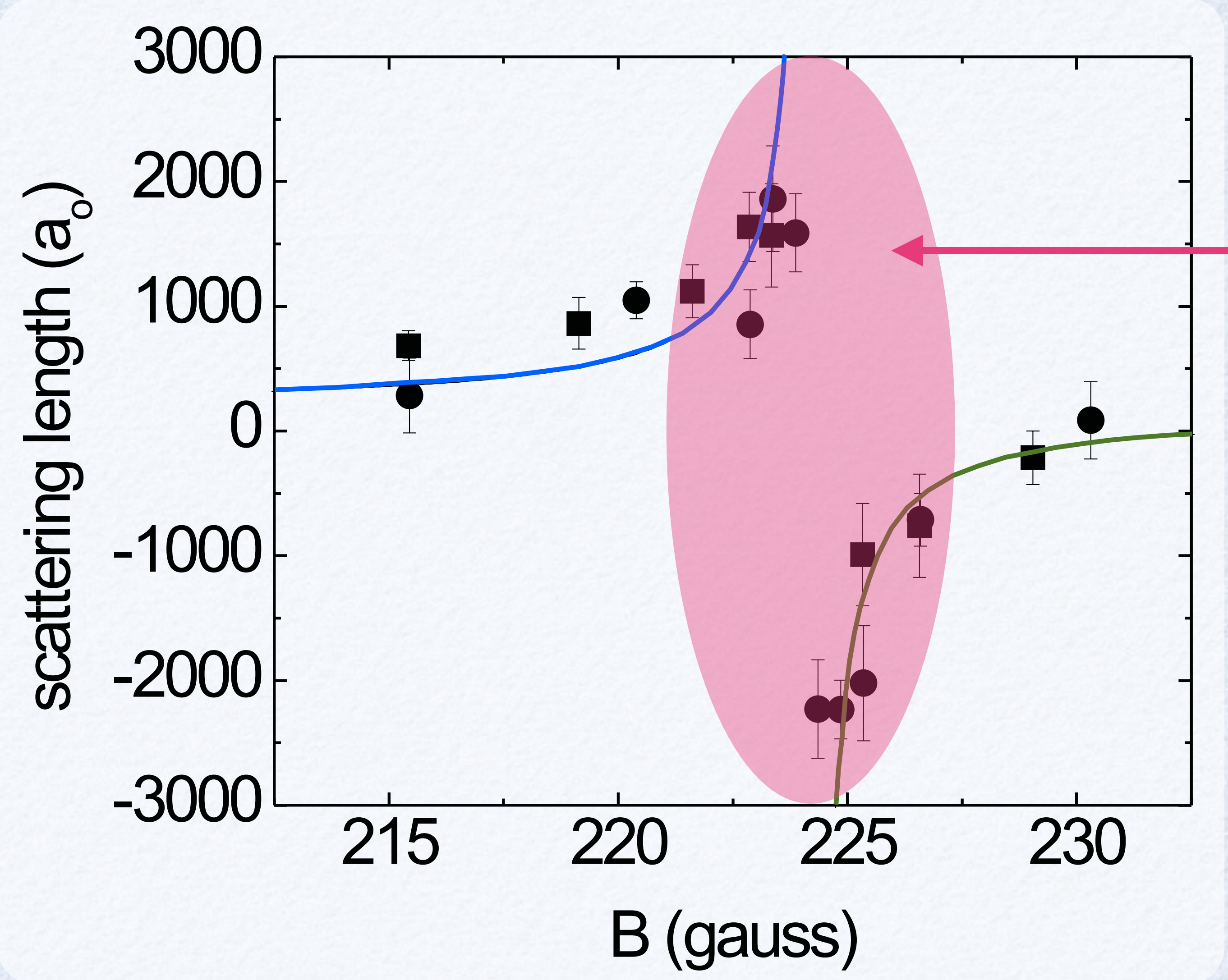
Ultracold atom experiments

Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

✓ Interaction strength by Feshbach resonances

10 ~ 100 a_0



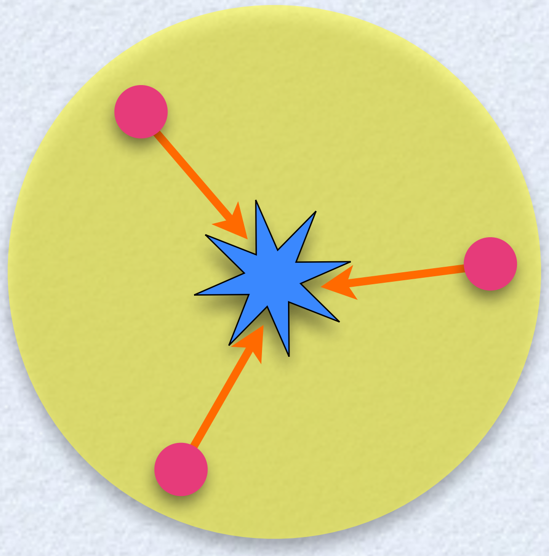


Universal regime

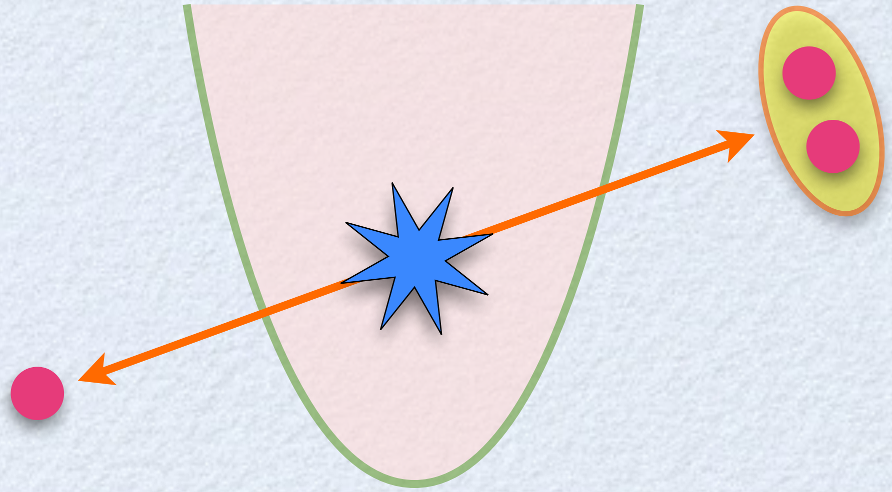
C.A. Regal & D.S. Jin
PRL90 (2003)

Ultracold atom experiments

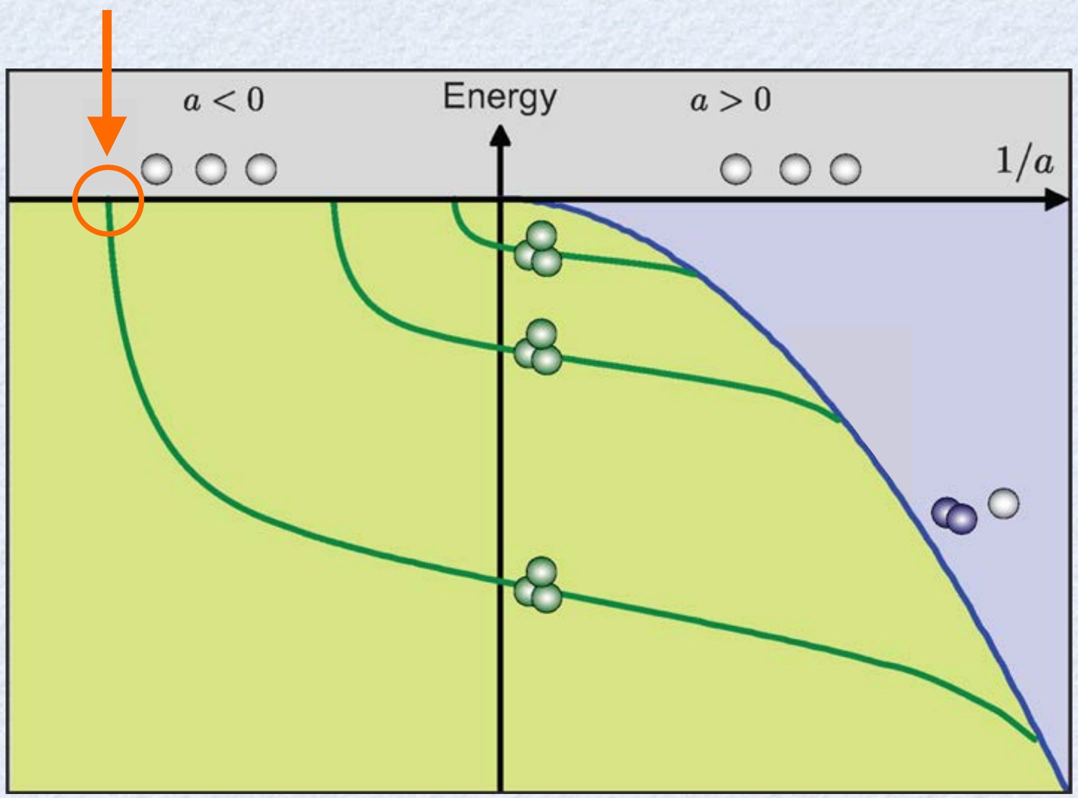
First experiment by Innsbruck group for ^{133}Cs (2006)

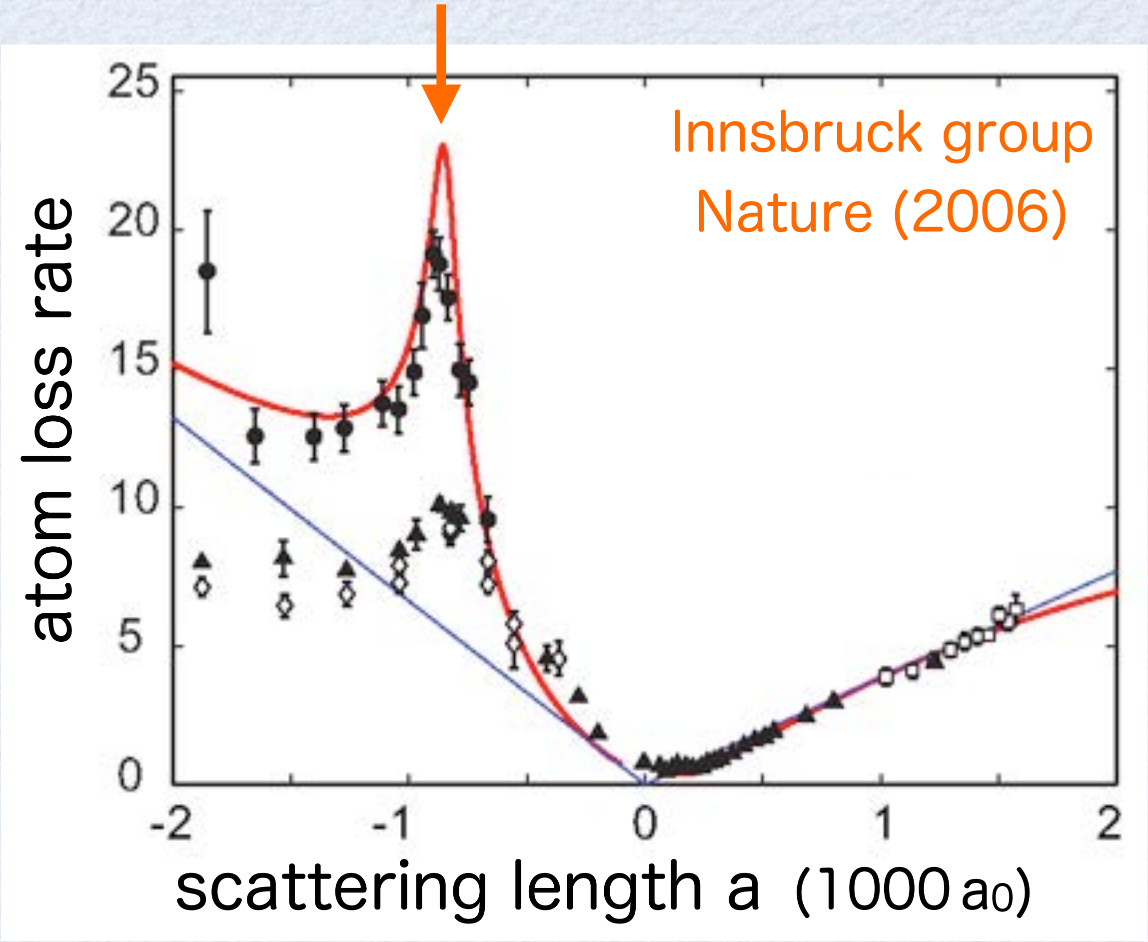


Trimer is unstable → atom loss

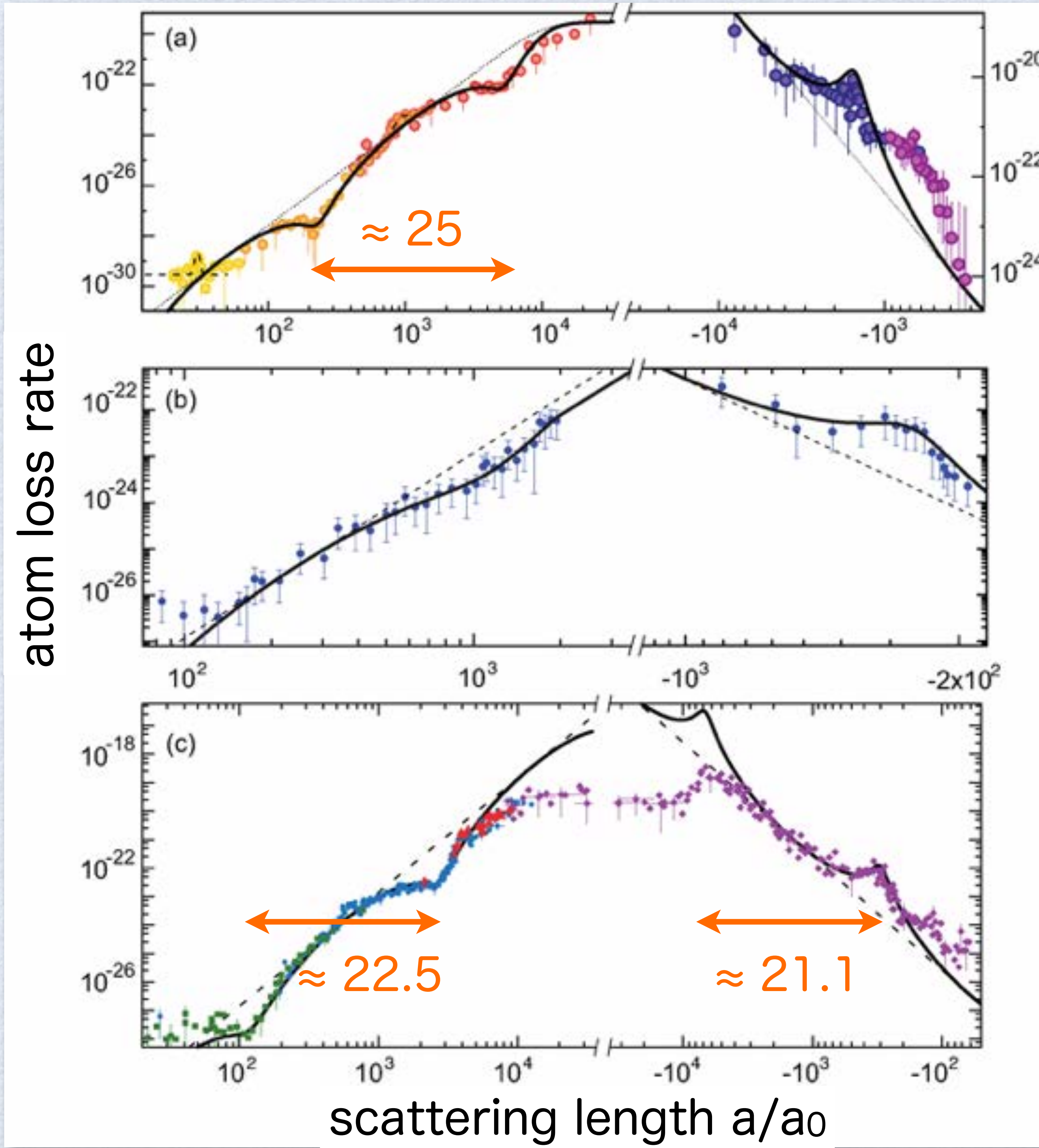


signature of trimer formation





Ultracold atom experiments



Florence group for ^{39}K (2009)

Bar-Ilan University for ^{7}Li (2009)

Rice University for ^{7}Li (2009)

Discrete scaling & Universality !

Short summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

nuclear physics	atomic physics	?
prediction (1970)	realization (2006)	

Where else can it be found ?

Beyond cold atoms

1. Universality in physics
2. What is the Efimov effect?
3. **Beyond cold atoms: Quantum magnets**
4. New progress: Super Efimov effect

Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\Big[\sum_{\hat{e}}\big(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z\big) + D(S_r^z)^2 - B S_r^z\Big]$$

exchange anisotropy (J, J_z)   single-ion anisotropy (D)

nature physics
ARTICLES
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Efimov effect in quantum magnets

Yusuke Nishida*, Yasuyuki Kato and Cristian D. Batista

Physics is said to be universal when it emerges regardless of the underlying microscopic details. A prominent example is the Efimov effect, which predicts the emergence of an infinite tower of three-body bound states obeying discrete scale invariance when the particles interact resonantly. Because of its universality and peculiarity, the Efimov effect has been the subject of extensive research in chemical, atomic, nuclear and particle physics for decades. Here we employ an anisotropic Heisenberg model to show that collective excitations in quantum magnets (magnons) also exhibit the Efimov effect. We locate anisotropy-induced two-magnon resonances, compute binding energies of three magnons and find that they fit into the universal scaling law. We propose several approaches to experimentally realize the Efimov effect in quantum magnets, where the emergent Efimov states of magnons can be observed with commonly used spectroscopic measurements. Our study thus opens up new avenues for universal few-body physics in condensed matter systems.

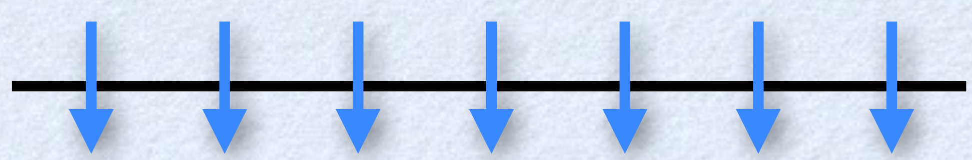
Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

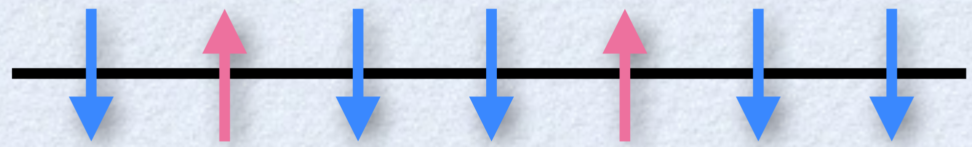
$$H = -\sum_{r}\left[\sum_{\hat{e}}\left(J S_{r}^{+} S_{r+\hat{e}}^{-} + J_{z} S_{r}^{z} S_{r+\hat{e}}^{z}\right) + D\left(S_{r}^{z}\right)^{2} - B S_{r}^{z}\right]$$

exchange anisotropy (J, J_z) — single-ion anisotropy (D)

Spin-boson correspondence

 ⇔

fully polarized state (B→∞) — No boson = vacuum

 ⇔ 

N spin-flips — N bosons = magnons

Quantum magnet

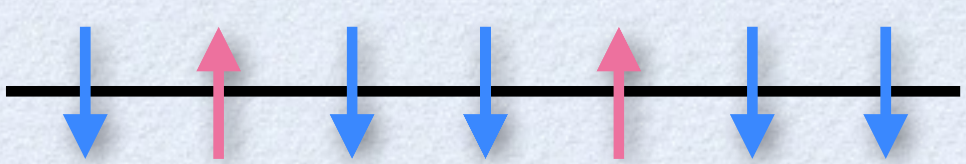
Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\left[\sum_{\hat{e}}\left(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z\right) + D(S_r^z)^2 - B S_r^z\right]$$

xy-exchange coupling (J)
⇔ hopping

z-exchange coupling (J_z)
⇔ neighbor attraction

single-ion anisotropy (D)
⇔ on-site attraction



⇔

N spin-flips ⇔ **N bosons = magnons**

Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_r \left[\sum_{\hat{e}} (J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z) + D(S_r^z)^2 - B S_r^z \right]$$

xy-exchange coupling
⇔ hopping

single-ion anisotropy
⇔ on-site attraction

z-exchange coupling
⇔ neighbor attraction

Tune these couplings to induce
scattering resonance between two magnons
⇒ Three magnons show the Efimov effect

Two-magnon resonance

Schrödinger equation for two magnons

$$E\Psi(r_1, r_2) = \Big[SJ\sum_{\hat{e}}(2 - \nabla_{1\hat{e}} - \nabla_{2\hat{e}})$$

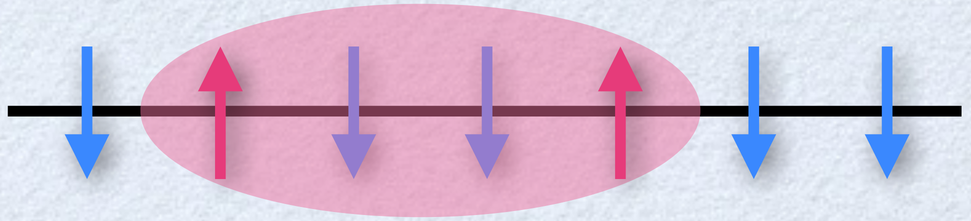
$$+ J\sum_{\hat{e}}\delta_{r_1,r_2}\nabla_{2\hat{e}} - J_z\sum_{\hat{e}}\delta_{r_1,r_2+\hat{e}} - 2D\delta_{r_1,r_2}\Big]\Psi(r_1, r_2)$$

hopping

neighbor / on-site attraction

Scattering length between two magnons

$$\lim_{|r_1-r_2|\to\infty}\Psi(r_1, r_2)\Big|_{E=0} \to \frac{1}{|r_1 - r_2|} - \frac{1}{a_s}$$



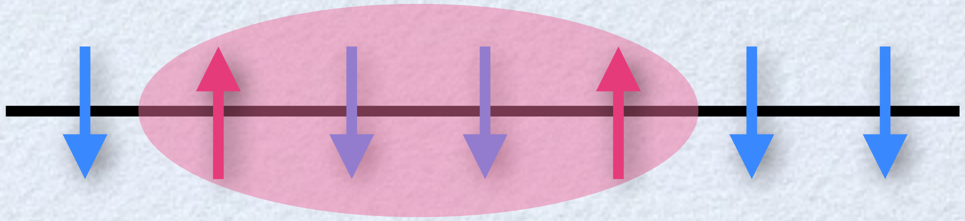
Two-magnon resonance

Scattering length between two magnons

$$\frac{a_s}{a} = \frac{\frac{3}{2\pi}\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}{2S - 1 + \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right) + 1.52\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}$$

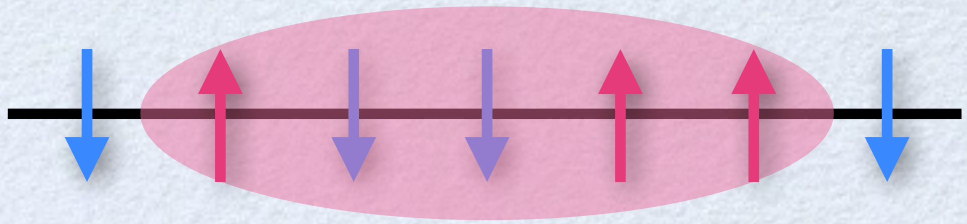
Two-magnon resonance ($a_s \to \infty$)

- $J_z/J = 2.94$ (spin-1/2)
- $J_z/J = 4.87$ (spin-1, D=0)
- $D/J = 4.77$ (spin-1, ferro J_z=J>0)
- $D/J = 5.13$ (spin-1, antiferro J_z=J<0)
- …



Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- **Spin-1/2**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- **Spin-1, D=0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

- **Spin-1, $J_z=J>0$**

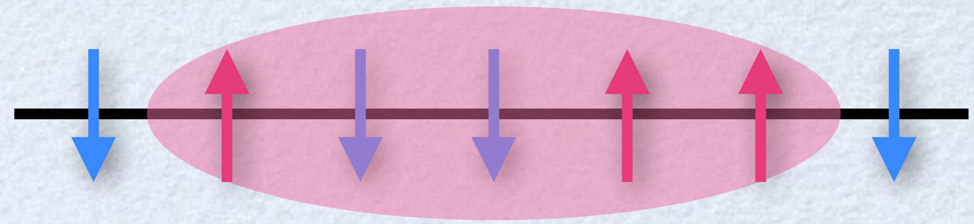
n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.50×10^{-2}	—
1	-1.16×10^{-4}	21.8

- **Spin-1, $J_z=J<0$**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-4.36×10^{-3}	—
1	-8.88×10^{-6}	22.2

Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



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2	-2.00×10^{-6}	22.7

→ Universal scaling law by ~ 22.7 confirms they are Efimov states !

New progress

1. Universality in physics
2. What is the Efimov effect ?
3. Beyond cold atoms: Quantum magnets
4. **New progress: Super Efimov effect**

Few-body universality

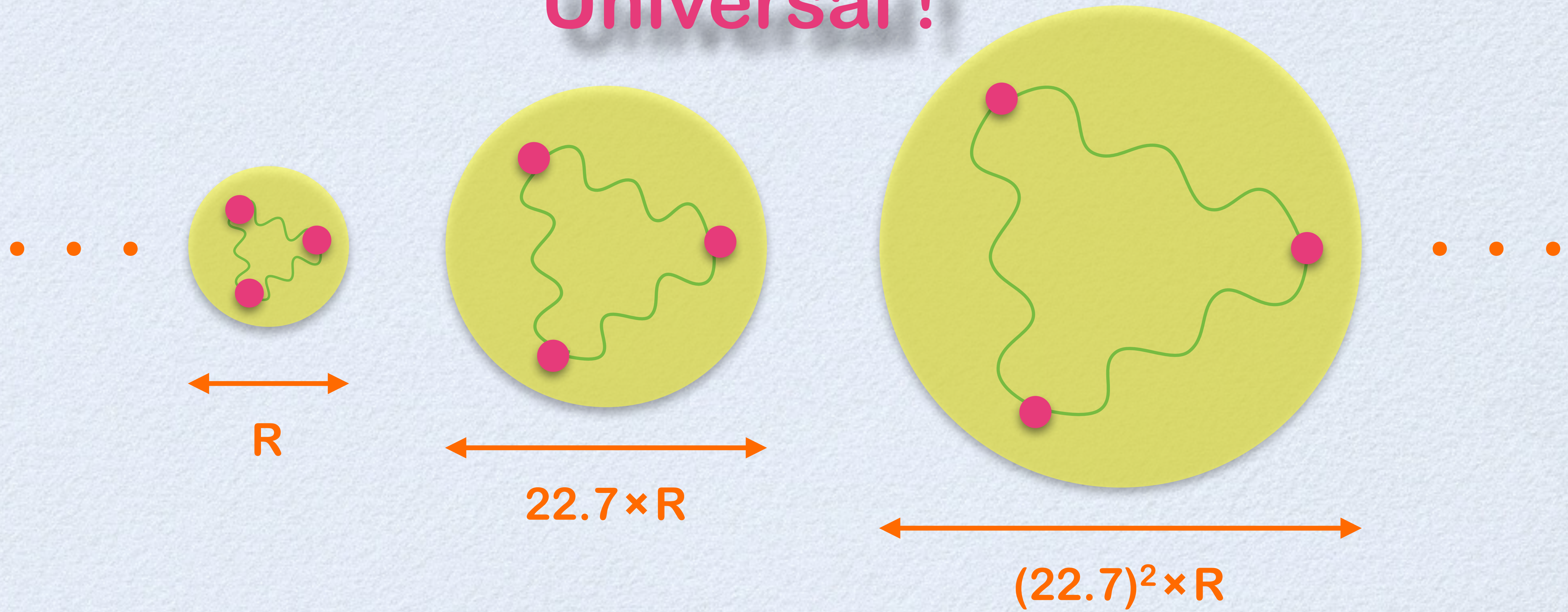
Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance

→ Infinite bound states with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Universal !



Few-body universality

Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance

→ Infinite bound states with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Efimov effect in other systems ?

No, only in 3D with s-wave resonance

	s-wave	p-wave	d-wave
3D	O	x	x
2D	x	x	x
1D	x	x	

Y.N. & S.Tan, Few-Body Syst

Y.N. & D.Lee Phys Rev A

Few-body universality

Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance

Infinite bound states with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Different universality in other systems ?

Yes, super Efimov effect in 2D with p-wave !

	s-wave	p-wave	d-wave
3D	O	x	x
2D	x	x !!!	x
1D	x	x	

Y.N. & S.Tan, Few-Body Syst

Y.N. & D.Lee Phys Rev A

Efimov vs super Efimov

Efimov effect

- 3 bosons
- 3 dimensions
- s-wave resonance

↓

exponential scaling

$$E_n \sim e^{-2\pi n}$$

Super Efimov effect

New !

- 3 fermions
- 2 dimensions
- p-wave resonance

↓

“doubly” exponential

$$E_n \sim e^{-2e^{3\pi n/4}}$$

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Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

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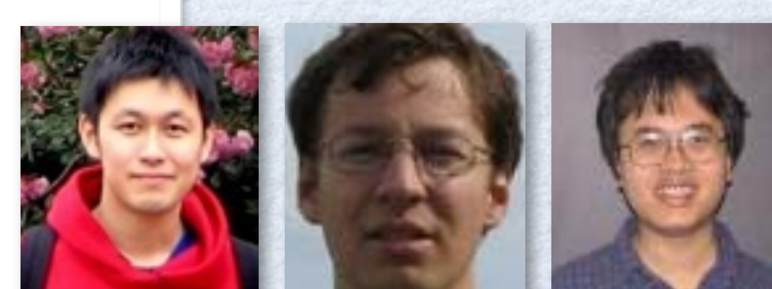
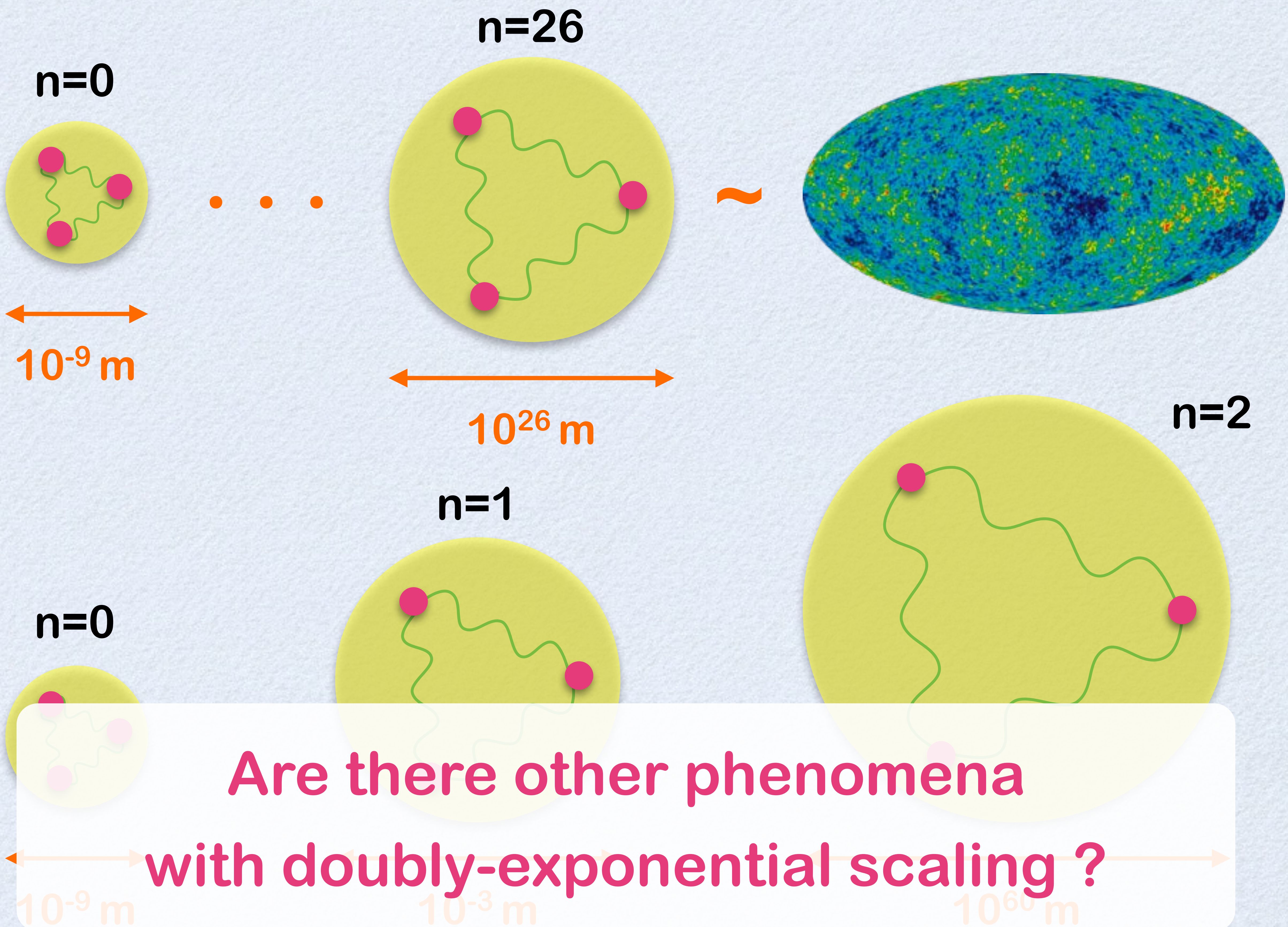
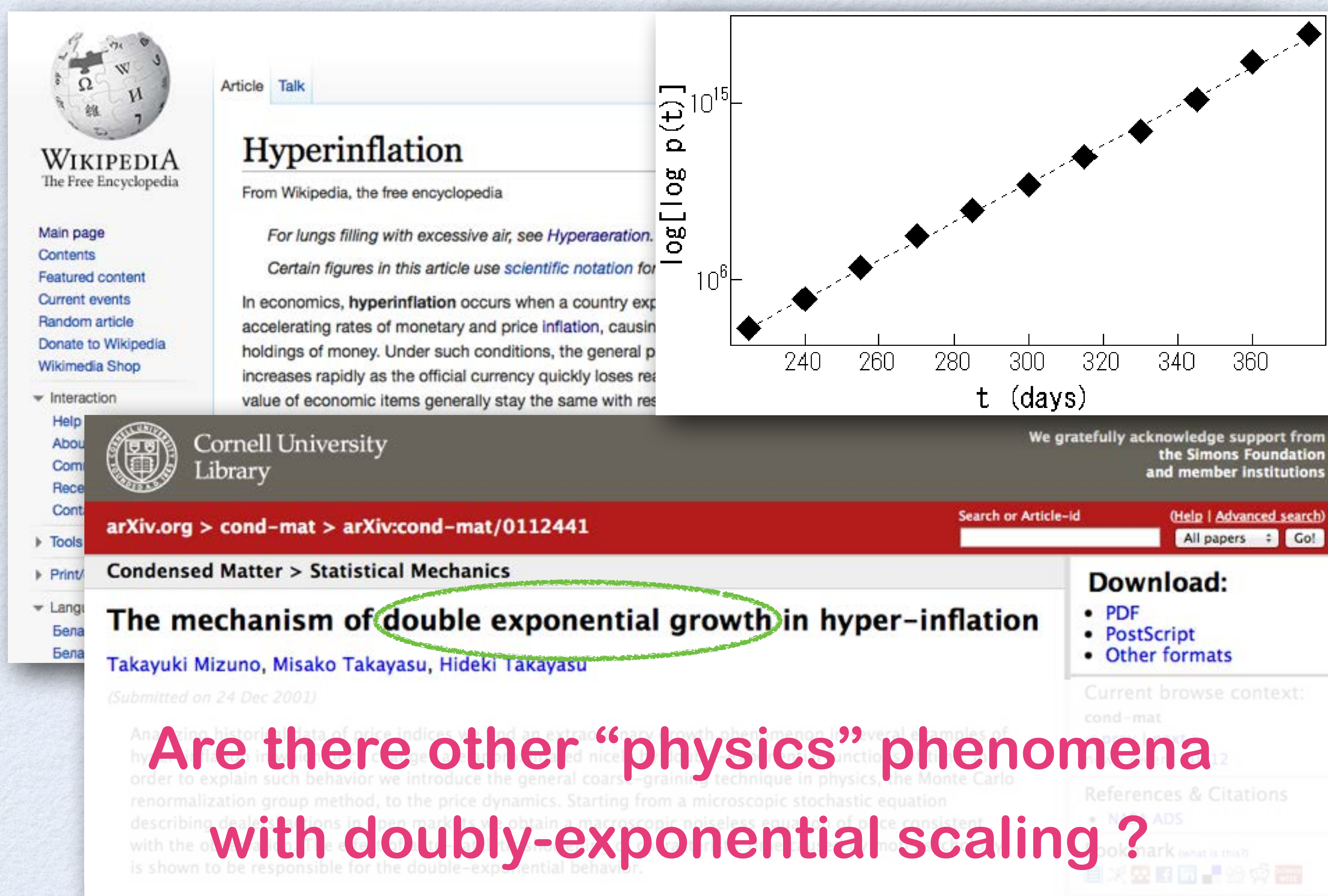


Efimov vs super Efimov



Are there other phenomena with doubly-exponential scaling ?

Efimov vs super Efimov



Are there other "physics" phenomena with doubly-exponential scaling ?

Summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

nuclear physics	atomic physics	condensed matter
prediction (1970)	realization (2006)	proposal (2013)

✓ Efimov effect in quantum magnets

Y.N, Y.K, C.D.B, Nature Physics 9, 93-97 (2013)

✓ Novel universality: Super Efimov effect

Y.N, S.M, D.T.S, Phys Rev Lett 110, 235301 (2013)